An analytical method for estimation of parameters of Self Mixing Interferometric phase equation over all feedback regimes

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Abstract. This paper presents a novel algorithm for measuring the linewidth enhancement factor of semiconductor lasers and the optical feedback level factor in a semiconductor laser with an external cavity. The proposed approach is based on analysis of the self-mixing phase equation to deduce equations for finding parameters given only knowledge of the perturbed phase. The effectiveness of the method has been validated with accuracy of 8.6% and 1.7% for C and α respectively while covering all feedback regimes.

1. Keywords

Linewidth enhancement factor (LEF), optical feedback, self-mixing interferometry, semiconductor lasers

2. Introduction

In optical feedback self-mixing interferometry(OFSMI), the instantaneous distance between the laser semiconductor diode driven by a constant injection current and a remote surface which back-scatters a small amount of optical power back into the laser diode cavity[7]. Linewidth enhancement factor (LEF) (denoted as α) is a fundamental parameter of self-mixing interferometry as it characterizes the linewidth, the chirp, the injection lock range, and the response to optical feedback [5].The influence of the parameters C and α on the emitted laser intensity has been extensively analyzed in [1] and [2]. Establishing an accurate measurement has been a challenging and active research topic that has attracted extensive research work during the past two decades [5]. Existing approaches include the direct physical measurement of the subthreshold optical spectrum as the injected current is varied [3] and techniques based on the analysis of the locking regimes induced by optical injection from a master laser [4]. Moreover, an analytic method, based on gradient descent approach was presented [6], which showed the accuracy of 6.7% and 4.63% for C and respectively.

3. Main theory

When the optical feedback phenomenon occurs, the laser wavelength is no longer the constant λ_o but is slightly modified and becomes a function of time $\lambda_f(t)$ when D(t) varies. The wavelength fluctuations can be found by solving the phase equation,

$$x_o(t) = x_f(t) + C\sin(x_f(t) + arctan(\alpha)) \tag{1}$$

x_o and x_f are referred as perturbed and unperturbed phase respectively, α is the linewidth enhancement factor and C is the coupling factor. x_o and x_f can be represented as,

$$x_o(t) = 2\pi\nu_o(t)\tau(t) \tag{2}$$

$$x_f(t) = 2\pi\nu_f(t)\tau(t) \tag{3}$$

where $\tau(t) = 2D(t)/c$ is the round trip time, with c as speed of light. $\nu_f(t)$ and $\nu_o(t)$ represents optical frequencies with and without feedback. Laser feedback output optical power(LDOOP) P(t) depends on the SM phenomenon and written as:

$$P(t) = P_o[1 + m\cos(x_f(t))]$$

where P_o is the power emitted by the free-running state laser diode and m is the modulation index. Therefore, for purpose of displacement measurement, we track from SM signal P(t) measurement toward perturbed phase $x_f(t)$ to $x_o(t)$ toward displacement D(t) measurement.Due to environmental fluctuations and uncertainty, it is desirable to estimate parameters C and α for a given condition of the interferometer for robust displacement measurement.

4. Theoretical derivation

4.1. Estimation of alpha

Taking derivative on both sides of (1),

$$\frac{dx_o}{dt} = \frac{dx_f}{dt}(1 + C\cos(x_f + \arctan(\alpha))) \tag{4}$$

$$\frac{\frac{dx_o}{dt}}{\frac{dx_f}{dt}} = 1 + C\cos(x_f + \arctan(\alpha)) \tag{5}$$

When extremas of x_o and x_f are reached simultaneously, then their derivatives approach zero simultaneously and their ratio approaches 1 by taking limit on time,i.e, when $t = t_{ext}$,

$$lim_{t\to t_{ext}} \frac{\frac{dx_o}{dt}}{\frac{dx_f}{dt}} = 1 \tag{6}$$

Considering when $t = t_{ext}$,then $x_f = x_{fe}$ and $x_o = x_{oe}$, Putting in (5),

$$\cos(x_{fe} + \arctan(\alpha)) = 0 \tag{7}$$

$$x_{fe} + \arctan(\alpha) = k\pi - \frac{\pi}{2} \tag{8}$$

Putting equation(8) in equation(1),as $\sin(k\pi - \frac{\pi}{2}) = \pm 1$

$$x_{oe} = x_{fe} \pm C \tag{9}$$

From equation(1), $x_o - x_f$ is maximally bounded by constant C,when x_f is allowed to increase from 0 to some value (less than $\frac{\pi}{2} - \arctan(\alpha)$).Therefore, on local maximas of displacement, x_o swings x_f by C. Lets denote maximas of x_o and x_f as x_{om} and x_{fm} respectively,then on local maximas of displacement,

$$x_{om} = x_{fm} + C \tag{10}$$

Putting in equation(1),

$$x_{fm} + C = x_{fm} + C\sin(x_{fm} + \arctan(\alpha)) \tag{11}$$

$$\sin(x_{fm} + \arctan(\alpha)) = 1 \tag{12}$$

$$x_{fm} + \arctan(\alpha) = \tfrac{\pi}{2} + 2k\pi \tag{13}$$

where, k is an integer.

$$\alpha = \tan(\tfrac{\pi}{2} + 2k\pi - x_{fm}) \tag{14}$$

Due to periodicity of tan function by $2k\pi$, equation (12) becomes,

$$\alpha = \tan(\tfrac{\pi}{2} - x_{fm}) \tag{15}$$

$$\alpha = \tan(\arcsin(1) - x_{fm}) \tag{16}$$

4.2. Estimation of C

Now, the perturbed phase (x_f) being modified form of x_o exhibits sharp transitions (from moderate to high feedback regime but it can be extended to all regimes once the algorithm is developed), then at those transitions derivative of x_f approaches infinity. Let x_f be represented as x_{ft} on transitions of perturbed phase, then equation (5) becomes,

$$1 + C\cos(x_{ft} + \arctan(\alpha)) = 0 \tag{17}$$

$$C = \tfrac{-1}{\cos(x_{ft}+\arctan(\alpha))} \tag{18}$$

5. Algorithm Design

5.1. Algorithm for alpha estimation

Since derivative of $\arcsin(t)$ is given by,

$$\tfrac{d(\arcsin(t))}{dt} = \tfrac{1}{\sqrt{1-t^2}} \tag{19}$$

Around t=1, $\frac{d\arcsin(t)}{dt}$ becomes quite large and sensitive, and slight deviations can lead to huge errors.Therefore, we can add counter sensitivity term as ϵ which has following properties.

$$\epsilon << 1 \text{ and } \epsilon \geq 0 \tag{20}$$

So, equation(14) becomes modified as,

$$\alpha = \tan(\arcsin(1-\epsilon) - x_{fm}) \tag{21}$$

If we are given information that α is bounded by some threshold '$\tau \geq 0$' and 0, then we find α by following algorithm

Step 1: Initially choose $\epsilon = 0$ and step size of varying it as $\delta\epsilon$

Step 2: For a given sample of perturbed phase x_f, find the maximum positive element as x_{fm}, which would be global maxima of $x_f(t)$.

Step 3: Plug the values into equation(21).If α is outside $[0, \tau]$, then increase ϵ to $\epsilon + \delta\epsilon$ and repeat Step 3.Otherwise, α would be required estimation.

5.2. Algorithm for C estimation

Due to sharpness of SM signal,a slight deviation of arguments of equation(18), can create huge errors. Due to this non-smooth property,we have to use statistic of finding globally sharpest transition and exploiting equation (18).

Step 1: For suitable magnitude threshold M, calculate derivative of sample of x_f as $\frac{dx_f}{dt}$ and find the set X as,

$$X = \{x_f(t) | t = arg[\frac{dx_f}{dt} \geq M]\} \tag{22}$$

Step 2: Plug the values into equation(18), then estimated value of C is,

$$C = max(\frac{-1}{cos(x + \arctan(\alpha))}) \ , \forall x \in X \tag{23}$$

6. Simulation Results

6.1. Estimation of C and alpha

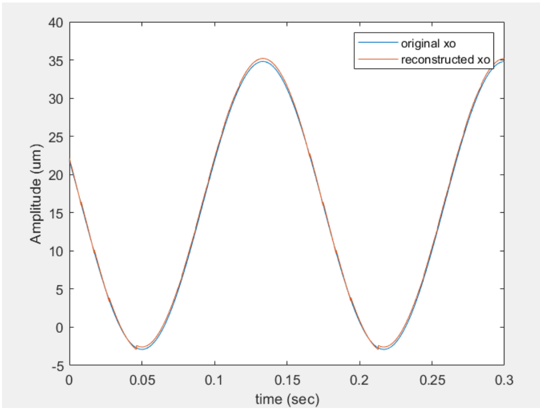

Figure 1. Reference unperturbed phase and reconstructed unperturbed phase for a displacement of 6kHz

6.1.1. Remark The above-mentioned algorithm can be extended and conjugated with an optimization algorithm, as depicted in [6], which would decrease the number of iterations for convergence and increase accuracy. Moreover, an obvious observation is that an arbitrary smoothing digital filter could increase accuracy when reconstructing displacement.

6.2. Reconstruction of unperturbed phase and displacement

For a sinusoidal displacement of 3 peak to peak μm and 6kHz, sampled at 10kHz with reference values of C and α as 4 and 5 respectively, on applying mentioned algorithms with $\tau = 0.02$ and $M = 10000$, we get a reconstructed unperturbed phase which gives mean square error of 32.4nm, by utilizing equation(1), as shown in figure(2) and (3) and reconstructed displacement is mentioned in figure(4). Displacement is reconstructed by using equation (2).
Similarly, for a sinusoidal displacement of 3 peak to peak μm and 6kHz,12kHz and 18kHz, sampled at 10kHz with reference values of C and α as 4 and 5 respectively, on applying mentioned algorithms with $\tau = 0.19$ and $M = 10000$, we get a mean square error of 96.1nm and results are mentioned in figure(5), (6) and (7), after utilizing equation (1) and (2). To work with the mentioned method, the "feedback regimes" term would be ignored and values of C and α would be randomly experimented with, from 1 to 9 for C and 2 to 5 for α respectively. For simulation, a sample of the perturbed phase corresponding to a sinusoidal displacement of 3 peak to peak

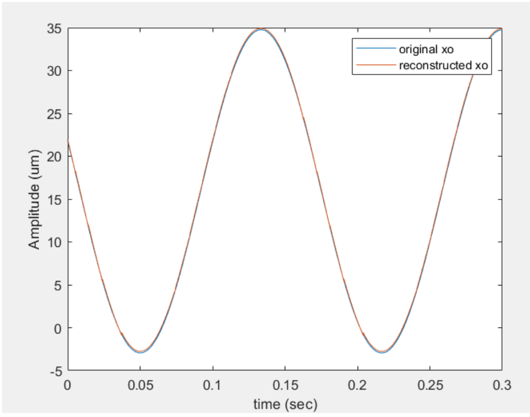

Figure 2. Plot of reference unperturbed phase and reconstructed unperturbed phase based on estimated C and α for a displacement of 6kHz

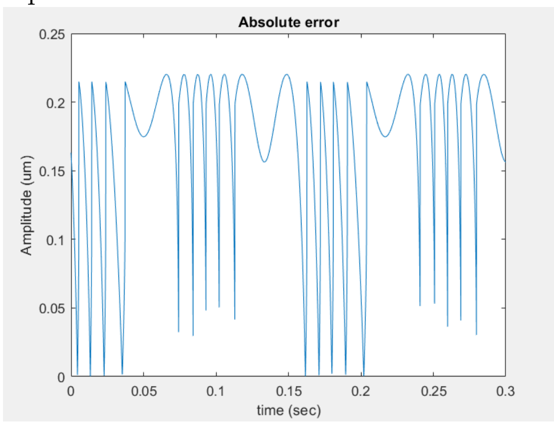

Figure 3. Plot of absolute error between reference unperturbed phase and reconstructed unperturbed phase based on estimated C and α for a displacement of 6kHz

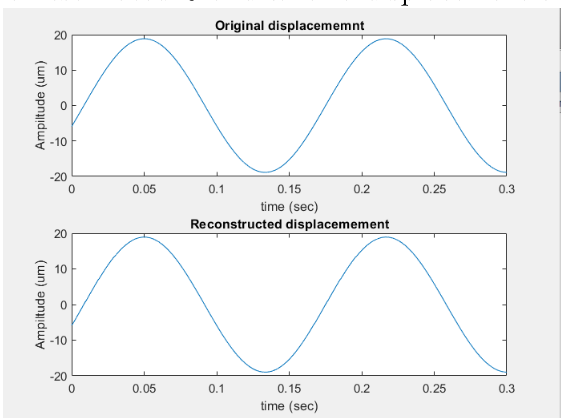

Figure 4. Reference displacement and reconstructed displacement for a displacement of 6kHz

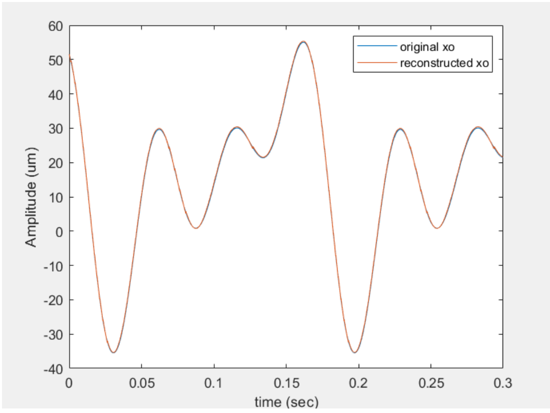

Figure 5. Plot of reference unperturbed phase and reconstructed unperturbed phase based on estimated C and α for a displacement having 6kHz,12kHz and 18kHz components

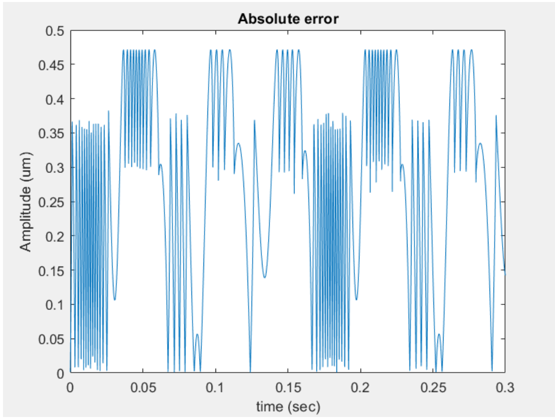

Figure 6. Plot of absolute error between reference unperturbed phase and reconstructed unperturbed phase based on estimated C and α for a displacement having 6kHz,12kHz and 18kHz components

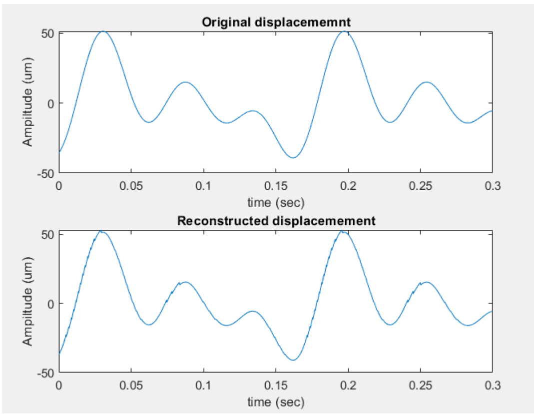

Figure 7. Reference displacement and reconstructed displacement for a displacement having 6kHz,12kHz and 18kHz components

μm and 6kHz, sampled at 10kHz, was used. The following table represents values of estimated values of C corresponding to reference C values, with fixed $\alpha = 5$, along with $\tau = 0.02$ and $M = 10000$ found by hit and trial.

Estimated C($\alpha = 5$)	Reference C($\alpha = 5$)	Percentage error %
1.1708	1	17
2.3401	2	17
4.0779	4	1.9475
5.4588	5	9.17
6.6384	7	5.7
9.1011	9	1.11

The average error for above table is 8.67%, for estimating C independently.
Similarly, following table presents estimated values of α corrosponding to reference values, when C is kept constant at 5

Estimated α(C=5)	Reference α(C=5)	Percentage error %
2.1098	2	5.49
3.0080	3	0.2667
4.0013	4	0.0325
5.0620	5	1.24

The average error for above table is 1.75%, for estimating α independently.
Simultaneous prediction of values of C and α corresponding to reference values are presented in following table.

Estimated α,C	Reference α,C	Percentage error %
3.4005, 4.2041	3,4	13.35, 5.1
4.1603, 5.1343	3,5	38.6, 2.6
5.2962, 6.1946	4,6	32.25, 3.16
5.2674, 7.1031	5,7	5.2, 1.42

The average error for estimating C and α simultaneously is 3.07% and 22.35% respectively. It should be noted that the above-presented data covers all feedback regimes.
Even though some errors appear large, for example, for reference C=6, α=4, give percentage errors of 3.16% and 32.25% respectively, when used for reconstruction of unperturbed phase from equation(1) gives means square error of 100nm, which is quite reasonable, and its depiction is presented in figure(1).

7. Conclusion

Therefore, based on SMI phase equation (1), the algorithm has been developed for estimation of C and α and is useful for reconstruction of unperturbed phase from sole knowledge of perturbed phase, for measurement of displacement of the target, under situations where C and α are known or vary with the accuracy of 8.67% and 1.75% respectively, when independent estimation and 22.35% and 3.07% for simultaneous estimation. But even still, the mean square error bound for reconstructed displacement is less than 100nm.

8. Acknowledgment

This study was not funded.

9. Conflict of Interest

The author declares that he has no conflict of interest.

10. References

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