Supplementary material: additional heat paper

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1 Stability of the EBM to sea ice perturbations

From equation (5) in the main text, the temperature response to a forcing is

$$\delta\langle\overline{T}\rangle = \frac{F_{ghg}}{B - \langle\overline{S\partial a/\partial T}\rangle}. \tag{1}$$

Since $\partial a/\partial T > 0$ (Wagner & Eisenman, 2015), equation (5) in the main text has a solution only if $\langle\overline{T}\rangle$ is of the same sign as F_{ghg}. If it is of opposite sign, there is no equilibrium solution when a forcing F_{ghg} is applied. Therefore, a stable equilibrium solution of the EBM has the property

$$B > \langle\overline{S\partial a/\partial T}\rangle. \tag{2}$$

In other words, sea ice perturbations cannot be self sustaining in a stable climate.

2 Pattern scaling calculation

Blackport and Kushner (2017) show that for a simulation representing a future warmed climate with LLW $\delta T_{l,ghg}$ and SIL δI_{ghg}, and a sea ice perturbation simulation with LLW $\delta T_{l,pert}$ and SIL δI_{pert}, the sensitivities of some field Z to these two parameters are given by

$$\begin{pmatrix} \left.\frac{\partial Z}{\partial T_l}\right|_I \\ \left.\frac{\partial Z}{\partial I}\right|_{T_l} \end{pmatrix} = \frac{1}{\delta I_{pert}\delta T_{l,ghg} - \delta I_{ghg}\delta T_{l,pert}} \begin{pmatrix} -\delta I_{ghg} & \delta I_{pert} \\ \delta T_{l,ghg} & -\delta T_{l,pert} \end{pmatrix} \cdot \begin{pmatrix} \delta Z_{pert} \\ \delta Z_{ghg} \end{pmatrix}. \tag{3}$$

Considering the EBM, the partial temperature response to LLW is

$$\frac{\partial T}{\partial T_l} = \frac{\delta I_{pert}\delta T_{ghg} - \delta I_{ghg}\delta T_{pert}}{\delta I_{pert}\delta T_{l,ghg} - \delta I_{ghg}\delta T_{l,pert}} \tag{4}$$

Assuming the sea ice perturbation method accurately achieves the target, $\delta I_{pert} = \delta I_{ghg}$ and $\delta(aS)_{pert} = \delta(aS)_{ghg}$. We also assume that there is little LLW in the sea ice perturbation simulation, i.e. $\delta T_{l,pert} \ll \delta T_{l,ghg}$, to simplify the denominator. This gives

$$\frac{\partial T}{\partial T_l} \approx \frac{\delta T_{ghg} - \delta T_{pert}}{\delta T_{l,ghg} - \delta T_{l,pert}} \tag{5}$$

From equation (2), the global mean annual temperature response in the FUTURE EBM simulation is

$$\delta\langle\overline{T}\rangle_{ghg} = B^{-1}\left(\delta\langle\overline{aS}\rangle_{ghg} + F_{ghg}\right), \tag{6}$$

and the temperature response in the perturbation simulation is

$$\delta\langle\overline{T}\rangle_{pert} = B^{-1}\left(\delta\langle\overline{aS}\rangle_{ghg} + \langle\overline{F_{pert}}\rangle\right), \tag{7}$$

where F_{pert} is the artificial heat flux in any of the perturbation methods. Taking the global and annual mean of (5) and substituting these expressions, we obtain

$$\frac{\partial\langle\overline{T}\rangle}{\partial T_l} \approx \frac{B^{-1}\left(F_{ghg} - \langle\overline{F_{pert}}\rangle\right)}{\delta T_{l,ghg}}. \tag{8}$$

$\partial T/\partial I$ is obtained by the same procedure. Assuming $\delta I_{pert} = \delta I_{ghg} \equiv \delta I$ yields

$$\frac{\partial T}{\partial I} = \frac{\delta T_{l,ghg}\delta T_{pert} - \delta T_{l,pert}\delta T_{ghg}}{\delta I(\delta T_{l,ghg} - \delta T_{l,pert})}. \tag{9}$$

Assuming little LLW in the perturbation simulation, taking the global mean, and substituting equations (6) and (7) gives

$$\frac{\partial\langle\overline{T}\rangle}{\partial I} \approx \frac{1}{B}\frac{\delta\langle\overline{aS}\rangle_{ghg} + \delta\langle\overline{F_{pert}}\rangle - \left(\delta\langle\overline{aS}\rangle_{ghg} + F_{ghg}\right)\left(\delta T_{l,pert}/\delta T_{l,ghg}\right)}{\delta I\left(1 - \delta T_{l,pert}/\delta T_{l,ghg}\right)}. \tag{10}$$

Assuming $\delta T_{l pert} \ll \delta T_{l ghg}$, this becomes

$$\frac{\partial \langle \overline{T} \rangle}{\partial I} \approx \frac{B^{-1}\left(\delta \langle \overline{aS} \rangle_{ghg} + \delta \langle \overline{F_{pert}} \rangle\right)}{\delta I}. \tag{11}$$

We obtain the EBM sensitivities to the new parameters F_{ice} and F_{ghg} the same way, except that the only assumption required to obtain the expressions in the text is that the perturbation simulation accurately achieves the target sea ice state, so that $\delta(aS)_{pert} = \delta(aS)_{ghg}$.

3 LLW vs. F_{ghg} as a scaling parameter

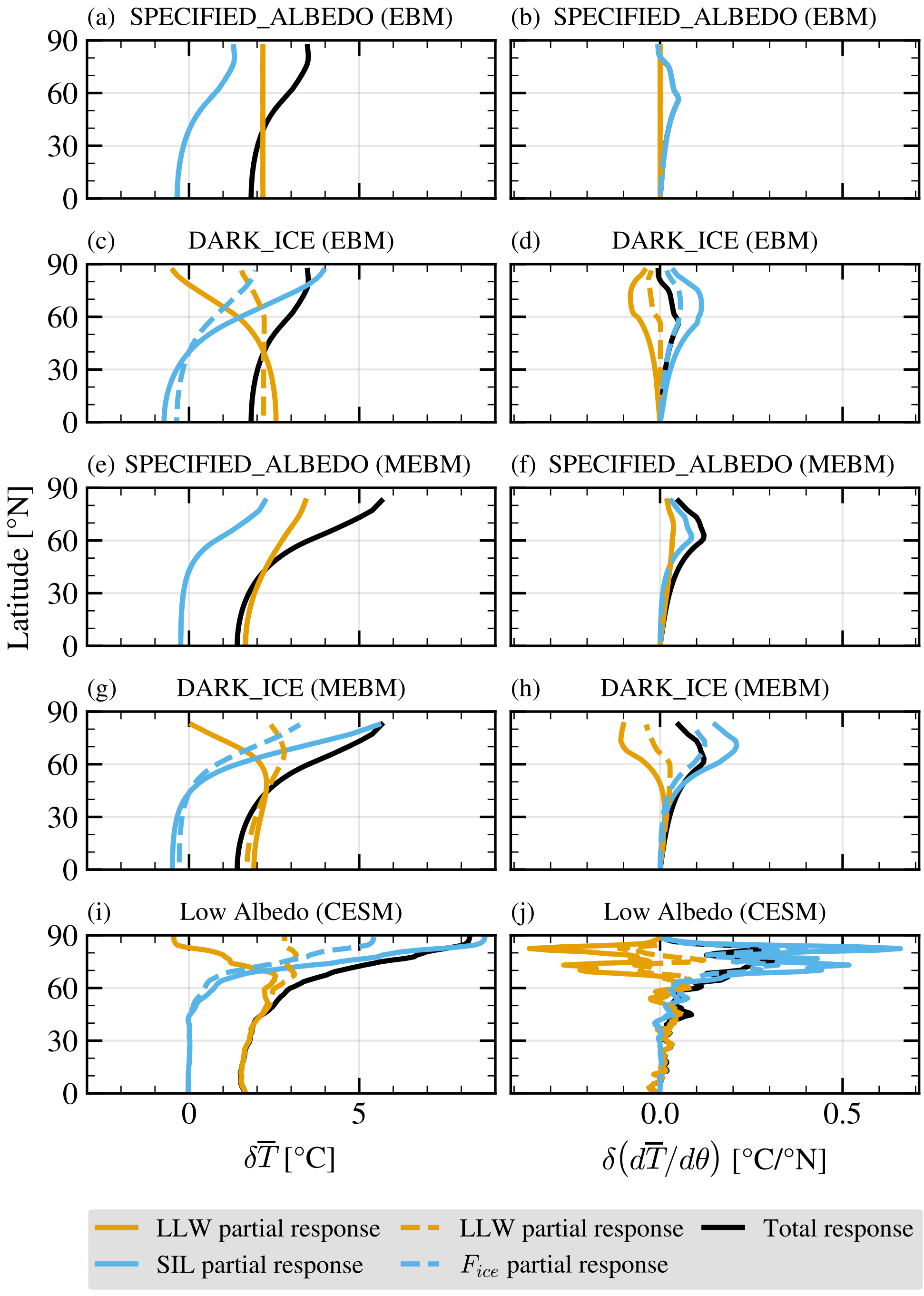

Figure S1: As in Figure 2, but dashed gold and blue curves show the partial responses to LLW and F_{ice} (as opposed to F_{ghg} and F_{ice}), respectively. The main difference between the two sets of plots is a global mean offset in the dashed curves, which has no bearing on our conclusions.

4 Accounting for additional heat in nudging simulations

In addition to the modified albedo simulations, we repeated our analysis on nudging simulations in the EBMs and in CESM. In this case, we define F_{ice} differently from the albedo modification case. In nudging simulations, we cannot define F_{ice} as the simple change in net TOA shortwave - this would only reflect physical changes in albedo and would not capture the artificial heat added by nudging. Instead, we add the nudging heat flux to the TOA shortwave change, giving $F_{ice} = S\delta a + F_{nudge}$. In the hybrid nudging scheme (Audette & Kushner, 2022), $F_{nudge} = \delta F_{hyb} + L_f h_{thin} \delta SIC$, where F_{hyb} is the heat flux applied to all categories of sea ice in each grid cell, L_f is the latent heat of fusion of seawater, and h_{thin} is the mean thickness of the thinnest category of sea ice in each grid cell. Using this parameter to account for the additional heat is not as clean as our definition of F_{ice} in albedo modification simulations, because $S\delta a$ and F_{nudge} represent different processes. In comprehensive models, the nudging flux is seen only by the sea ice model, while the net TOA shortwave directly affects the entire atmospheric column and the surface. This is in contrast to $F_{ice} = S\delta a$ in albedo modification simulations, where we used the change in TOA shortwave to capture both the shortwave forcing from the physical albedo feedback and from artificial darkening of the ice, both of which are seen by the whole model.

Nonetheless, using F_{ice} as a scaling parameter successfully accounts for the artificial heat in the EBMs (top four rows of Figure S2). This is because the EBM is too simple for a nudging flux to be applied only to the sea ice component, so the nudging flux directly affects the surface energy balance, and the above-mentioned caveat does not apply in this model. In contrast, scaling by F_{ice} in the WACCM hybrid nudging simulations does not properly account for the artificial heat (bottom row of Figure S2). The new scaling parameter attributes nearly the entire surface temperature response to LLW, and almost no warming to SIL. This feature is also present in the air temperature and zonal wind fields (not shown).

Examining the F_{nudge} and $S\delta a$ fields in the hybrid nudging simulations reveals that they should not be added on equal footing. Figure S3 shows that the total nudging flux from 70-90°N in pa-futArcSIC is more than twice the total change in TOA shortwave integrated over the same region, so that artificial heat accounts for about 70% of F_{ice}. By comparison, we estimate that artificial heat accounts for about 30% of F_{ice} in Low Albedo. One interpretation of this large nudging flux is that

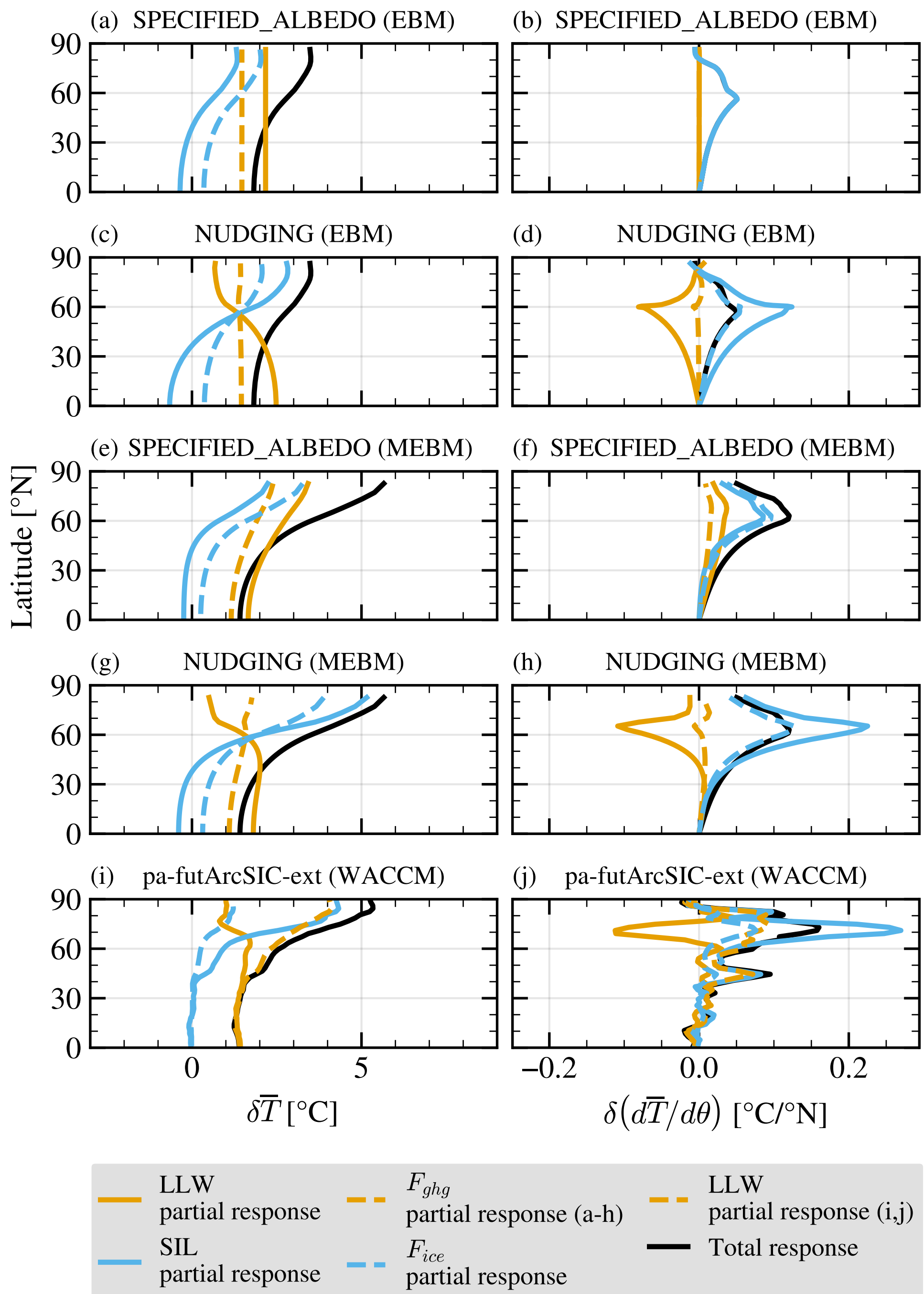

Figure S2: As in Figure 3, but for the nudging simulations in the EBM (top four rows) and the CESM-WACCM hybrid nudging simulations (bottom row).

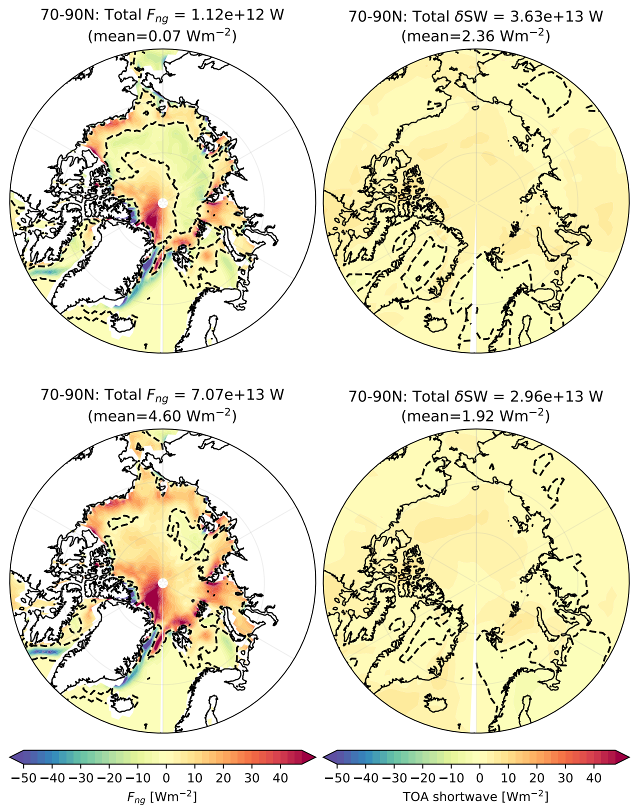

Figure S3: The heat flux added by the hybrid nudging method (a) compared to the change in net TOA shortwave (b). Both quantities are differences from the pa-pdSIC control simulation (a nonzero nudging flux is added in that simulation to achieve the desired control ice conditions). In the hybrid nudging method, F_{nudge} is the sum of a heat flux added to the bottom of the sea ice and implicit latent heat added by directly converting thinnest category ice to freshwater (Audette & Kushner, 2022).

the artificial heat added by the nudging method is inducing a huge spurious response, responsible for almost the entire climate response according to pattern scaling (Figure S2). This is unlikely, given that nudging methods give similar climate responses to the albedo modification method (Sun et al., 2020). Rather, it seems that we have not chosen the correct scaling parameter for the nudging method. Because it is only seen by the sea ice model, a unit of nudging flux probably does not have as great an influence on the climate system as a unit change in net TOA shortwave. It would be interesting if a scaling parameter that properly accounts for the heat added by all perturbation methods could be found, but that is not the focus of this work.