Exact Non-Linear Solutions for Fully-Developed Flow and Heat Transfer of Viscoelastic Fluids in Micro Scale Channels with Viscous Dissipation

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January 19, 2018

Abstract

In the current study, using both linear and exponential forms of the Simplified Phan-Thien–Tanner constitutive equation (SPTT) for simulation of the polymeric stress tensor, exact analytical solutions for flow, temperature distributions and heat transfer of viscoelastic fluids passing through micro-scale channels, i.e. incorporating the effect of slip, are derived. The problem is solved with a constant heat flux at the. To analyze the effects of velocity slip in the vicinity of the wall region, a slip coefficient parameter is used to illustrate the important roles of slip phenomena on the enhancement of flow rate and consequently heat transfer of viscoelastic fluids in micro-scale geometries. In order to analyze the effect of mechanical energy in heating and cooling scenarios, results for both positive and negative values of the dimensionless Brinkman Number cases are obtained and investigated in detail. To study separately the contributions from entropy and the (internal) energy elasticity of mechanical energy in the heat transfer analysis of viscoelastic fluids a dimensionless parameter is employed. It is shown that this parameter can significantly influence the heat transfer modeling and neglect these parts of the energy equation can cause a considerable deviation.

Nomenclature

r^* – radial direction

r – dimensionless radial direction

Introduction

There are many theoretical works concentrated on non-slip situation for Newtonian cases. Seminal work by Shah [1] focused on the effect of making rounded corner of cross section of conduits in heat transfer. It is shown that this issue can slightly influence on flow and heat transfer of the problem. In following, Shah and London [2] extended the heat transfer analysis to the scenario of bended pipes. Using a finite difference method it is observed that centrifugal force arising from curvature of pipe can influence on heat transfer of the problem. Rich applications of slip flow studies in microscale technologies have attracted the attention of an increasing number of investigators dues to its wide range of applications in biophysical related regimes, medical diagnosis, refrigeration systems, chemical reactors, rocket thermal sciences and electronic component cooling mechanisms in recent years. The soaring request in the industry to design the microscale equipment with highest possible performance has induced lots of researchers to conduct a considerable amount of studies based on experiment, analytical or numerical methods to principally analyze the slip phenomenon in contraction area of solid and fluids in last decade. As it is known, slip flow phenomenon generally plays a significant role in both micro-scale and polymeric melt-related surveys. One should note that in polymer melts discussion, temperature dependency of viscosity is an important issue and the uncoupling of flow from temperature is questionable and can cause inaccuracy. Hettiarachchi et al. [3] used a finite element method to numerically analyze the effect of slip situation on laminar flow and heat transfer of fluids passing through rectangular conduits. In the end, a correlation is presented to estimate the friction factor as a function of Knudsen number and aspect ratio. The results indicated that increment in slip coefficient is followed by an increment on flow rate and consequently heat transfer. In a following paper, Lee and Garimella [4] extended the problem to the entrance region of micro rectangular conduits using a three dimensional numerical method. The solutions were related to the laminar fully developed flow and local and average Nusselt numbers as functions of dimensionless axial distance and channel aspect ratio. Generally speaking, analytical solutions can be used as a benchmark solution to validate the accuracy of numerical solutions due to their high reliability. But, unfortunately due to the non-linear style of governing equations it is hard and even in some cases impossible to derive an exact analytical solution for all the problems. Consequently these methods have attracted less attention in comparison to other methods and majority of the presented analytical solutions are either limited estimations like perturbations methods or are derived for some simple regimes. Among analytical solutions, interesting work of Hooman et al. [5] obtained a correlation for fully developed flow in both circular pipes and between parallel plates. They studied the scaling effects of variable property, viscous dissipation and velocity slip on forced convection of gaseous flow. To simulate the temperature dependency of fluid properties with temperature a perturbation parameter is suggested to uncouple the momentum equation from temperature solution. Duan and Muzychka [6] analyzed the flow and heat transfer of micro annular channels. In slip flow regimes, it is found that the Nusselt number variation plummets with increment in the Knudsen number. Yu and Ameel [7,8] used an integral transient technique to investigate over laminar forced convection in thermally developing slip flow for both constant wall temperature and heat flux boundary conditions.

In recent decades, non-Newtonian fluids have stimulated substantial interest, owing to their wide range of applications in biomechanics and chemical engineering [9-11]. The interesting influences of first and second normal-stresses [12-13] and relaxation and retardation characteristic times of rheological fluids [14-18] on the flow behavior and also heat transfer of these fluids has grown into a major field in its own right. In industry, heat transfer has been playing a major role in production rates and characteristics of manufactured materials. In this regimes, viscoelastic behavior and high viscosity of polymeric materials along with large deformation rates result in the transformation of a large amount of mechanical energy and therefore variation in fluid temperature. For instance in extruders, the dissipated part of the mechanical energy is so significant that can be used to enhance the melting process in the industry. The stated issue induces investigators to incorporate these parts into energy equation as an important parameter. Unfortunately lots of heat transfer related studies concerning viscoelastic fluids have omitted mechanical energy contribution on heat transfer part investigation. Braun and Friedrich [19] and Ko and Lodge [20] have conducted some interesting studies in this field. Peters and Baiijens [21] have investigated this issue in a great detail to determine which part of the supplied mechanical energy is irreversibly dissipated and which part can be released into mechanical energy and is stored as energy elasticity. In viscoelastic materials, generally speaking, the limiting cases are ideal elastic and viscous materials. In this work, it is shown that mechanical energy is partly stored as elastic energy and partly dissipated. In general, elastic energy of viscoelastic fluids itself stores in two different ways; as entropy that is attributed to the Helmholtz free energy and can increase the temperature of the fluid and also as internal energy elasticity attributed to the internal energy part.

Of the relatively small amount of studies which have been conducted in the field of a viscoelastic fluid in a non-slip situation, several are interesting. Oliveira and Pinho [17] derived an exact analytical solution for the non-linear SPTT constitutive equation model. They analyzed the effect of different parameters including Deborah number and parameter of the Phan-Thien-Tanner model on velocity distribution and showed an increment in elastic properties of the fluid can increase the flow rate. Following works ([14] and [22]), for the linear form of temperature distribution, showed in simplified Phan-Thien-Tanner viscoelastic fluids, heat which is generated via viscous dissipation has an important influence on fluid elasticity and heat transfer by imposed heating at the wall. Forgoing method is applied to other problems, Coelho et al [23] extended the problem to flow and heat transfer in a dynamic and thermally fully developed channel flow of the SPTT fluid induced by combined electro-osmosis and pressure gradient. Also, Hashemabadi and Mirnajafizadeh [15] extended the method to investigate the effects of viscous dissipation on heat transfer by implementing Nahme-Griffith number as perturbation parameter. They observed that viscous dissipation may considerably alter isothermal flow characteristics and enhances Nusselt number. Unfortunately in all of the above-mentioned works only effect of entropy elasticity were included in heat transfer analysis and the effect of energy elasticity which particularly illuminate the elastic effects of viscoelastic materials were omitted that can cause inaccuracy in this types of studies.

Motivated by the industrial relevance of viscoelastic flows, an important area which requires further analysis is the slip hydrodynamics in proximity to the pipe (conduit) boundary. Therefore, in the current study, exact analytical solutions for laminar viscoelastic fluid moving through a microchannel is developed to elucidate the effects of slip situation on flow and heat transfer characteristics for both linear and exponential forms of SPTT fluids. In the flow part, in order to simulate the behavior of viscoelastic material the simplified form of Phan-Thien_Tanner constitutive equation is used. Using the definition of normal-stress differences, the coefficient of first normal-stress difference is calculated and the effect of this parameter is also in flow and heat transfer investigation are studied. In order to accurately have discussion over the effect of mechanical energy on heat transfer of the problem both energy elasticity and entropy part are considered on energy equation.

Mathematic Modeling

The flow is considered to be fully developed both thermally and hydrodynamically and the pipe is subjected to constant heat flux at the wall. It is also assumed that the flow is steady and laminar. The fluid properties and model parameters are considered to be constant and independent of temperature. The geometry of the problem is presented in figure 1.

In order to use the dimensionless form of the governing equations the following parameters are used:

$$r = \frac{r^*}{r_0}, z = \frac{z^*}{r_0}, U = \frac{U^*}{W_0}, \tau = \frac{\tau^* \eta W_0}{r_0}, De = \frac{\lambda W_0}{r_0}, T = \frac{T^* - T_w}{q r_0 / k}, Br = \frac{\eta W_0^2}{q r_0} \tag{1}$$