Total Factor Productivity and Energy Consumption: Theoretical Reconsideration

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Abstract

The paper theoretically derives a framework, which expresses energy demand as a function of Total Factor Productivity (TFP), employing optimization techniques.

Please note the following is work in progress and must not be referred to without our consent. However, we would be happy to receive any comments or suggestions via email.

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Introduction

After the pioneering papers by Solow (1956, 1957) it is recognized that total factor productivity (TFP), as a representative of the set of improvements in technology, efficiency gains, improvements of labor skills, plays crucial role in the economic growth of the economy. Number of studies (Abramovitz, 1956; Kendrick, 1961; Hisnanick and Kymn, 1992; Easterly and Levine, 2000, inter alia) concluded that TFP has an explanatory power on production inputs. Many studies have been devoted to the investigation of the relationship between TFP and economic growth. In addition, there is a vast literature examining the relationship between energy consumption and economic growth (Soytas and Sari, 2003; Narayan and Smyth, 2008; Ozturk et al., 2010; Belke et al., 2011; Sadorsky, 2009; Apergis and Payne, 2011; Menegaki, 2011; Tugcu et al., 2012; Mukhtarov et al., 2018, inter alia). In addition, some papers reviewed the studies devoted to energy consumption-economic growth nexus.

From the theoretical point of view TFP also has an explanatory power on energy consumption, as one of the production factors, and this is confirmed by empirical examination (Boyd and Pang, 2000; Tugcu, 2013; Ladu and Meleddu, 2014, inter alia). Unlike the vast literature on theoretical and empirical research devoted to the TFP and the economic growth-energy consumption nexus, the relationship between TFP and energy consumption, especially empirical investigation of that relationship has not been studied much. Although, it was shown in early 80's (Schurr, 1983; Jorgenson, 1984) that energy consumption has a positive impact on TFP, seems that the reverse impact has not attracted the attention much. The existing literature, mainly focused on the causality examination and the few papers studied the impact of energy consumption on TFP, in terms of estimating of coefficients of that relationship. In addition, to the best of our knowledge, there is not a study estimating the impact of TFP on energy consumption. Moreover, the theoretical framework of the impact of TFP on energy consumption has not been clearly stated in previous research. In this regard, the objective of the current paper is to derive an explicit functional relationship between TFP and energy consumption, i.e., what is the impact of TFP on energy consumption, what is the expected sign and magnitude of that impact, and empirically investigate that relationship on the individual country case.

The contribution of the paper to the existing literature is twofold: first, it derives an explicit functional form for energy consumption where TFP plays a role one of the drivers of energy consumption, which allows econometrically estimate the impact of TFP an energy consumption; second the derived relationship tested in the individual country case and the proposed hypotheses about the impact of TFP on energy consumption tested.

For the derivation of the above-mentioned energy consumption-TFP relationship the cost-minimization approach of constrained optimization is used, applying the Lagrange Multiplier method. It is mathematically shown that TFP has negative impact on energy consumption, which is in line with the theoretical expectations, since TFP as a representative of technological improvements, efficiency gains, improvements in R&D and labor skills and etc. is expected to decrease energy consumption. In addition, the magnitude of the coefficient of TFP in the derived specification allows to make some conclusions of the production scale of the economy of interest.

The rest of the paper is structure as follow: Section 2 presents the short literature review on TFP research and on the energy consumption-TFP nexus; Section 3 devoted to the derivation of the functional relationship between energy consumption and its drivers, TFP being one of them; the used data in empirical estimations described in Section 4, while Section 5 presents the econometric methodology. Results of empirical estimations are given in Section 6, Sections 7 and 8 provide the discussion of the findings and Conclusion of the research, respectively.

2. Literature Review

After the seminal papers Solow (1956, 1957) and Swan (1956) the TFP phenomenon and the relationships between economic growth and production inputs have gained increasing attention, and later many others contributed to the TFP theory (Kendrick, 1961; Solow, 1962;; Denison, 1962, 1967; Griliches and Jorgenson,1967; Christensen et al., 1973; Hulten, 1992; Abramowitz, 1993, inter alia) and a plenty of studies have devoted to the investigation of this relationship(s) in case of different economies. Nordhaus (1975), Tsvetanov and Nordhaus (1975), Beenstock and Willcocks (1981), Beenstock and Dalziel (1986), inter alia, have derived the specifications for energy demand equations and discussed some details of them in terms of possible drivers of energy demand and possible proxies for them. Kraft and Kraft (1978) paper was a starting point for the investigation of the economic development-energy consumption nexus and was followed by many other case studies, only few of which mentioned in the Introduction section.

Papers devoted to the relationship between energy consumption and TFP mainly focused on the causality analyses. Since, the objective of our study is to investigate the impact of TFP on energy consumption we will not review these type of papers. Only limited number of previous research studied the relationship in terms of coefficient estimations, and all of them studied the impact of energy or energy types on TFP. In other words, they have not investigated the impact of TFP on energy demand. The only papers we are aware of are Hisnanick and Kymn (1992) for US, Tugcu (2013) for Turkey, Ladu and Meleddu (2014) for Italy, Moghaddasi and Pour (2016) for Iran, Haider and Ganaie (2017) for Indian case, studied the impact of aggregated/disaggregated energy consumption on TFP.

As the reviewed literature shows the functional specification relating energy demand to TFP has not been derived explicitly and there is not an empirical study investigating this relationship. Hence, the current paper contributes to the literature addressing these two issues.

3. Derivation of the Functional Form for Energy Demand as a Function of TFP

In order to derive a functional specification for energy demand as a function of TFP the set up can be formalized as follow. The Cobb-Douglas production function, which related the output to the production factors is given as below:

$$Q = AK^{\alpha}L^{\beta}E^{\gamma}M^{\delta} \text{ (1)}$$

Where: *Q*, *K*, *L*, *E*, and *M* are output, capital, labor, energy consumption and materials, respectively, and A is a *TFP*.

The target is to minimize the total cost, in other words to define the quantities *of Q*, *K*, *L*, *E*, and *M* which gives minimum value to the following total cost function:

$$C = p_k K + p_l L + p_e E + p_m M \text{ (2)}$$

Where: C is total cost and, p_k, p_l, p_e, p_m are capital, labor, energy and material prices, respectively.

Then treating the total cost function as an objective function and the production function as a constraint the exercise can be formulated as a constrained optimization problem:

$$C = p_k K + p_l L + p_e E + p_m M \rightarrow Min \quad (3)$$

Subject to

$$Q = f(K, L, E, M) = AK^{\alpha} L^{\beta} E^{\gamma} M^{\delta} \quad (4)$$

Using the Lagrange multipliers method for constrained optimization we can modify our optimization set up as follow (let's call it G):

$$G = C + \lambda(Q - f(K, L, E, M)) \rightarrow Min \ (5)$$

Which becomes:

$$G = p_k K + p_l L + p_e E + p_m M + \lambda(Q - AK^{\alpha} L^{\beta} E^{\gamma} M^{\delta}) \rightarrow Min \ (6)$$

Now we have an unconstructed optimization problem with objective function G, as given above.

Based on Lagrange multipliers method next we should calculate the partial derivatives of the function *G* with respect *K*,*L*, *E*, *M* and λ. As it known form the calculus in order to get partial derivative of a function with respect to one variables, other variables are considered as constants. For example, if we are taking partial derivative of the function *G* with respect to *K*, then all other variables (*L*, *E*, *M* and λ) are considered as constants. The partial derivatives are given below:

$$\begin{cases} G'_{\lambda} = Q - AK^{\alpha} L^{\beta} E^{\gamma} M^{\delta} \\ G'_K = p_k - \lambda A \alpha K^{\alpha-1} L^{\beta} E^{\gamma} M^{\delta} \\ G'_L = p_l - \lambda A \beta K^{\alpha} L^{\beta-1} E^{\gamma} M^{\delta} \\ G'_E = p_e - \lambda A \gamma K^{\alpha} L^{\beta} E^{\gamma-1} M^{\delta} \\ G'_M = p_m - \lambda A \delta K^{\alpha} L^{\beta} E^{\gamma} M^{\delta-1} \end{cases} \quad (7)$$

As a next step in order to find the extremum point we should equate all the above derivatives to zero.

$$\begin{cases} Q - AK^{\alpha} L^{\beta} E^{\gamma} M^{\delta} = 0 \\ p_k - \lambda A \alpha K^{\alpha-1} L^{\beta} E^{\gamma} M^{\delta} = 0 \\ p_l - \lambda A \beta K^{\alpha} L^{\beta-1} E^{\gamma} M^{\delta} = 0 \\ p_e - \lambda A \gamma K^{\alpha} L^{\beta} E^{\gamma-1} M^{\delta} = 0 \\ p_m - \lambda A \delta K^{\alpha} L^{\beta} E^{\gamma} M^{\delta-1} = 0 \end{cases} \quad (8)$$

Let's take the second terms of each equation to the right side of the equation:

$$\begin{cases} Q = AK^{\alpha}L^{\beta}E^{\gamma}M^{\delta} \\ p_k = \lambda A\alpha K^{\alpha-1}L^{\beta}E^{\gamma}M^{\delta} \\ p_l = \lambda A\beta K^{\alpha}L^{\beta-1}E^{\gamma}M^{\delta} \\ p_e = \lambda A\gamma K^{\alpha}L^{\beta}E^{\gamma-1}M^{\delta} \\ p_m = \lambda A\delta K^{\alpha}L^{\beta}E^{\gamma}M^{\delta-1} \end{cases} \quad (9)$$

Now let's take logs of both sides and use some properties of the logarithmic function, namely the following properties:

$$a)\ln(x*y) = \ln(x) + \ln(y) \quad b)\ln(x^n) = n*\ln(x)$$

then we will get the following system of equations:

$$\begin{cases} lnQ = lnA + \alpha lnK + \beta LnL + \gamma lnE + \delta lnM \\ lnp_k = ln\lambda\alpha A + (\alpha-1)lnK + \beta lnL + \gamma lnE + \delta lnM \\ lnp_l = ln\lambda\beta A + \alpha lnK + (\beta-1)lnL + \gamma lnE + \delta lnM \\ lnp_e = ln\lambda\gamma A + \alpha lnK + \beta lnL + (\gamma-1)lnE + \delta lnM \\ lnp_m = ln\lambda\delta A + \alpha lnK + \beta lnL + \gamma lnE + (\delta-1)lnM \end{cases} \quad (10)$$

Since, our purpose is to derive a formula for Energy demand function, we should express all other variables in terms of energy demand, namely *E*.

Let's express *K*, *L* and *M* in terms of *E* and then consider this expression in the first equation of the last system of equations (10). Subtracting side by side, the fourth equation of the system (10) from the second one yields:

$lnp_k - lnp_e = ln\lambda\alpha A + (\alpha-1)lnK + \beta lnL + \gamma lnE + \delta lnM - (ln\lambda\gamma A + \alpha lnK + \beta lnL + (\gamma-1)lnE + \delta lnM)$ (11)

Opening parenthesis with the opposite sign, combining the coefficients of the same variables we get:

$lnp_k - lnp_e = (ln\lambda\alpha A - ln\lambda\gamma A) + [(\alpha-1)lnK - \alpha lnK] + [\beta lnL - \beta lnL] + [\gamma lnE - (\gamma-1)lnE] + [\delta lnM - \delta lnM]$ (12)

Now let's do some simplifications and use one property of logarithmic function, namely: $\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$, then we get:

$$ln\frac{p_k}{p_e} = ln\frac{\alpha}{\gamma} - lnK + lnE \quad (13)$$

Modifying this equation, a little we can derive a formula relating *K* to *E*:

$$lnK = -ln\frac{p_k}{p_e} + ln\frac{\alpha}{\gamma} + lnE \text{ or}$$

$$lnK = ln\frac{p_e}{p_k}\frac{\alpha}{\gamma}E \quad \text{and therefore: } K=\frac{p_e}{p_k}\frac{\alpha}{\gamma}E \quad (14)$$

In a similar way subtracting the fourth equation consequently from the third and fifth equations we get the formulas relating L and M to E:

$$\mathrm{L}=\frac{p_e}{p_l}\frac{\beta}{\gamma}E \text{ (15) and } \mathrm{M}=\frac{p_e}{p_m}\frac{\delta}{\gamma}E \quad (16)$$

Considering (14), (15) and (16) in the first equation of the system (10) we end up with:

$$lnQ = lnA + \alpha\ln(\frac{p_e}{p_k}\frac{\alpha}{\gamma}E) + \beta\ln(\frac{p_e}{p_l}\frac{\beta}{\gamma}E) + \gamma lnE + \delta\ln(\frac{p_e}{p_m}\frac{\delta}{\gamma}E) \quad (17)$$

Using the above-mentioned properties of the logarithmic function (17) can be modified as follow:

$$lnQ = lnA + \alpha\ln\left(\frac{p_e}{p_k}\right) + \alpha\ln\left(\frac{\alpha}{\gamma}\right) + \alpha lnE + \beta\ln\left(\frac{p_e}{p_l}\right) + \beta\ln\left(\frac{\beta}{\gamma}\right) + \beta lnE + \gamma lnE + \delta\ln\left(\frac{p_e}{p_m}\right) + \delta\ln\left(\frac{\delta}{\gamma}\right) + \delta lnE \quad (18)$$

Rearranging (18) and combining the constant terms and coefficient of *lnE* we get:

$$lnQ = lnA + \left[\alpha\ln\left(\frac{\alpha}{\gamma}\right) + \beta\ln\left(\frac{\beta}{\gamma}\right) + \delta\ln\left(\frac{\delta}{\gamma}\right)\right] + \alpha\ln\left(\frac{p_e}{p_k}\right) + \beta\ln\left(\frac{p_e}{p_l}\right) + \delta\ln\left(\frac{p_e}{p_m}\right) + [\alpha+\beta+\gamma+\delta]lnE \quad (19)$$

Let's find *lnE* from this expression:

$$[\alpha+\beta+\gamma+\delta]lnE = lnQ - lnA\left[\alpha\ln\left(\frac{\alpha}{\gamma}\right) + \beta\ln\left(\frac{\beta}{\gamma}\right) + \delta\ln\left(\frac{\delta}{\gamma}\right)\right] - \alpha\ln\left(\frac{p_e}{p_k}\right) - \beta\ln\left(\frac{p_e}{p_l}\right) - \delta\ln\left(\frac{p_e}{p_m}\right) \quad (20)$$

Taking into account the fact that $(-\ln(\frac{x}{y})) = ln\frac{y}{x}$ we can modify (20) as follow:

$$[\alpha+\beta+\gamma+\delta]lnE = lnQ - lnA + \left[\alpha\ln\left(\frac{\gamma}{\alpha}\right) + \beta\ln\left(\frac{\gamma}{\beta}\right) + \delta\ln\left(\frac{\gamma}{\delta}\right)\right] + \alpha\ln\left(\frac{p_k}{p_e}\right) + \beta\ln\left(\frac{p_l}{p_e}\right) + \delta\ln\left(\frac{p_m}{p_e}\right) \quad (21)$$

Dividing both sides by the coefficient of *lnE* and using some mathematical properties of logarithmic function we get:

$$lnE = \frac{1}{[\alpha+\beta+\gamma+\delta]}lnQ - \frac{1}{[\alpha+\beta+\gamma+\delta]}lnA + \frac{\left[\alpha\ln\left(\frac{\gamma}{\alpha}\right)+\beta\ln\left(\frac{\gamma}{\beta}\right)+\delta\ln\left(\frac{\gamma}{\delta}\right)\right]}{[\alpha+\beta+\gamma+\delta]} + \frac{\alpha}{[\alpha+\beta+\gamma+\delta]}\ln p_k + \frac{\beta}{[\alpha+\beta+\gamma+\delta]}\ln p_l + \frac{\delta}{[\alpha+\beta+\gamma+\delta]}\ln p_m + \frac{(-\alpha-\beta-\delta)}{[\alpha+\beta+\gamma+\delta]}p_e \quad (22)$$

(22) can be written as below, with denoting coefficients with new letters:

$$lnE = \alpha_0' + \alpha'\ln p_k + \beta'\ln p_l + \delta'\ln p_m + \gamma' lnp_e - \eta' lnA + \eta' lnQ \quad (23)$$

Where:

$$\alpha_0' = \frac{\left[\alpha\ln\left(\frac{\gamma}{\alpha}\right)+\beta\ln\left(\frac{\gamma}{\beta}\right)+\delta\ln\left(\frac{\gamma}{\delta}\right)\right]}{[\alpha+\beta+\gamma+\delta]}, \alpha' = \frac{\alpha}{[\alpha+\beta+\gamma+\delta]}, \beta' = \frac{\beta}{[\alpha+\beta+\gamma+\delta]}, \delta' = \frac{\delta}{[\alpha+\beta+\gamma+\delta]}, \gamma' = -\frac{(\alpha+\beta+\delta)}{[\alpha+\beta+\gamma+\delta]}, \eta' = \frac{1}{[\alpha+\beta+\gamma+\delta]}.$$

As a result, we obtained a formula for energy demand which expresses it as a function of capital, labor, material and energy prices, TFP and total output.

As discussed in Nordhaus (1975) the demand functions of the economy for each product can be written as:

$$Q_i = f^i(P_1, P_2, P_3, \dots, P_n, Y), i = 1, \dots, n \quad (24)$$

Where P_i's are prices and Y is the total income. This function (with variables in logs) can be written explicitly as:

$$lnQ = \theta_0 + \theta_1 \ln p_k + \theta_2 \ln p_l + \theta_3 \ln p_m + \theta_4 lnp_e + \theta_5 lnY \quad (25)$$

If we substitute lnQ in (23) with its expression in (25) it yields to:

$$lnE = \alpha_0' + \alpha' \ln p_k + \beta' \ln p_l + \delta' \ln p_m + \gamma' lnp_e - \eta' lnA + \eta'(\theta_0 + \theta_1 \ln p_k + \theta_2 \ln p_l + \theta_3 \ln p_m + \theta_4 lnp_e + \theta_5 lnY) \quad (26)$$

and making some modifications in terms of combining similar coefficients we end up with:

$$lnE = (\alpha_0' + \eta'\theta_0) + (\alpha' + \eta'\theta_1) \ln p_k + (\beta' + \eta'\theta_2) \ln p_l + (\delta' + \eta'\theta_3) \ln p_m + (\gamma' + \eta'\theta_4) lnp_e - \eta' lnA + \eta'\theta_5 lnY \quad (27)$$

(27) can be expressed in the following form:

$$lnE = \alpha_0'' + \alpha'' \ln p_k + \beta'' \ln p_l + \delta'' \ln p_m + \gamma'' lnp_e - \eta' lnA + \eta'' lnY \quad (28)$$

where

$$\alpha_0'' = \alpha_0' + \eta'\theta_0, \alpha'' = \alpha' + \eta'\theta_1, \beta'' = \beta' + \eta'\theta_2, \delta'' = \delta' + \eta'\theta_3, \gamma'' = \gamma' + \eta'\theta_4 \; and \; \eta'' = \eta'\theta_5$$

Equation (28) is the energy demand equation. As can be seen from (28), the energy demand can be expressed as a function of TFP, in addition to prices of production factors and the total income.

Equations (23) and (28) can be used as theoretical framework for econometrically estimating demand for energy as a function of TFP, in (23) with output and in (28) with income variable.

As can be seen from (28) the coefficient of TFP, which is equal to the ratio of two positive numbers, but with negative sign, is negative by definition. The negative sign of TFP coefficient is in line with the economic point of view, since TFP as a representative of technological progress, institutional development and innovations is expected to reduce the energy use over time.

Another useful feature of the coefficient of TFP in energy demand equation is that it allows to evaluate how the economy/sector under consideration performs, especially if the TFP is calculated using methods other than production function approach. That is, since the coefficient of TFP is equal to $-\frac{1}{[\alpha+\beta+\gamma+\delta]}$, the denominator is the sum of the coefficients of production inputs. Hence, we can suggest the following procedure. If the coefficient of TFP variable in absolute terms (η')

a) $\eta' > 1$, there is decreasing return to scale,
b) $\eta' = 1$, there is constant return to scale,
c) $\eta' < 1$, there is increasing return to scale.

3.1. Some other Useful Modifications of TFP Calculation

Let's consider the Cobb-Douglas production function with four factors:

$$Q_t = A_t K_t^{\alpha} L_t^{\beta} E_t^{\gamma} M_t^{\delta} \quad (29)$$

Where A is a technology parameter. Assuming constant return to scale; namely: $\alpha + \beta + \gamma + \delta = 1$, and expressing $\beta = 1 - (\alpha + \gamma + \delta)$ then we will get the production function as follow:

$$Q_t = A_t K_t^{\alpha} L_t^{1-(\alpha+\gamma+\delta)} E_t^{\gamma} M_t^{\delta} \quad (30)$$

Now let's take logarithm of both sides of equation (30) and consider that $\log(x)^n = n * \log(x)$. Then we get:

$$lnQ_t = lnA_t + \alpha lnK_t + [1 - (\alpha + \gamma + \delta)]lnL_t + \gamma lnE_t + \delta lnM_t \quad (31)$$

Then let's separate the expression in front of lnL_t, it yields to:

$$lnQ_t = lnA_t + \alpha lnK_t + lnL_t - \alpha lnL_t - \gamma lnL_t - \delta lnL_t + \gamma lnE_t + \delta lnM_t \quad (32)$$

With little modification (32) can be written as:

$$lnQ_t - lnL_t = lnA_t + \alpha lnK_t - \alpha lnL_t + \gamma lnE_t - \gamma lnL_t + \delta lnM_t - \delta lnL_t \quad (33)$$

Taking into account that $\ln(x) - \ln(y) = ln\frac{x}{y}$ (34) can be expressed as follow:

$$ln\frac{Q_t}{L_t} = lnA_t + \alpha ln\frac{K_t}{L_t} + \gamma ln\frac{E_t}{L_t} + \delta ln\frac{M_t}{L_t} \quad (35)$$

In (35) now we have a production function in three variables, and the variables have special meanings/interpretations. Namely, $\frac{Q_t}{L_t}$ is labor productivity, $\frac{K_t}{L_t}, \frac{E_t}{L_t}\ and\ \frac{M_t}{L_t}$ are labor intensities of capital, energy and materials. As it well known from the calculus:

$$\Delta Y_t \approx dY_t = Y_t' dt = Y_t' \Delta t \text{ (36)}$$

and

$$\Delta t = t - (t-1) = 1 \text{ (37)}$$

(Here Δ is difference operator and " $'$ "-"prime" stands for derivative), then we get:

$$Y_t' = \Delta Y_t \text{ (38)}$$

Taking derivative of both sides of (35) with respect to time and considering (38), yields to:

$$\frac{L_t}{Q_t} * \Delta\left(\frac{Q_t}{L_t}\right) = \frac{\Delta A_t}{A_t} + \alpha \frac{L_t}{K_t} * \Delta\left(\frac{K_t}{L_t}\right) + \gamma \frac{L_t}{E_t} * \Delta\left(\frac{E_t}{L_t}\right) + \delta \frac{L_t}{M_t} * \Delta\left(\frac{M_t}{L_t}\right) \text{ (39)}$$

With a little modification (39) can expressed as follow:

$$\frac{\Delta\left(\frac{Q_t}{L_t}\right)}{\frac{Q_t}{L_t}} = \frac{\Delta A_t}{A_t} + \alpha \frac{\Delta\left(\frac{K_t}{L_t}\right)}{\frac{K_t}{L_t}} + \gamma \frac{\Delta\left(\frac{E_t}{L_t}\right)}{\frac{E_t}{L_t}} + \delta \frac{\Delta\left(\frac{M_t}{L_t}\right)}{\frac{M_t}{L_t}} \text{ (40)}$$

With some notations (40) will be:

$$\dot{P}_L = \dot{A}_t + \alpha \dot{K}_L + \gamma \dot{E}_L + \delta \dot{M}_L \text{ (41)}$$

Where, $\dot{P}_L = \frac{\Delta\left(\frac{Q_t}{L_t}\right)}{\frac{Q_t}{L_t}}$ is labor productivity growth, $\dot{A}_t = \frac{\Delta A_t}{A_t}$ is total factor productivity growth, $\dot{K}_L = \frac{\Delta\left(\frac{K_t}{L_t}\right)}{\frac{K_t}{L_t}}$ is growth of capital intensity, $\dot{E}_L = \frac{\Delta\left(\frac{E_t}{L_t}\right)}{\frac{E_t}{L_t}}$ is growth of labor intensity of energy and $\dot{M}_L = \frac{\Delta\left(\frac{M_t}{L_t}\right)}{\frac{M_t}{L_t}}$ is growth of labor intensity of materials.

(41) can be used to calculate the TFP growth ($\dot{A}$) as:

$$\dot{A}_t = \dot{P}_L - (\alpha \dot{K}_L + \gamma \dot{E}_L + \delta \dot{M}_L) \text{ (42)}$$

Now, if we subtract lnK_t from both sides of (31), it yields to:

$$lnQ_t - lnK_t = lnA_t + \alpha lnK_t - lnK_t + [1 - (\alpha + \gamma + \delta)]lnL_t + \gamma lnE_t + \delta lnM_t \text{ (43)}$$

With a little modification (43) can be expressed as below:

$$ln\frac{Q_t}{K_t} = lnA + (\alpha - 1)lnK_t - (\alpha - 1)lnL_t + \gamma lnE_t - \gamma lnL_t + \delta lnM_t - \gamma lnL_t \quad (44)$$

Which yields to:

$$ln\frac{Q_t}{K_t} = lnA_t + (\alpha - 1)ln\frac{K_t}{L_t} + \gamma ln\frac{E_t}{L_t} + \delta ln\frac{M_t}{L_t} \quad (45)$$

After performing the same procedures as done for (42) we get the equation below:

$$\dot{P}_K = \dot{A}_t - (\beta + \gamma + \delta)\dot{K}_L + \gamma\dot{E}_L + \delta\dot{M}_L \ (46)$$

Where we considered that $\alpha - 1 = -(\beta + \gamma + \delta)$ and $\dot{P}_K = \frac{\Delta\left(\frac{Q_t}{K_t}\right)}{\frac{Q_t}{K_t}}$ capital productivity and $\dot{A}_t, \dot{K}_L, \dot{E}_L, \dot{M}_L$ are as defined above, in the case of (42). (46) also can be used to calculate TFP growth, namely:

$$\dot{A}_t = \dot{P}_K + (\beta + \gamma + \delta)\dot{K}_L - \gamma\dot{E}_L - \delta\dot{M}_L \ (47)$$

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