Every Prime Number Greater than Three Has Finitely Many Prime Friends

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Abstract

The sums of prime components of composite numbers lead to prime numbers, a remarkable phenomena in number theory that is examined in this paper. We present a constructive proof methodology to show that adding the prime components of the resulting composite numbers iteratively converges to prime numbers for any composite number higher than four. By applying logical reasoning and the fundamentals of number theory, we prove that all prime numbers larger than three have a finite number of "prime friends," or composite numbers whose prime factors add up to the prime number itself. We reveal the method by which prime numbers emerge from this iterative process by analyzing partitions of prime numbers and applying the function $2k + 3$. This abstract summarizes our investigation into the convergence of prime numbers through the sums of prime factors of composite numbers, demonstrating a significant relationship between primes and composites in number theory.

NOTE : We have created the term "prime friend"; it is not a term found in the traditional lexicon of mathematics.

1 Introduction

The interaction between prime and composite numbers in number theory frequently reveals fascinating patterns and correlations. A particularly interesting question is the one concerning the relationship between prime numbers and the recursive summing of prime elements in composite numbers. This study explores this territory and attempts to provide a constructive demonstration by use of the sum of prime components of composite numbers, stating that all prime numbers larger than three are covered.

Investigating Composite Sum and Prime Friends

The first premise presents the idea of "prime friends" in composite numbers, where a composite number is created by adding the prime factors, starting a chain reaction of summations. We set out to discover the underlying physics behind this phenomena through an analytical lens based in number theory and logical deductions.

Drawing Parallels with Ruth-Aaron Numbers

A conceptual link to this inquiry can be found by drawing comparisons with Ruth-Aaron numbers, in which the sum of prime components in two different numbers equals itself. But although Ruth-Aaron numbers concentrate on pairs of single numbers, we extend our investigation to include all composite and prime numbers, providing more profound understanding of their relationship.

Constructive Proof and Theorem

By rigorous logic and rigorous mathematics, we develop a convincing demonstration that shows that the recursive accumulation of prime factors inside composite numbers covers all prime numbers larger than three. We clarify the finite character of prime friends and the general trend towards prime numbers by utilizing the function 2k + 3 and the complex partitions of prime numbers.

This work aims to elucidate the underlying relationship between prime and composite numbers by demonstrating how the prime factor summation of composites inevitably leads to the prime number discovery. Through clarifying this complex relationship, we add to the larger discussion on number theory and provide new insights into the basic characteristics of integers.[1][2][3]

2 Theorem first

LET

$$u = \prod_{i=1}^{k} p_i$$

$$v = \sum_{i=1}^{n} p_i$$

If v is composite, then

$$v = p_1 \times p_2 \times \cdots \times p_{n-1} \times p_n$$

$$w = p_1 + p_2 + p_3 + \cdots + p_{n-1} + p_n$$

If w is composite,than

$x = p_{r_1} \cdot p_{r_2} \cdot p_{r_3} ... \cdot p_{r_n}$

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$y = p_1 \times p_2 \times \cdots \times p_{n-1} \times p_n$

$p = p_1 + p_1 + p_1 + ... + p_n$

where u, w,..., y are the additional composite numbers for each step to the prime p, and n is the composite number we began with.

We can see that $p \neq 2, 3$ due to their partitions.

$2 = 2 = 1 + 1$
$3 = 2 + 1 = 1 + 1 + 1$,
so a composite number does not exist.
where 2 or 3 is the total of its prime components.

The only composite number on which the theorem is in applicable is 4. It is caught up in an endless circle with itself.

$4 = 2 \cdot 2$
$2 + 2 = 4$

Prior to fully proving the theorem, we demonstrate that n's prime factor sum is always less than n. This seems insignificant even in the absence of proof, but processing the full proof requires caution and diligence.

3 Lemma one

If n is more than four, the total of the prime elements of a composite number n is always smaller than n.

Proof

Let sum(u) represent the total of the composite number n's prime factors. Decide which prime factor (p) is the least of u. It shows

$$sum(u) < u$$

such that it can be proven that

$$\frac{u}{p} \geq p$$

and that

$$p + sum(\frac{u}{p}) < n$$

So it can be shown

$$p + sum(\frac{u}{p}) < p + \frac{u}{p} \leq 2(\frac{u}{p}) \leq p(\frac{u}{p}) = u$$

4 Proof of Theorem one

We know from Lemma 1 that any given composite number larger than four has a prime factor sum that is less than that number. This indicates that Theorem 1's prime factor summation process is a terminating one. Thus

$$u < w < ... < y$$

Following each sum of their prime factors, the composite numbers must always get smaller and smaller until they terminate. This implies that we have to arrive at a prime number after t composite numbers. Lemma 1 states that the process cannot be true if this were untrue since it would require a composite number to conclude. A composite number larger than four can never be the conclusion of the process since it always has prime elements that can be added up to produce a number smaller than the composite number itself. Consequently, the process illustrated by Theorem 1 must always end with a prime number bigger than three, assuming that any composite number n is greater than four. The proof is not yet complete. For each prime number bigger than three, we need to demonstrate that this is accurate. With the assistance of our close friends, we succeed.

5 Definition

A composite number that is equal to a prime number when the sum of its prime elements is called a prime friend1. This is how it is expressed mathematically.

$$u = \prod_{i=1}^{k} p_i$$

u is a prime friend if and only if

$$p = \sum_{i=1}^{n} p_i$$

Examples:

6 is a prime friend of 5.
$6 = 2 \cdot 3$
$\Rightarrow 2 + 3 = 5$

12 is a prime friend of 7.
$12 = 2 \cdot 2 \cdot 3$
$\Rightarrow 2 + 2 + 3 = 7$

28 is a prime friend of 11.
$28 = 2 \cdot 2 \cdot 7$
$\Rightarrow 2 + 2 + 7 = 11$

20 is a prime friend of 13.
$20 = 2 * 2 * 5$
$\Rightarrow 2 + 2 + 5 = 13$

34 is a prime friend of 17.
$34 = 2 * 17$
$\Rightarrow 2 + 17 = 19$

48 is a prime friend of 13.
$48 = 2 * 2 * 2 * 2 * 3$

$\Rightarrow 2+2+2+2+3 = 11$

88 is a prime friend of 17.
88 = 2 * 2 * 2 * 11
$\Rightarrow 2+2+2+11 = 17$

6 Lemma two

(i).There are infinitely many prime friends.
(ii).There is a finite number of prime friends for every prime number after 3.

proof

We inspect the function $2k + 3$, where $k \in N$.

A multiple of two is always the difference between two prime numbers greater than two. This means that, given the proper input of k (i.e., k cannot be a multiple of three since 2k + 3 would be divisible by three), 2k + 3 will produce every prime integer larger than 2. This allows us to divide any prime number into k summands of 2 and a summand of 3.

Given that 2 and 3 are prime numbers and that we can express the sum of k twos and a three in this way, it follows that every prime number larger than three has at least one prime friend.

2 · 1 + 3 = 2 + 3 = 5
$\Rightarrow 2 \cdot 3 = 6$
2 · 2 + 3 = 4 + 3 = 2 + 2 + 3 = 7
$\Rightarrow 2 \cdot 2 \cdot 3 = 12$
2 · 4 + 3 = 8 + 3 = 2 + 2 + 2 + 2 + 3 = 11
$\Rightarrow 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 48$

A prime number can only be written as the sum of prime numbers in a finite number of ways because it has a finite number of divisions. Because of our earlier reasoning, every prime number larger than three can only have a limited number of prime friends, which implies that there can only be an infinite number of prime friends in total.

Lemma 1 tells us that every prime number bigger than three has a finite number of prime companions. Consequently, every other composite number must terminate into a prime number. As a result, every composite number bigger than four will eventually result in a prime number, making sure that no prime number after three is omitted.

7 Conclusion

Ultimately, our research examined the intriguing connection between prime and composite numbers, focusing on the situation where the sum of the prime components of a composite number yields a prime number. We proved that for each composite number greater than four, combining the prime components of the composite number iterative converges to prime numbers using a constructive proof methodology.

In order to prove that every prime number greater than three has a limited number of "prime friends," or composite numbers whose prime factors add up to the prime number itself, our research relies on logical reasoning and basic number theory concepts. We discovered the mechanism by which prime numbers emerge from this repeated process by examining partitions of prime numbers and utilizing the formula 2k + 3.

Number theory is still a relatively unexplored field. Exploring this field could add a great deal to our encyclopedia of numerical knowledge, perhaps adding several pages to our understanding. We encourage readers to delve deeper into this area of mathematics and related topics. It is important to note that findings in this field might provide information that helps resolve theories like the Twin Prime hypothesis.

References

[1] Goldstein, Larry J. "A history of the prime number theorem." The American Mathematical Monthly 80.6 (1973): 599-615.

[2] Ribenboim, Paulo. The book of prime number records. Springer Science and Business Media, 2012.

[3] Curtis, Matthew, and Gurudeo Anand Tularam. "The importance of numbers and the need to study primes: The prime questions." Journal of Mathematics and Statistics 7.4 (2011): 262-269.