Unified Power Conservative Equivalent Circuit for DC Networks

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ABSTRACT This paper introduces a new equivalent circuit for linear direct current networks consisting of independent voltage and current sources, and resistors, which represents all the power dissipated internally in the resistors. It is demonstrated that the internal losses of any network have two components. One is variable and dependent on the internal resistances of the actual circuit and the power transferred to the pair of accessible terminals. The other is constant and dependent only on the internal voltage and current sources and the resistances of the actual network. It is also demonstrated theoretically and validated by numerical simulation that the traditional Thevenin and Norton equivalent circuits are particular cases of the proposed equivalent circuit in this paper. The proposed equivalent circuit can be used to analyze power and efficiencies of the actual network.

INDEX TERMS equivalent circuit, power conservation, Thévenin's theorem, Thévenin equivalent circuit, Norton's theorem, Norton equivalent circuit, efficiency of equivalent circuits, general network theorem.

I. INTRODUCTION

THE Thevenin [1] and Norton [4] equivalent circuits are important to the theory of DC networks and are commonly used in the analysis of electrical circuits basically because they provide simplicity in many practical applications.

However, as it is well known, they are limited in representing the phenomena that occur in the actual circuit, since they can only be used to determine voltage and current at a pair of accessible terminals of the network.

Since they do not represent the totality of the internal losses of the actual circuit, they cannot be used to determine efficiency and for power analysis.

As presented in [16], a review of publications since 1883 presenting the DC network classical theorems [1]–[13] reveals that none of them address the analysis of the internal losses and how to include them in an equivalent circuit.

In [16] it is demonstrated that any DC network formed by independent voltage sources and resistors, with a pair of accessible terminals ab, can be represented by the equivalent circuit shown in Fig. 1.

In the equivalent circuit shown in Fig. 1(b), V_T and R_T are the voltage and resistance of the traditional Thévenin equivalent circuit. The resistor R_X is constant, depen-

dent only on the network current internal parameters, and represents the internal constant power dissipation that is independent of the power transferred to the terminals ab.

This paper extends the analysis presented in [16], for networks formed by independent voltage and current sources, and resistors. A unified equivalent circuit is introduced. It represents and can be used to determine not only the power transferred to the external terminals, but also the internal losses and the actual network efficiency.
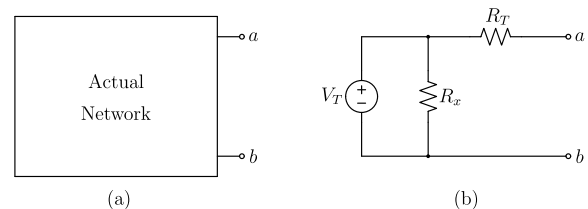
FIGURE 1. The equivalent circuit for DC networks consisting of independent voltage sources and resistors [16].

II. THE EQUIVALENT CIRCUIT FOR DC NETWORKS FORMED BY CURRENT SOURCES AND RESISTORS

A linear resistive network formed by independent current sources and resistors, represented by N, is shown in Fig. 2. There are two accessible external terminals, designated by ab, to which an external voltage source V_o is connected.
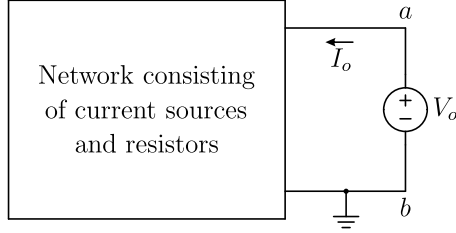
FIGURE 2. Direct current network consisting of independent current sources and resistors.

The node voltage equations in matrix form are given by

$$\begin{bmatrix} I_1 \\ I_2 \\ ... \\ I_n \\ I_a \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & ... & G_{1n} & G_{1a} \\ G_{12} & G_{22} & ... & G_{2n} & G_{2a} \\ ... & ... & ... & ... & ... \\ G_{n1} & G_{n2} & ... & G_{nn} & G_{na} \\ G_{a1} & G_{a2} & ... & G_{an} & G_{aa} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ ... \\ V_n \\ V_a \end{bmatrix} \tag{1}$$

where

I_1, I_2, ..., I_n are the algebraic sums of all the source currents at principal nodes 1, 2, ..., n, a of the network. The node b is the reference node.

V_1, V_2, ..., V_n are voltages between the nodes 1, 2, ..., n, a, and the reference node b.

G_{11}, G_{22}, ..., G_{nn}, G_{aa} are the sums of all the conductances connected to nodes 1, 2, ..., n, a, respectively.

G_{ij} is the sum of all the conductances connecting nodes i and j. Note that $G_{ij} = G_{ji}$.

In order to reduce the size of the equations, without loss of generality, we will assume that the original network contains only three principal nodes, designated by 1, 2 and a. From (1) we can write (2).

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = G \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a \tag{2}$$

where

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix} \tag{3}$$

From (2) we find

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a \tag{4}$$
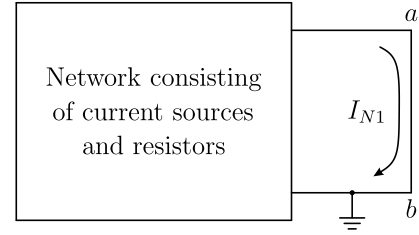
From (1) we can obtain

$$I_a = \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + G_{aa} V_a \tag{5}$$

Substituting (4) in (5) gives

$$I_a = \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - \\ - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a + G_{aa} V_a \tag{6}$$

A. NORTON EQUIVALENT CIRCUIT

Let us analyze the case where the terminals ab are connected in short circuit causing $V_{ab} = 0$ and $I_a = I_N$, as shown in Fig. 3.

FIGURE 3. Network with terminals connected in short circuit , where $V_{ab} = 0$ and $I_a = -I_N$.

From (6) we obtain

$$I_a = \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{7}$$

Substituting (2) in (7) we find

$$I_N = - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{8}$$

where I_N represents the current value of the current source of the Norton equivalent circuit.

Next, we will set equal to zero all currents from the internal current sources of the network and apply a voltage V_a between terminals ab, as shown in Fig. 4.
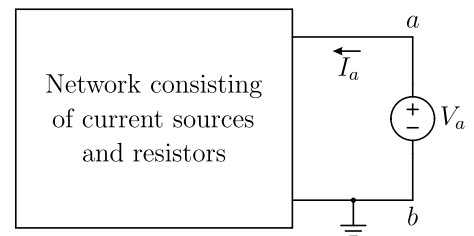
FIGURE 4. Measurement of the resistance of the Norton equivalent circuit.

When $I_1 = I_2 = 0$, from (6) we find

$$I_a = G_{aa} V_a - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a \tag{9}$$

The conductance of the Norton equivalent circuit is defined by

$$G_N = \frac{I_a}{V_a} \tag{10}$$

Therefore, from (9) we find

$$G_N = G_{aa} - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \tag{11}$$

Substitution of (2), (8) and (11) into (6) yields

$$I_a = G_N V_a + \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{12}$$

or yet

$$I_a = G_N V_a - I_N \tag{13}$$

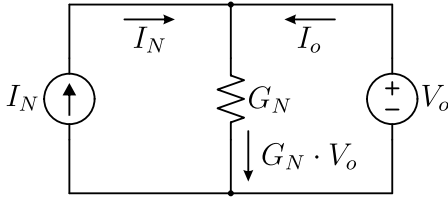Equation (13) represents the Norton equivalent circuit shown in Fig. 5.

FIGURE 5. Norton's traditional equivalent circuit for the DC network shown in Fig. 1

B. INTERNAL DISSIPATED POWERS IN THE NETWORK WHEN THE TERMINALS ab ARE CONNECTED IN SHORT CIRCUIT ($V_A = 0$)

The power dissipated internally in the network when the external terminals ab connected in short circuit is defined by

$$P_y = \begin{bmatrix} I_1 & I_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{14}$$

For this condition, in which $V_a = 0$, from equation (2) we obtain

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{15}$$

Substitution of (15) in (14) gives

$$P_y = \begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{16}$$

C. INTERNAL POWER DISSIPATED IN THE NETWORK WHEN $V_a \neq 0$

The power dissipated internally in the original network when $V_a \neq 0$ is given by

$$P = \begin{bmatrix} I_1 & I_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + V_a I_a \tag{17}$$

Substituting (4) in (17) we find

$$P = \begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - \\ - \begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a + V_a I_a \tag{18}$$

Multiplying (6) by V_a we obtain

$$I_a V_a = \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} V_a - \\ - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a^2 + G_{aa} V_a^2 \tag{19}$$

Substitution of (19) in (18) gives

$$P = \begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} - \\ \begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} V_a + \\ \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I \end{bmatrix} V_a - \\ \begin{bmatrix} G_{aa} - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \end{bmatrix} V_a^2 \tag{20}$$

Since

$$\begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \\ \begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \tag{21}$$

from (20) we find

$$P = \begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \\ \begin{bmatrix} G_{aa} - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \end{bmatrix} V_a^2 \tag{22}$$

Substituting (11) and (16) in (22), we find

$$P = P_y + G_N V_a^2 \tag{23}$$

Equation (23) indicates that there are two components of internal losses in the actual network, one of which is constant and represented by P_y, which depends only on the internal parameters of the actual network, and another variable given by the term $G_N V_a^2$ that, in addition to depending on the internal parameters represented by G_N, also depends on the voltage (or current) at the external terminals ab. The invariant losses are not represented in the traditional Norton equivalent circuit, shown in Fig. 5. Equation (23) represents the equivalent circuit shown in Fig. 6.

The resistor R_y accounts for the invariant losses, and can then be defined by
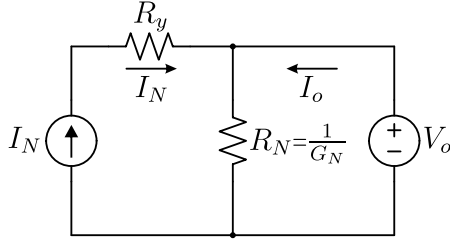
$$P_y = R_y I_N^2 \tag{24}$$

Thus,

FIGURE 6. Power conservative Norton equivalent circuit of networks consisting of independent current sources and resistors.

$$R_y = \frac{P_y}{I_N^2} \qquad (25)$$

Substitution of (8) and (16) into (25) yields

$$R_y = \frac{\begin{bmatrix} I_1 & I_2 \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}{\left(\begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} G^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \right)^2} \qquad (26)$$

The presence of the resistor R_y in series with the current source I_N does not interfere with the behavior observed from the external terminals ab. We can then conclude that the Norton equivalent circuit shown in Fig. 5 is a particular case of the equivalent circuit shown in Fig. 6, when $R_y = 0$. The power transferred to the terminals ab is independent of R_y. However, the power transmitted to the circuit by the current source I_N depends on it. We can also conclude that the traditional Norton equivalent circuit efficiency is always less than the original DC network. Thus, its equivalence is limited, and it cannot be used for analysis of powers, losses and efficiency.

D. NUMERICAL EXAMPLE

Let us consider the circuit shown in Fig. 7, which will be used to illustrate the use of the analysis results presented in the previous sections, to determine the parameters of the proposed equivalent circuit.
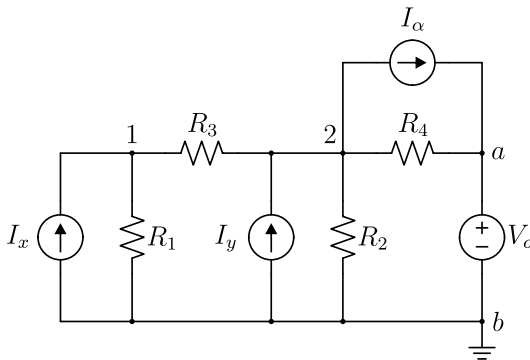

FIGURE 7. Electric circuit for the numerical example.

The parameters of the circuit are $I_x = 10A$, $I_y = 4A$, $I_\alpha = 3A$, $R_1 = 10\Omega$, $R_2 = 5\Omega$, e $R_4 = 7\Omega$.

The node voltage equations in matrix form are

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} G_1 + G_3 & -G_3 & 0 \\ -G_3 & G_2 + G_3 + G_4 & -G_4 \\ 0 & -G_4 & G_{aa} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_a \end{bmatrix} \qquad (27)$$

Thus,

$G_{11} = G_1 + G_3 = \frac{1}{R_1} + \frac{1}{R_3} = 0.6S$

$G_{22} = G_2 + G_3 + G_4 = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = 0.843S$

$G_{12} = -G_3 = -\frac{1}{R_3} = -0.5S$

$G_{21} = -G_3 = -\frac{1}{R_3} = -0.5S$

$G_{1a} = 0S$

$G_{2a} = -G_4 = -\frac{1}{R_4} = -0.143S$

$G_{aa} = G_4 = \frac{1}{R_4} = 0.143S$

$I_1 = I_x = 10A$

$I_2 = I_y - I_\alpha = 1A$

The current I_N of the equivalent circuit is given by

$$I_N = I_\alpha - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (28)$$

Substituting the values of the circuit parameters in (28) we obtain $I_N = 6.128A$.

The admittance value G_N of the Norton equivalent circuit is given by

$$G_N = G_{aa} - \begin{bmatrix} G_{1a} & G_{2a} \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix}^{-1} \begin{bmatrix} G_{1a} \\ G_{2a} \end{bmatrix} \qquad (29)$$

Substitution of network parameters in (29) yields $G_N = 0.095S$ and $R_N = 10.529\Omega$. The power dissipated internally with the terminals ab connected in short circuit is given by

$$P_y = \begin{bmatrix} I_1 & I_2 \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (30)$$

Substituting in (30) the parameters of the network of Fig. 7, we find $P = 371.061W$ and substituting the values of P_y and I_N in (25) we obtain $R_y = 9.88\Omega$. The resulting equivalent circuit with the obtained parameters is shown in Fig. 8.

The internal losses as a function of the voltage V_a for the actual circuit, the proposed equivalent circuit and the Norton equivalent circuit are given respectively by equations (31), (32) and (33).
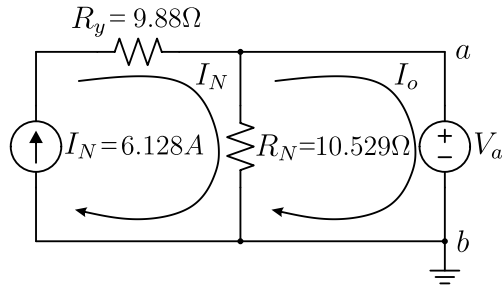
$$\Delta P_1(V_a) = V_1 I_x + V_2 I_y + (V_a - V_2)I_\alpha + V_a I_a \qquad (31)$$

$$\Delta P_2(V_a) = R_y I_N^2 + \frac{V_a^2}{R_N} \qquad (32)$$

$$\Delta P_N(V_a) = \frac{V_a^2}{R_N} \qquad (33)$$

In Fig. 9 the three curves are shown as a function of V_a. It can be seen that the proposed equivalent circuit internal

FIGURE 8. Power conservative equivalent circuit for the network shown in Fig. 7.

losses are equal to the actual network internal losses. In turn, the Norton equivalent circuit internal losses are less than the losses of the actual network losses. For this reason, the Norton equivalent circuit efficiency is always greater than the original circuit one.
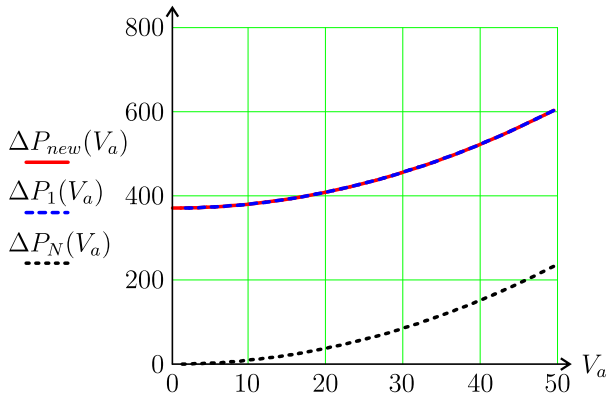
FIGURE 9. Internal losses: (a) actual network ($\Delta P_1(V_a)$); (b) proposed equivalente circuit ($\Delta P_2(V_a)$); (c) Norton equivalente circuit ($\Delta P_N(V_a)$)

III. THE EQUIVALENT CIRCUIT FOR DC NETWORKS FORMED BY INDEPENDENT VOLTAGE AND CURRENT SOURCES, AND RESISTORS

Let us consider a network N formed by independent voltage and current sources, and resistors shown in Fig. 10, with a pair of accessible terminals ab, to which a voltage source V_o is connected.

A. EQUIVALENT CIRCUIT WITH ALL INTERNAL CURRENT SOURCES EQUAL TO ZERO
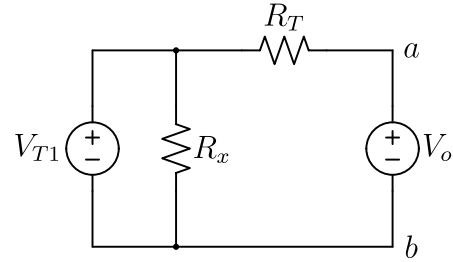
When all internal current sources in the network are set equal to zero, the equivalent circuit is shown in Fig. 11 [16].

The power dissipated internally in the this equivalent circuit resistors is given by

$$P_a(V_a) = \frac{V_{T1}^2}{R_x} + \frac{(V_o - V_{T1})^2}{R_T} \qquad (34)$$
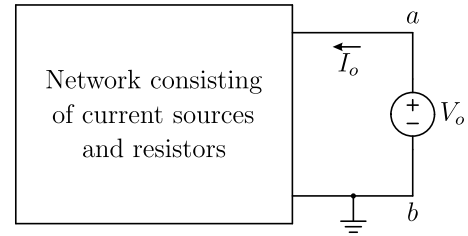

FIGURE 10. Network formed by independent current and voltage sources, and resistors.

FIGURE 11. Power conservative equivalent circuit of the network with internal current sources set equal do zero.

B. EQUIVALENT CIRCUIT WITH ALL INTERNAL VOLTAGE SOURCES EQUAL TO ZERO

As demonstrated in the previous section, the power conservative equivalent circuit for the network formed only by current sources and resistors is shown in Fig. 12.
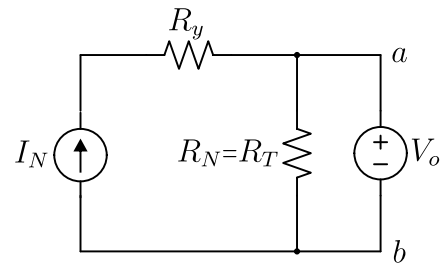
FIGURE 12. Power conservative equivalent circuit of the network with internal voltage sources set equal do zero.

The power dissipated internally in the resistors of the equivalent circuit is given by

$$P_b = R_y I_N^2 + \frac{V_o^2}{R_N} \qquad (35)$$

C. THE UNIFIED POWER CONSERVATIVE EQUIVALENT CIRCUIT

The superposition principle dictates (and the intuition suggests) that the complete network unified equivalent circuit with all voltage and current sources is the result of an appropriate combination of the partial circuits shown in Fig. 11 and Fig. 12, resulting in the power conservative equivalent circuit shown in Fig. 13.

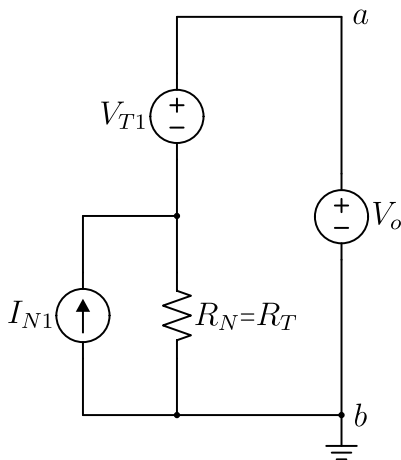The power dissipated in the internal resistors is given by
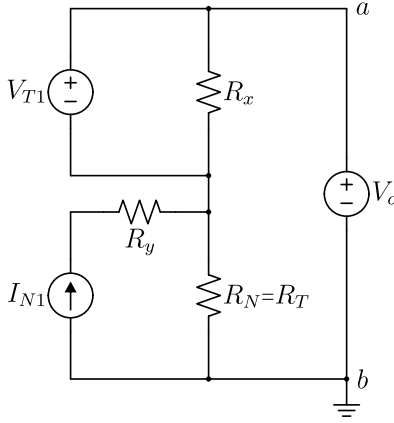
FIGURE 13. Power conservative equivalent circuit for the network formed by independent voltage and current sources, and resistors.

$$P = \frac{V_{T1}{}^2}{R_x} + R_y I_N{}^2 + \frac{(V_o - V_{T1})^2}{R_T} \qquad (36)$$

For a network with only voltage sources and resistors, $I_{N1} = 0$ and equation (36) equals equation (34). For networks formed only by current sources and resistors, $V_{T1} = 0$ and the equation (36) equals equation (35).

D. THEVENIN AND NORTON EQUIVALENT CIRCUITS DERIVED FROM THE UNIFIED EQUIVALENT CIRCUIT
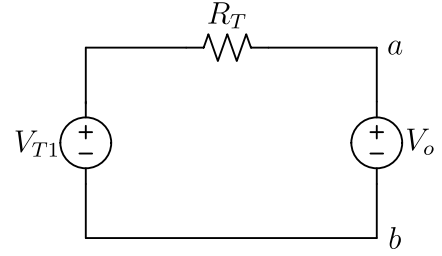
In the equivalent circuit shown in Fig. 13, the resistors R_x and R_y do not interfere in the phenomena observable from the external terminals ab. In applications where the power dissipated internally can be ignored, the resistors R_x and R_y can be removed, which results in the equivalent circuit shown in Fig. 14.

FIGURE 14. Non-conservative equivalent circuit for the network formed by independent voltage and current sources, and resistors.

From the circuit shown in Fig. 14, the traditional Thevenin equivalent circuit shown in Fig. 15 is obtained, where
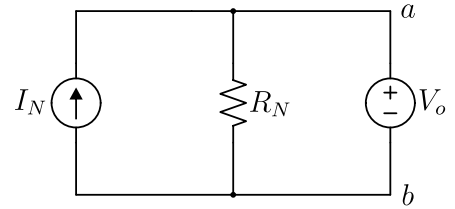
$$V_T = V_{T1} + R_T I_{N1} \qquad (37)$$

FIGURE 15. Traditional Thevenin equivalent circuit.

Likewise, from the unified equivalent circuit shown in Fig. 13, the Norton equivalent circuit shown in Fig. 16 is obtained, where
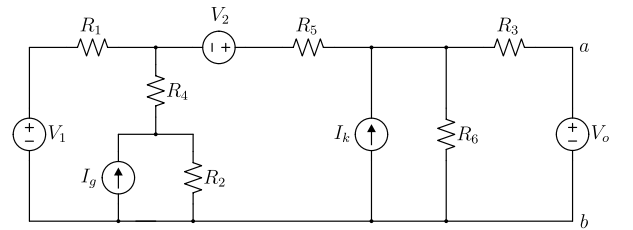
$$I_N = I_{N1} + \frac{V_{T1}}{R_T} \qquad (38)$$

FIGURE 16. Traditional Norton equivalent circuit.

E. NUMERICAL EXAMPLE

To illustrate the determination of the proposed equivalent circuit parameters, we will use the circuit shown in Fig. 17, with $V_1 = 100V$, $V_2 = 35V$, $I_g = 10A$, $I_k = 3.5A$, $R_1 = 5\Omega$, $R_2 = 7\omega$, $R_3 = 3\Omega$, $R_4 = 2\Omega$ and $R_5 = 2\Omega$.

FIGURE 17. Circuit for the numerical example.

1) Measurement of Resistance R_T

To determine R_T (or R_N), the voltage sources V_1 and V_2 are replaced by short circuit and the internal current sources I_g and I_k are replaced by open circuit. The resistance R_T is measured from the terminals ab, as shown in Fig. 18.

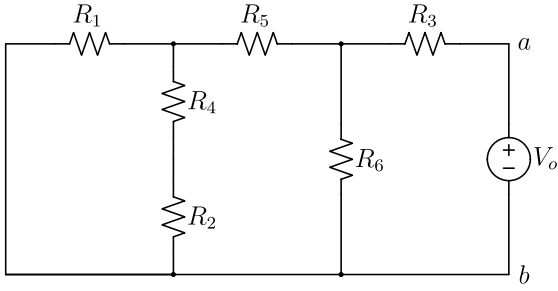The value of R_T found by simulation and given by $R_T = \frac{V_\alpha}{I_\alpha}$ is $R_T = 7.177\Omega$.

FIGURE 18. Circuit for measurement of the resistance RT from the terminals ab.

2) Measurement of Resistance R_X and Voltage V_{T1}

To determine R_X, the power P_X consumed by the actual circuit with the terminals ab open and the current sources I_g and I_k set equal to zero is determined. The corresponding equivalent circuit is shown in Fig. 19.
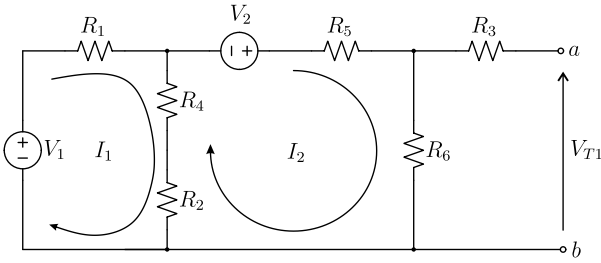

FIGURE 19. Actual circuit with the terminals ab open and the current sources set equal to zero, for measurement of power P_X and voltage V_{T1}

The values of currents obtained by simulation are $I_1 = 9.577A$ and $I_2 = 3.787A$. The power P_X is given by

$$P_X = V_1 I_1 + V_2 I_2 \tag{39}$$

Substituting the obtained values for V_1, V_2, I_1 and I_2 we obtain $P_X = 1090.63W$. The voltage at the terminals ab obtained by simulation is $V_{T1} = 79.54V$. The resistance R_X is determined by

$$R_X = \frac{V_{T1}^2}{P_X} \tag{40}$$

The values substitution for V_{T1} and P_X into (40) gives $R_X = 5.804\Omega$.

3) Measurement of Resistance R_y and Current I_{N1}

To determine R_y, the voltage sources V_1 and V_2 are set equal to zero and the terminals ab are connected in short circuit, as shown in Fig. 20.

The dissipated power in the circuit resistors shown in Fig. 20 is given by

$$P_Y = V_g I_g + V_k I_k \tag{41}$$

The voltages value obtained by simulation are $V_g = 30V$ and $V_K = 155V$. Substituting the values of I_g, I_k, V_g and
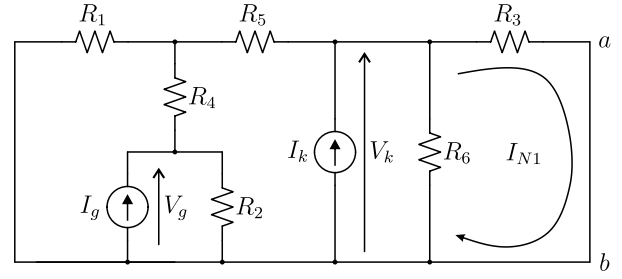

FIGURE 20. Circuit with the terminals ab in short circuit and the internal voltage sources set equal to zero for measurement of P_y, R_y and I_{N1}.

V_K into (41) we find $P_y = 354.25W$. The simulation also yields $I_{N1} = 4.827A$. The resistance R_y is determined by

$$R_y = \frac{P_y}{I_{N1}^2} \tag{42}$$

Substitution of P_y and I_{N1} values in (42) yields $R_y = 15.20\Omega$.

4) Internally Dissipated Power

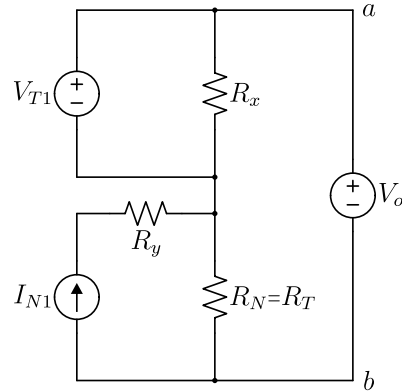The equivalent circuit with the obtained parameters is shown in Fig. 21.

FIGURE 21. Equivalent circuit of the network shown in Fig. 17, with $V_{T1} = 79.54V$, $R_X = 5.804\Omega$, $R_Y = 15.20\Omega$ and $I_{N1} = 4.827A$.

Fig. 22 shows the plot of the power dissipated internally in the equivalent circuit shown in Fig. 21 and calculated using equation (36), and the power dissipated internally in the original network shown in Fig. 17 obtained by numerical simulation, against voltage V_a.

IV. ON THE VALUE OF R_X AND R_Y

It was demonstrated in [16] that the value of R_X for networks formed by independent voltage sources and resistors, depends on the voltage sources and the actual circuit internal resistances. For the particular case where the network has only one voltage source, the value of R_X depends only on the internal resistances and is independent of the voltage source.

In Section II of this article it was shown that for networks consisting of independent current sources and resistors, the
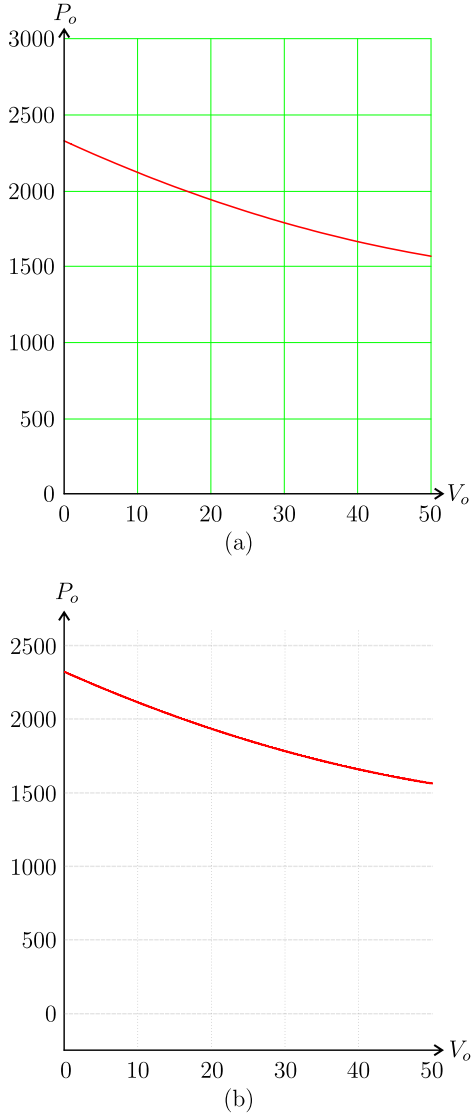
FIGURE 22. Power dissipated internally against voltage V_a: (a) for the proposed equivalent circuit and (b) for the actual network.

Theorem: Any linear DC network consisting of independent voltage and current sources, and resistors, with two accessible terminals (Fig. 23(a)) can be replaced by an equivalent circuit with a DC voltage source V_{T1}, a DC current source I_{N1} and three resistors R_X, R_y and R_T (Fig. 23(b)). The voltage V_{T1} is measured at the open terminals, with all internal current sources of the network set equal to zero. The current I_{N1} is the current through a short circuit applied to the external terminals, with all network internal voltage sources set equal to zero. The resistance R_X associated in parallel with the voltage source $VT1$ is given by the equation $R_X = \frac{V_{T1}{}^2}{P_X}$, where P_X is the power supplied by the internal voltage sources and dissipated internally in the circuit with the external terminals open and all internal current sources set equal to zero. The resistance R_y associated in series with the current source I_{N1} is given by $R_y = \frac{P_y}{I_{N1}{}^2}$, where P_y is the power supplied by the internal current sources and dissipated in the network, with a short circuit applied to the external terminals and all internal voltage sources set equal to zero. The resistance R_T is that measured at the terminals with all internal voltage replaced by a short circuit and all internal current sources replaced by an open circuit.

FIGURE 23. The power conservative unified equivalent circuit of DC networks consisting of independent voltage and current sources, and resistors.

value of R_y is determined by (26). When all internal current sources are set equal to zero except I_1, the resistance R_y is determined by
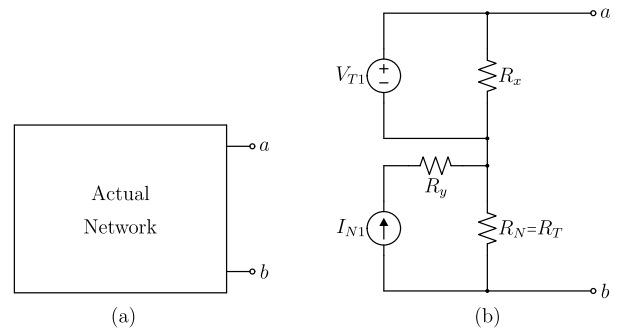
$$R_y = \frac{G_{22}\left(G_{11}G_{22} - G_{12}{}^2\right)}{\left(G_{12}G_{2a} - G_{22}G_{1a}\right)^2} \qquad (43)$$

This result then shows that the value of R_y, for networks containing only one current source and resistors, is also dependent only on the internal resistances and independent of the current source.

V. A GENERAL NETWORK THEOREM
In the previous sections, the theorem proof formulated below was presented, which unifies the extension of Thevenin theorem presented in [16], with the extension of the Norton theorem presented in section II of this paper.

VI. CONCLUSIONS
This paper proposes a new equivalent circuit for DC networks formed by independent voltage and current sources, and resistors. that is power conservative in the sense that its internal losses are equal to the actual network internal losses. It is demonstrated that the traditional Thevenin and Norton equivalent circuits are particular cases of the proposed circuit and can obtained form it by ignoring the internal resistors responsible for the dissipated power components that are independent of the power transferred to the component, device or system connected at the accessible external terminals. The proposed equivalent circuit is universal for DC networks consisting of independent voltage and current sources, and resistors and can be used to determine the actual network power and efficiency.

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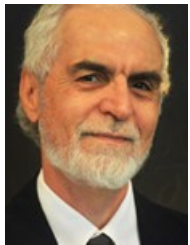
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