Na-K-CL: Mathematical model of Na-K-Cl homeostasis

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The dynamics $[K]\_{o}$, $[Cl]\_{in}$ and $[Na]\_{in}$ is governed by the state variables of the excitatory population, which are the mean voltage $U\_{mean}^{E}$, the mean total potassium current $I\_{K−tot}^{E}$, the total GABAergic current $I\_{GABA−tot}^{E}$ and the firing rate $ν^{E}$. The main equations are as follows:

$$\begin{matrix}\frac{d[K]\_{o}}{dt}&=&\frac{k\_{K}}{F d}(I\_{K}+I\_{K−pump}−I\_{KCC2})+G+k\_{Diff}([K]\_{ext}−[K]\_{o});\\\frac{d[Cl]\_{in}}{dt}&=&\frac{k\_{Cl}}{F}(I\_{Cl,leak}+I\_{KCC2});\\\frac{d[Na]\_{in}}{dt}&=&\frac{k\_{Cl}}{F}(−I\_{Na,leak}+q\_{Na}ν^{E}+\frac{3}{2}I\_{K−pump});\end{matrix}$$

$$\begin{matrix}I\_{KCC2}=I\_{kcc2}^{max}(V\_{K}−V\_{CL})/((V\_{K}−V\_{CL})+V\_{1/2});\end{matrix}$$

$$\begin{matrix}I\_{K,leak}&=&g\_{L,K}(U\_{mean}^{E}−V\_{K});\\I\_{Cl,leak}&=&g\_{L,Cl}(U\_{mean}^{E}−V\_{Cl});\\I\_{Na,leak}&=&g\_{L,Na}(U\_{mean}^{E}−V\_{Na});\end{matrix}$$

$$\begin{aligned}
 I\_{K-pump}&=&\frac{-2 I\_{Na-K-pump}^{max}}{(1+[K]\_{o}^\alpha/[K]\_{o})^2 ~(1+[Na]\_{in}^\alpha/[Na]\_{in})^3}; \\
 I\_{K} &=& I\_{K, leak} + I^E\_{K-tot} \\ I\_{Cl}}&=&I\_{Cl, leak} - I^E\_{GABA-tot}; \end{aligned}$$

$$\begin{aligned}
 k\_2&=&{k\_1}/{(1+\exp(-(K-15\hbox{mM})/1.15\hbox{mM})};\\
 \frac{dB}{dt}&=&k\_1 (B\_{max}-B) - k\_2 B; \\
 G &=& k\_1 (B\_{max}-B)/k\_{1N} - k\_2 B; \end{aligned}$$

$$\begin{aligned}
 V\_{Cl} &=&0.0266 ~\ln(([Cl]\_{in}-[Cl]\_{shift}) /[Cl]\_o); \\
 V\_{K} &=&0.0266 ~\ln([K]\_{o} /[K]\_i); \\
 V\_{Na} &=&0.0266 ~\ln([Na]\_o/[Na]\_{in}); \\
 V\_{GABA}&=&0.0266 ~\ln((4[Cl]\_{in}+[HCO\_3]\_{in})/(4 [Cl]\_o+[HCO\_3]\_{out})) $$

$$\begin{aligned}
I\_{K, leak}&=&g\_{L,K}(\overline{U^{\E}}-V\_K}), \\
I\_{Cl, leak}&=&g\_{L,Cl}(\overline{U^{\E}}-V\_{Cl}}), \\
I\_{Na, leak}&=&g\_{L,Na}(\overline{U^{\E}}-V\_{Na}}), \end{aligned}$$

$$\begin{aligned}
\overline{U^{\E}}=\int\_0^{\infty} U(t,\ts) ~\rho(t,\ts) ~d\ts\end{aligned}$$

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