Direct Position Determination of Moving Targets Based on DOA

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In this letter, we focus on the problem of direct position determination (DPD) for moving targets. Compared with the traditional two-step localization methods, the DPD methods is more robust at low signal-to-noise ratio (SNR). However, to guarantee the optimal location results the computational complexity of DPD with grid search is too high, especially for moving targets. Therefore, we propose a new DPD method for moving targets with low computational complexity. First, using a proposed cost function, we obtain the position information from the received array signals directly. Second, we use the method of successional difference which averages the difference result of position over time to extract the velocity. It avoids the multi-dimensional parameters grid search and reduces the the computational complexity greatly. Simulation results demonstrate that the proposed algorithm outperforms other methods in a wide range of scenarios, especially at low SNR.

Introduction: The method of passive position determination has been studied since World War II. It plays a more and more important role in martial and civilian fields including navigation, reconnaissance, emergency rescue, and wireless sensor networks [1**?** , 2]. Traditional localization systems of the target are based on two steps [3]. First, the direction of arrival (DOA), the time difference of arrival (TDOA) and the frequency difference of arrival (FDOA) of the received signals are measured. Second, the measured parameters are used to estimate the target position by solving a set of nonlinear equations [4]. Although the two-step method is relatively simple, its performance is poor at low signal-to-noise ratio (SNR). Because it doesn't consider that all measured parameters obtained in the first step must be consistent with a geolocation of the same target. The location results are not guaranteed to be optimal. To overcome the defect, the single-step location method called direct position determination (DPD) [5] has been proposed and developed over the past few decades. It completes the target position estimation directly from the received signals. In this way, the implicit constraint that all measured parameters correspond to the same target is inherently satisfied. However, to guarantee that global optimum, the computationally expensive of grid search used in DPD is usually high. The DPD method in [5] gets the single target position from the DOA position information first. Next, a DPD method named the subspace data fusion (SDF) method is presented in [6], which reduces the complexity of the multi-target direct localization effectively. Then a DPD method for a narrow-band radio target based on the Doppler frequency shift only is presented in [7]. To enhance the accuracy of the localization, some scholars introduce the characteristic of the received signals into the DPD method. For example, the algorithms in [8] and [9] utilize the characteristics of constant mode signals and strictly noncircular signals, respectively.

Although there are plenty of DPD methods appropriate for the stationary targets, the DPD methods of the moving targets have not been studied extensively. Because instead of estimating only the two position parameters, i.e., the x and y coordinates, it is also required to estimate the velocity parameters $\dot{x}$ and $\dot{y}$. The consequence is that a 4-dimensional grid search is required which renders the methods computationally cumbersome. To conquer the difficulties, some direct tracking methods of analogous thoughts in distributed multi-sensor contexts have been proposed to avoid the multi-dimensional parameters grid search. In [10] a particle filter uses the received radio signals directly to determine the location and velocity of the moving target. Based on the method of [10], the author in [11] uses multiple particle filtering (MPF) to reduce the computational complexity further. In [12], the Gaussian-Newton type iterative DPD (GN-DPD) based on the Doppler frequency shifts is presented. Considering the effect of time the delay or Doppler frequency shift on the received signal, the aforementioned methods use the particle filter or GN iteration method and they can work in a lower noise level environment. However, the position information from TDOAs or FDOAs may be not available in some situations. For instance, when we use only one single moving observer (OB) to locate the moving target, there are no TDOAs between the OBs. Also when the speed of the moving target is slow, the Doppler effect will be too small to be observed. In these cases, DOA information is particularly important. To the best of our knowledge, there are few DPD methods for moving targets that take advantage of DOA information now.

With these observations in mind, we propose a new DPD method of moving targets using array signals processing in this letter. First, we obtain the position information from the received array signals based on DOA directly. To enhance the accuracy of localization, we proposed a new cost function which gets a sharper spectral peak of the position cepstrum diagram than that of traditional MUSIC-type algorithms in moving targets. Second, we use the method of successional difference to extract the velocity which avoids the multi-dimensional parameters grid search and reduces complexity greatly.

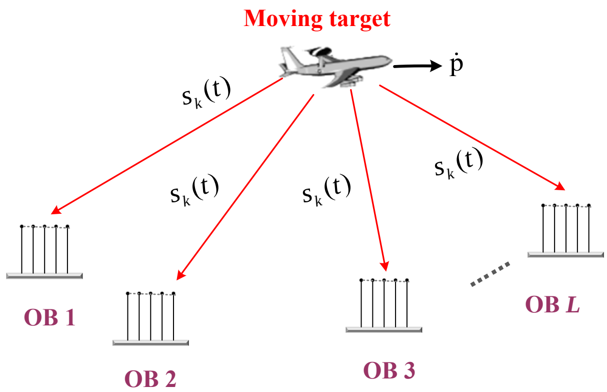


Fig 1 *Observers and the moving target geometry.*

System Model: The geometrical relationship between the moving target and OBs is shown in Fig. 1. Consider the target moves with unknown constant velocity $\mathbf{p} = [v_x, v_y]^T$. Its initial position vector is $\mathbf{p} = [x_0, y_0]^T$. The signals transmitted from the target are intercepted by L stationary OBs with the position $\mathbf{q}_l = [x_l, y_l]^T (l = 1, \cdots, L)$ at K short intervals. Each OB is equipped with an M-elements (M is the number of array elements) uniform linear array (ULA). The element spacing d is half-wavelength of the carrier frequency. To describe the scene and locate the target, some assumptions are listed below:

Assumption 1: At each interception point, the observation time T is short enough. Thus the position and the velocity of the moving target remains constant over the T.

Assumption 2: The signals from the target is narrowband, the bandwidth B is smaller than $1/\tau_{\max}$ ($\tau_{\max}$ is the maximum propagation time between the target to each OB). As a result, the complex signal envelope is the same at all the spatially separated OBs, which means there is no TDOAs between the OBs.

Let $\mathbf{p}_k = [x_k, y_k]^T$ denote the position of the moving target at the kth interception point. It can be expressed as

$$\mathbf{p}_k = \mathbf{p} + (k-1) \cdot T \cdot \mathbf{p},\ k = 1, \cdots, K. \tag{1}$$

Based on the assumptions stated above, the complex signal $\mathbf{r}_{l,k}(t) \in \mathbb{C}^{M\times 1}$ observed by the lth OB during the kth interception interval at the time $0 \le t \le T$ is modeled as

$$\mathbf{r}_{l,k}(t) = \beta_{l,k}\mathbf{a}_{l,k}(\mathbf{p}_k)s_k(t)e^{j2\pi f_{l,k}(\mathbf{p}_k)t} + \omega_{l,k}(t), \tag{2}$$

where $\beta_{l,k}$ is an unknown complex path attenuation. $\mathbf{a}_{l,k}(\mathbf{p}_k) \in \mathbb{C}^{M\times 1}$ is the lth OB's array response to signals transmitted from the position $\mathbf{p}_k$. It can be expressed as

$$\mathbf{a}_{l,k}(\mathbf{p}_k) = \left[e^{-j2\pi\frac{d}{\lambda}\mathbf{m}\sin\theta_{l,k}(\boldsymbol{p}_k)}\right]^T, \mathbf{m} = [0, 1, \cdots, M-1]^T, \quad (3)$$

and $\sin\theta_{l,k}(\mathbf{p}_k) = \frac{x_k - x_l}{\|\mathbf{p}_k - \mathbf{q}_l\|_2}$, $\theta_{l,k}$ is the DOA of $\mathbf{p}_k$ to the lth OB which contains the location position of the target. $s_k(t)$ is the signal complex envelope during the kth interception interval. $\omega_{l,k}(t) \in \mathbb{C}^{M\times 1}$ is a Gaussian noise with zero mean. $f_{l,k}(\mathbf{p}_k)$ is the Doppler frequency shift observed by the lth OB during the kth interception interval. After down conversion, the $f_{l,k}(\mathbf{p}_k)$ can be expressed as

$$f_{l,k}(\mathbf{p}_k) = -f_c\frac{\mathbf{p}^T(\mathbf{p}_k - \mathbf{q}_l)}{c\|\mathbf{p}_k - \mathbf{q}_l\|_2}, \quad (4)$$

where f_c is the carrier frequency and c is the speed of light. During each interception interval, we collect N samples $\widetilde{\mathbf{r}}_{l,k} \in \mathbb{C}^{MN\times 1}$ of these down converted signals with the sampling period $T_s = T/(N-1)$. The nth sampled date $\mathbf{r}_{l,k}[n] \in \mathbb{C}^{M\times 1}$is given by

$$\begin{aligned}\mathbf{r}_{l,k}[n] &= \beta_{l,k}\mathbf{a}_{l,k}(\mathbf{p}_k)s_k[n]\,e^{\mathrm{j}2\pi f_{l,k}(\mathbf{p}_k)nT_s} + \omega_{l,k}[n] \\ n &= 0, 1, \cdots, N-1,\end{aligned} \quad (5)$$

where

$$\begin{cases}\mathbf{r}_{l,k}[n] \triangleq \mathbf{r}_{l,k}(nT_s) \\ s_k[n] \triangleq s_k(nT_s) \\ \omega_{l,k}[n] \triangleq \omega_{l,k}(nT_s).\end{cases} \quad (6)$$

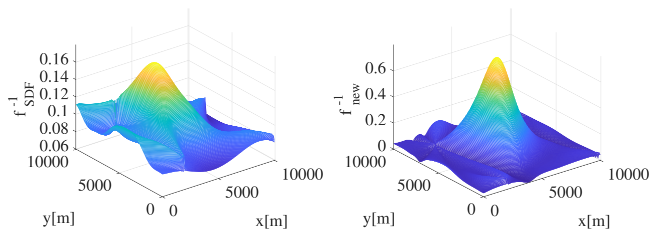
Composite all the samples of the lth OB during the kth interception interval into a vector $\widetilde{\mathbf{r}}_{l,k} \in \mathbb{C}^{MN\times 1}$ and obtain

$$\widetilde{\mathbf{r}}_{l,k} = \beta_{l,k}\mathbf{A}_{l,k}(\mathbf{p}_k)\mathbf{s}_k + \omega_{l,k}, \quad (7)$$

where

$$\begin{cases}\widetilde{\mathbf{r}}_{l,k} \triangleq \left[\mathbf{r}_{l,k}{}^T[0], \cdots, \mathbf{r}_{l,k}{}^T[n], \cdots, \mathbf{r}_{l,k}{}^T[N-1]\right]^T \\ \mathbf{A}_{l,k} \triangleq \mathbf{D}_{l,k}(\mathbf{p}_k) \otimes \mathbf{a}_{l,k}(\mathbf{p}_k) \\ \mathbf{D}_{l,k}(\mathbf{p}_k) \triangleq \mathrm{diag}\left[e^{\mathrm{j}2\pi f_{l,k}(\mathbf{p}_k)(0)T_s}, \cdots, e^{\mathrm{j}2\pi f_{l,k}(\mathbf{p}_k)(N-1)T_s}\right] \\ \mathbf{s}_k \triangleq [s_k[0], \cdots, s_k[n], \cdots, s_k[N-1]]^T.\end{cases} \quad (8)$$

Note that TDOAs and FDOAs are embedded in observation vectors (7), in which there are initial position $\mathbf{p} = [x_0, y_0]^T$ and velocity $\mathbf{p} = [v_x, v_y]^T$ information. Given the vectors (7) together with the positions and velocities of OBs, the problem remains to estimate the initial position $\mathbf{p} = [x_0, y_0]^T$ and velocity $\mathbf{p} = [v_x, v_y]^T$ of the moving target.



(a) (b)

Fig 2 *The position cepstrum diagram. (a) The SDF; (b) The proposed.*

The proposed algorithm: The received signal covariance matrix of the lth OB at the kth observation time interval $\mathbf{R}_{l,k} \in \mathbb{C}^{M\times M}$ generally estimated as

$$\hat{\mathbf{R}}_{l,k} = \frac{1}{N}\mathbf{X}_{l,k}\mathbf{X}_{l,k}{}^H, \quad (9)$$

where $\mathbf{X}_{l,k} = \left[\mathbf{r}_{l,k}[0], \cdots, \mathbf{r}_{l,k}[n], \cdots, \mathbf{r}_{l,k}[N-1]\right]$. The eigenvalue decomposition of $\hat{\mathbf{R}}_{l,k}$ is expressed as

$$\hat{\mathbf{R}}_{l,k} = [\mathbf{u}^s_{l,k}\mathbf{U}^n_{l,k}]\,\boldsymbol{\Sigma}_k[\mathbf{u}^s_{l,k}\mathbf{U}^n_{l,k}]^H, \quad (10)$$

where $\mathbf{u}^s_{l,k} \in \mathbb{C}^{M\times 1}$ shows the signal subspace, $\boldsymbol{\Sigma}_k \in \mathbb{C}^{M\times M}$ is a diagonal matrix composed of eigenvalues from the largest one to the smallest one. $\mathbf{U}^n_{l,k} \in \mathbb{C}^{M\times(M-1)}$ denotes the noise subspace.

The traditional MUSIC-type algorithm is suitable for stationary targets. Its principle is to synthesize the noise subspace of all OBs to estimate the target position. One of the classical MUSIC-type algorithm is the SDF method [6]. Note that the SDF considerably outperforms other traditional localization methods especially at low SNR, we take advantage of the SDF method from stationary target to the moving target firstly. At the kth observation time interval, the cost function of the SDF method is expressed as

$$f_{\mathrm{SDF}}(x, y) = \sum_{l=1}^{L}\left|\mathbf{a}_{l,k}(\mathbf{p}_k)^H\mathbf{U}^n_{l,k}\right|^2. \quad (11)$$

When the value of $f^{-1}{}_{\mathrm{SDF}}$ reaches its maximum, the value of coordinate corresponds to the position of target. In order to improve accuracy even further, a new cost function is proposed. On the basis of the SDF, we take advantage of the equivalent form of signal space and array response space. The proposed cost function shows following

$$f_{\mathrm{proposed}}(x, y) = \sum_{l=1}^{L}\frac{\left|\mathbf{a}_{l,k}(\mathbf{p}_k)^H\mathbf{U}^n_{l,k}\right|^2}{\left|\mathbf{a}_{l,k}(\mathbf{p}_k)^H\mathbf{u}^s_{l,k}\right|^2}. \quad (12)$$

When the grid point (the value of coordinate corresponds to the position) is equal to the target position, not only the numerator $\left|\mathbf{a}_{l,k}(\mathbf{p}_k)^H\mathbf{U}^n_{l,k}\right|$ has a minimum value but the denominator $\left|\mathbf{a}_{l,k}(\mathbf{p}_k)^H\mathbf{u}^s_{l,k}\right|$ has a maximum value. Because the noise space is orthogonal to the signal space and the form of signal space and array response space are equivalent. Therefore, the position cepstrum diagram (the inverse of the cost function) of proposed method will be sharper than that of SDF. As shown in Fig. 2, under the same simulation conditions, the proposed cost function has sharper spectral peak than that of SDF.

Considering the relationship between signal space and array response space, the proposed cost function has stronger robustness and we can get the estimation position of the moving source at each interception point $\hat{\mathbf{p}}_k$ more precisely. By averaging the difference result of position over time, the estimation velocity $\hat{\mathbf{p}}$ can be obtained from the following equation (13) which avoid the multi-dimensional parameters grid search and reduce the computational complexity greatly.

$$\hat{\mathbf{p}} = \frac{\sum_{k=1}^{K}(\hat{\mathbf{p}}_{k+1} - \hat{\mathbf{p}}_k)}{(K-1)\cdot T}. \quad (13)$$

Simulations and discussions: The simulation results and analyses are based on a five-element half-wave-spaced ULA system. The location geometry is as presented in Fig. 1. We consider the scenario of four OBs, they are located at the positions: $[0, 0]^T$[m], $[8000, 0]^T$[m], $[0, 8000]^T$[m], $[8000, 8000]^T$[m]. The baseband signal waveforms are generated as narrow band ping with identical power δ_s^2. The noises are with power δ_n^2, the SNR is expressed as

$$\mathrm{SNR} = 10\lg\left(\delta_s^2\big/\delta_n^2\right)(\mathrm{dB}). \quad (14)$$

For the moving target, we assume that $\mathbf{p} = [2000, 1000]^T$ [m], $\mathbf{p} = [200, 200]^T$ [m/s], $c = 3.0\times 10^8$ [m/s], $F_s = 2\times 10^3$ [Hz], $N = 100$, $K = 5$ and $f_c = 0.03$ G[Hz]. The position root mean square error (RMSE) are defined by

$$\mathrm{RMSE}(\mathbf{p}) = \sqrt{\frac{1}{I}\sum_{i=1}^{I}\|\mathbf{p} - \hat{\mathbf{p}}_i\|^2}, \quad (15)$$

where I is the number of Monte Carlo trials and $\hat{\mathbf{p}}_i$ denote the estimation of $\mathbf{p}$ in the ith Monte Carlo trial. The similar method can be applied to RMSE($\mathbf{p}$). To obtain statistical results we used $I = 200$. The simulation results are as follows. To the best of our knowledge, there are few DPD methods for moving targets take advantage of DOA informations now. In this part, we compare the performance of the proposed method with the traditional differential Doppler (DD)[7], the GN-DPD method [12] and CRLB at different SNRs. The effect of the number of samples N is also presented. For grid search methods, the target's position is searched within a 4000 × 4000 grid in both methods. It is the same to target's velocity for DD. FDOAs is estimated by Cross Ambiguity

Table 1. Computational Complexity Comparison.

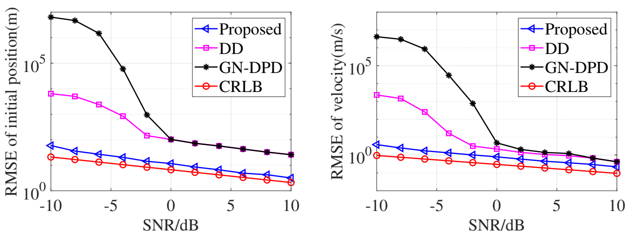
Algorithm	Computational Complexity
DD	$O(LNM^2 + N^2) + N_P N_v KL(4KL + K)$
Proposed	$KLM^2(N + M) + O(N_p KL(M^2(M + 2) + 2M))$

Table 2. Average runtime Comparison.

Number of samples	100	150	200	250	300
DD(s)	1.47	2.04	2.54	2.97	3.52
Proposed(s)	0.85	1.37	1.46	1.53	1.57

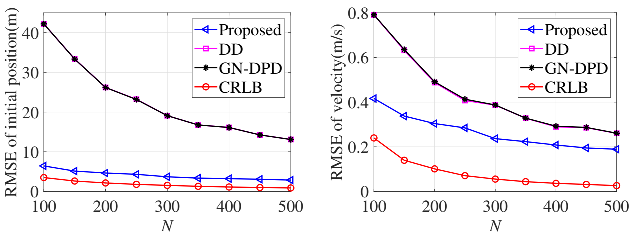
Function (CAF). The initial values of the GN-DPD is the result of the DD.

First, Fig. 3 show the RMSEs of the initial position and velocity versus the SNR, respectively. It can be seen that as the SNR increases, all the methods approach the CRLB gradually. The GN-DPD only used the FDOAs and it results depend on the choice of initial values. The proposed method always outperforms the other methods, especially at low SNR. The proposed method shows great performance robustness to SNR. Then, we continue to performing simulations by changing the number of samples N at each interception interval. The SNR is fixed at 10 dB. The results are shown in Fig. 4. It can be seen that as the number of samples increases all the methods obtain lower RMSEs. Because the DOA position information and the Doppler frequency shift position information are both related to the number of samples. However, the proposed method outperforms the other metonds, particularly, for a small number of samples N.



(a) (b)

Fig 3 *The RMSEs versus the SNR. (a) The initial position; (b) The velocity.*



(a) (b)

Fig 4 *The RMSEs versus samples N. (a) The initial position; (b) The velocity*

Complexity analysis: The complexity of the GN-DPD method is related to the number of iterations, the number of iterations has a great relationship with the selection of the initial value. In this part, we focus on the computational complexity of the proposed method and the DD method in grid search. Table 1 lists the computational complexity of the two algorithms. N_p and N_v represent the number of grids in position search and speed search, respectively. In addition, in order to see the computational overhead of each algorithm clearly, we compare the running time of the two algorithms under the same simulation platform. The result is shown in Table 2, it can be observed that the proposed method has less running time all the time. Because as shown in Table 1 the proposed method doesn't need multi-dimensional parameters grid search and it only needs to search in N_P grids (two dimensions) while the DD method needs in $N_P N_v$ grids (four dimensions). Thus, compared with the DD method the proposed algorithm reduces the computational complexity greatly which is excellent for engineering applications.

Conclusion: In this letter, we propose a new DPD method for moving target based on DOA. First, a new cost function is used to obtain the position information from the received signals directly. Then we use the method of successional difference to extract the velocity which avoids the multi-dimensional parameters grid search and reduces the computational complexity greatly. Numerical results demonstrate that the proposed algorithm outperforms the other methods in a wide range of scenarios especially at low SNR and a small number of samples N.

Acknowledgments: The funding supports from the National Key Research and Development Program of China under Grant 2018YFB1802300 and Marine Economy Development Project of Guangdong Province GDNRC[2024]01.

Received: xx xxx 2024 *Accepted:* xx xxx 2024
doi: 10.1049/ellxxx

Conflict of interest: The authors declare no conflict of interest.

Data availability statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

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