

Multi-UAV cooperative air combat target assignment method based on VNS-IBPSO algorithm in complex dynamic environment

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Abstract: Effective target assignment plays a crucial role in maximizing the efficiency and success of cooperative air combat involving multiple UAVs in complex and dynamic environments. Accurate target threat assessment is essential for successful target assignment. This study proposes a threat assessment method that considers multiple threat factors of UAV targets and introduces an uncertain information representation technique using interval-valued intuitionistic fuzzy number. To achieve the fusion of multi-moment target information, weights are assigned to the time series using the normal distribution method. Furthermore, a weight optimization model is presented to integrate the threat factor weights obtained through the AHP method and the entropy method. For solving the multi-weapon multi-target assignment problem, a target assignment method based on the VNS-IBPSO algorithm is introduced. This method improves upon the limitations of the BPSO algorithm, such as limited local search capability and premature convergence, by combining variable neighborhood search (VNS) and an improved binary particle swarm optimization algorithm (IBPSO). The effectiveness of the proposed method is validated through simulation experiments, which demonstrate its ability to quickly and accurately complete target assignment tasks. This method provides an effective solution for the coordination task allocation of multi-UAV cooperative air combat.

Key words: multi-weapon multi-target assignment, threat assessment, interval-valued intuitionistic fuzzy number, weight optimization model, VNS-IBPSO

1 Introduction

With the rapid development of artificial intelligence and UAV technology, multi-UAV cooperative air combat has gradually become an important development trend of future war. It can not only improve combat efficiency and reduce combat losses, but also reduce casualties, and has broad application prospects. Air combat decision making is the key for UAVs to win in air combat^[1], and is a dynamic process that changes with the battlefield environment, requiring real-time adjustments to maximize the effectiveness of cooperative operations based on external interference and various internal uncertainties^[2]. In this process, as the core components of the air combat decision system, target threat assessment and weapon target assignment are key technologies that affect the ultimate operational effectiveness.

In a multi-UAV cooperative air combat scenario, target threat assessment refers to the evaluation of the combat power elements of both sides based on their situational information, and thus the overall battlefield posture. Accurate and reasonable target threat assessment results provide an important basis for subsequent operational decisions, such as weapon target assignment, cooperative maneuver decisions and equipment configuration. Several research methods have been used in the past to address this problem. For example, Hierarchical analysis method(AHP)^[3, 4, 5], fuzzy sets theory^[6, 7], TOPSIS method^[8, 9] and multiple attribute decision making^[10, 11] and so on. However, the above methods do not take into account the effects of changes in air combat situational

information. Therefore, some methods for processing situational information under dynamic conditions have been proposed. For instance, Feng et al.^[12] proposed an improved generalized intuitionistic fuzzy soft set (GIFSS) method for dynamic assessment of air target threat. Wang et al.^[13] used dynamic Bayesian network inference to estimate the target threat at different time slices. Zhang et al.^[14] obtained time series weights by a Poisson distribution method based on multiple target posture data. However, these methods do not account for the uncertainty of air combat situational information. The complexity of target threat assessment is mainly reflected in two aspects. On the one hand, due to the need to use multi-sensors to detect the threat factor information of enemy UAVs, the information obtained by different sensors has errors due to their own performance limitations and external environmental influences, and it is difficult to use accurate values to represent. On the other hand, with the continuous improvement of the stealth performance of the UAV target, it is difficult for us to capture all the performance parameters of the enemy target^[15]. As a result, the target situational information is time-varying, uncertain and incomplete, which is manifested as mixed situational information. How to reasonably characterize and process the mixed situational information in a dynamic and complex environment is the main content of this study.

Weapon target assignment problem is a typical combinatorial optimization problem, and has been proved to be an NP-complete problem^[16]. There are two main types of algorithms to solve this: exact solution algorithm and approximation solution algorithm. The accurate solution algorithms include branch and bound method^[17, 18], Hungarian algorithm^[19, 20, 21], auction algorithm^[22] and so on. These algorithms can compute the exact optimal solution, have strong interpretability and are easy to implement. However, its limitation is that it is difficult to solve large-scale WTA problems. It is necessary to design corresponding accurate mathematical models according to specific problems, and the solving steps are cumbersome. The approximate solution algorithms include rule-based heuristics^[23], LaGrange relaxation methods^[24], metaheuristics^[25], and machine learning algorithms^[26, 27, 28]. Among them, the heuristic intelligent optimization algorithm is most widely used to solve the WTA problem, a large number of research applications verify the good optimization effect of the heuristic algorithm in this problem. Including genetic algorithm(GA)^[29], particle swarm optimization(PSO)^[30], artificial bee colony algorithm(ABC)^[31], ant colony optimization algorithm(ACO)^[32], grey wolf optimization algorithm(GWO)^[33] and hybrid intelligent search algorithm^[34], etc. The advantage of this kind of algorithm is that the algorithm framework is easy to implement, can search the solution space in a wide range, and can solve large-scale problems. However, its limitation is that the algorithm takes a long time to solve, the performance is not stable enough, and it easily falls into a local optimum.

Multi-UAV cooperative air combat is one of the typical operational forms of weapon target assignment problem applications, which belongs to the multi-weapon multi-target assignment problem and is a higher dimensional problem with more criterion. Zhen et al.^[35] proposed an improved cooperative target assignment scheme based on a contract network protocol for target attack mission of heterogeneous UAV swarm. Xing et al.^[36] proposed a self-organized offense-defense confrontation decision-making algorithm for a dynamic swarm versus swarm UAV combat problem. Song et al.^[37] established a realistic UAV-target assignment model and proposed a differential evolution algorithm. They also developed the corresponding gene coding method, which solved the two special situations of the UAV not performing the mission and the target not being attacked. Different UAVs can carry different weapons and have different attack capabilities. The focus of this study is on how to reasonably match and utilize different types of weapon resources to ensure maximum combat benefit at minimum cost, and how to quickly and accurately find the optimal allocation scheme through intelligent optimization algorithms.

In this study, to solve the UAVs target threat assessment and multi-weapon multi-target assignment (MWMTA) problem for cooperative UAV operations in the context of over-the-horizon attack. In terms of threat assessment. A UAV target threat assessment model is constructed, representative threat factors are selected, and interval-valued intuitionistic fuzzy number characterization is used to address the uncertainty and incompleteness in the threat factor information; For the time-varying nature of the enemy posture, time series weights are generated based on the normal cumulative distribution to fuse multi-moment posture information. At the same time, an indicator weight optimization model integrating AHP method and entropy weight method is proposed, and the subjective and objective weight characteristics are comprehensively considered; An evaluation process based on dynamic interval-valued intuitionistic fuzzy multi-attribute decision is given, and the effectiveness and rationality of this method is verified by simulation. Regarding the MWMTA problem, the global utility function is calculated based on the threat posture indicator parameters of enemy UAVs relative to our UAVs and the destructive effectiveness of our UAV weapon resources on enemy UAVs. A MWMTA model is constructed based on the global utility function; The update strategy of BPSO is analyzed in depth, and the reason why BPSO has strong global exploration capability and lacks local exploration capability is found based on the formula derivation. By improving the update strategy of the BPSO and introducing the VNS operator, the search capability of BPSO is improved; Through simulation analysis and comparative experiments, it is proved that the VNS-IBPSO algorithm has good stability, rapidity and convergence, and can solve the multi-UAV cooperative air combat target assignment problem in complex dynamic environment quickly and accurately.

The rest of this article is organized as follows. In the second section, the multi-UAV cooperative air combat target threat assessment and MWMTA problems are described and modeled, and the uncertain information representation method is introduced. In the third section, a time-series weighting model and a threat indicator weighting optimization model are proposed, and a specific threat assessment process is given. In the fourth section, an innovative VNS-IBPSO algorithm is proposed. In the fifth section, simulations are performed to verify the effectiveness of our proposed algorithm. Finally, conclusions are given in the sixth section.

2 Problem description and modeling

A brief description of a multi-UAV coordinated air combat scenario is given. To simplify the problem, it is reasonable to assume that enemy target situational information has been obtained through the search phase. Assume that the number of our UAVs is m and number of enemy UAVs is n . The total number of weapon resources carried by all UAVs is q and weapon resources carried by each UAV is $q_i, i \in \{1, 2, \dots, m\}$. Our UAVs continuously obtain K time slices of situational information, time set is noted as $t = \{t_1, t_2, \dots, t_k\}$. Considering the threat factors of the enemy target and establishing threat assessment model. Through the proposed threat assessment method, the dynamic threat assessment of the enemy UAV target is carried out. After obtaining the comprehensive threat assessment value, the next weapon target allocation can be carried out. A list of key symbols used hereafter is provided in Table 1.

Table 1. Symbol Definitions

Symbol	Definitions
m	The number of our UAVs
n	The number of enemy UAVs
q	The number of our weapons
s	The number of threat evaluation factors

q_i	The number of weapons carried by each UAV
q_j	The number of weapons assigned to each UAV
t_k	Moment t_k
$U_{q \times n}$	Weapon target assignment matrix
$Q_{q \times n}$	Target damage probability matrix
$V_{q \times n}$	The comprehensive threat assessment matrix of enemy UAVs to our UAVs
R_k^i	Target situational information matrix obtained by i^{th} UAV in moment t_k
p_{kj}	The damage probability of our k^{th} weapon to the j^{th} enemy target
S_{ij}	The threat assessment value of the j^{th} enemy UAV to our i^{th} UAV
o_{jl}	The l^{th} threat factor value of the j^{th} enemy UAV

Based on the above parameter information, the UAV cooperative air combat decision making flow is shown in Figure 1. Among them, the following key issues need to be addressed.

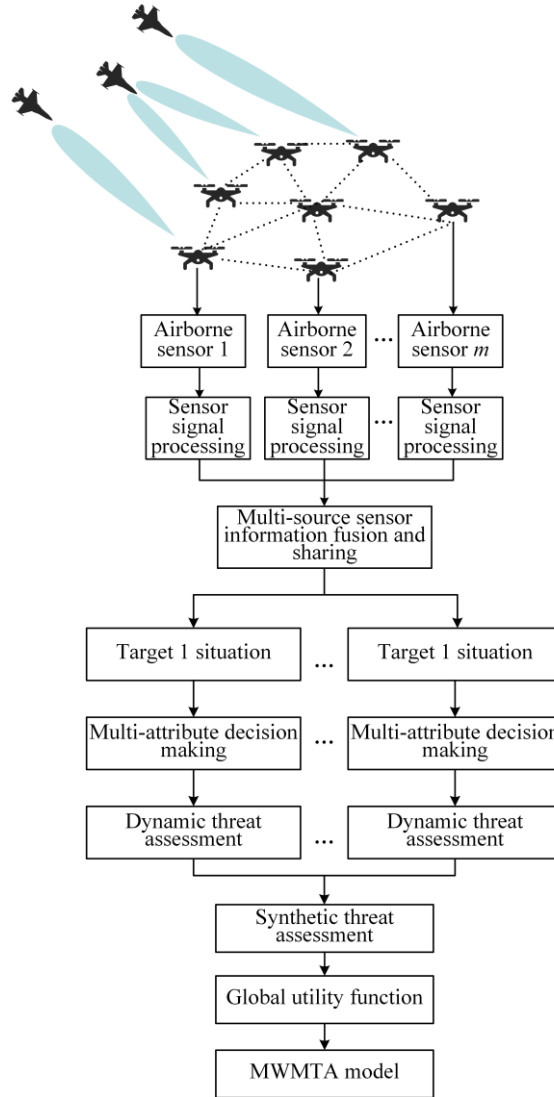


Figure 1 Multi-UAV cooperative air combat decision making flow

2.1 Selection of threat assessment factor for UAV targets

When conducting target threat assessment, the selection of threat assessment factor from target situational information should be considered first. Generally, it includes static threat assessment factor and dynamic threat assessment factor. Among them, the static threat assessment factor reflects the static attributes of the enemy UAV, such as target maneuverability, electronic countermeasure capability, combat radius, etc. which is generally a fixed value. The dynamic threat assessment factor mainly refers to the target motion situation information, which reflects the relative motion state of the enemy and our UAV, such as relative angle, speed, height, distance and so on. According to the actual characteristics of UAV cooperative air combat, the threat assessment factors are selected as follows.

1. Speed threat factor

The radial velocity of the enemy UAV target is selected as the Speed threat factor, and the near direction is positive and the far direction is negative. The faster the radial velocity of the UAV is, the stronger the maneuverability is, and the greater the offensive advantage is. At the same time, the faster the speed can make the UAV get rid of the target or complete the pursuit of the target. It belongs to benefit-oriented factor.

2. Height threat factor

The target height refers to the vertical nearest distance between the target height plane of the enemy UAV and our UAV. The UAV at a higher altitude will occupy a more favorable attack position, and the corresponding weapon load can also obtain higher kinetic energy through the conversion of potential energy. Therefore, the higher target, the greater threat to us. It belongs to benefit-oriented factor.

3. Distance threat factor

The target distance refers to the projection distance on the horizontal plane of the connection between the both sides. Usually, the closer the enemy UAV's target distance is, the more obvious the attack intention to us, the shorter defense time is, and the greater threat to us is. It belongs to cost-oriented factor.

4. Angle threat factor

The target angle threat can be described by the target entry angle. The target entry angle refers to the angle between the connecting line between the target and our UAV and target UAV speed direction. The smaller the entry angle is, the more obvious the target attack intention is and the greater the threat to us. It belongs to cost-oriented factor.

5. RCS threat factor

The enemy's stealth performance is directly related to whether it is detected by airborne sensors. The smaller the radar cross section (RCS), the better the enemy's stealth performance, the smaller the probability of being detected by airborne radar, and the greater the threat to us. It belongs to cost-oriented factor.

6. Type threat factor

The threat degree of different UAV types is different. In this study, the UAV targets types are considered according to the combat function, which can be divided into four categories: attack UAV, interference UAV, scout UAV and bait UAV.

2.2 Description of mixed situational information in treat assessment.

In a complex dynamic environment, due to the difficulty of data collection, the diversity of target behaviors, and the conflict of intelligence information, the target situational information is uncertain and incomplete. The use of fixed numerical forms to describe such uncertain information

in threat assessment may lead to inaccurate, oversimplified and misleading assessment results, which may affect the effectiveness of subsequent decision-making. Therefore, use interval-valued number, intuitionistic fuzzy number, exact number, categorical variables and other data forms to generate mixed situational information, so as to characterize the uncertainty and incompleteness of target situational information. Target situational information matrix obtained by l^{th} UAV in moment t_k is $R_{t_k}^i = [o_{jl}]_{n \times s}$, $j \in \{1, 2, \dots, n\}, l \in \{1, 2, \dots, s\}$. The l^{th} threat factor value of the j^{th} enemy UAV is o_{jl} , the representation of o_{jl} are as follows:

1. Interval number representation

Considering the detection error of the airborne sensor, the error is set to be $\Delta\tau$. Then it can be expressed by interval number: $o_{jl} \in [\underline{o}_{jl}, \bar{o}_{jl}]$, the upper limit of it is $\underline{o}_{jl} = \tilde{o}_{jl} - \Delta\tau$, the lower limit of it is $\bar{o}_{jl} = \tilde{o}_{jl} + \Delta\tau$. \tilde{o}_{jl} is the initial attribute value detected by the airborne sensor.

2. Intuitionistic fuzzy number representation

Considering that there are omissions or intelligence conflicts in the collection process of target situational information. Then it can be expressed by intuitionistic fuzzy number: $o_{jl} = \langle \mu_{jl}, \nu_{jl}, \pi_{jl} \rangle$ with μ_{jl} is membership degree and ν_{jl} is non-membership degree, $\pi_{jl} = 1 - \mu_{jl} - \nu_{jl}$ is the hesitancy degree. They calculated as follows:

If it belongs to benefit-oriented factor:

$$\mu_{jl} = \frac{\underline{o}_{jl}}{\sqrt{\sum_{j=1}^n (\bar{o}_{jl})^2}}, \nu_{ij} = \frac{\bar{o}_{jl}}{\sqrt{\sum_{j=1}^n (\underline{o}_{jl})^2}} \quad (1)$$

If it belongs to cost-oriented factor:

$$\mu_{jl} = \frac{1/\bar{o}_{jl}}{\sqrt{\sum_{j=1}^n (1/\underline{o}_{jl})^2}}, \nu_{ij} = \frac{1/\underline{o}_{jl}}{\sqrt{\sum_{j=1}^n (1/\bar{o}_{jl})^2}} \quad (2)$$

3. Classification variable representation

When the threat factor value is a categorical variable, it is usually described by linguistic variables such as 'very high', 'high', 'general', and 'low' based on domain knowledge. It needs to be transformed into the corresponding intuitionistic fuzzy number. The threat factor values corresponding to different types of UAV targets are shown in Table 2.

Table 2 The threat factor quantification value corresponding to different types of UAV

Target type	linguistic variables	intuitionistic fuzzy number
Attack UAV	very high	(0.90,0.05)
Interference UAV	high	(0.75,0.10)
Scout UAV	general	(0.50,0.25)
Bait UAV	low	(0.25,0.20)

4. Interval-valued intuitionistic fuzzy number representation

Interval-valued number can be regarded as a special fuzzy number. In order to facilitate the subsequent calculation, when threat factor values are described as intuitionistic fuzzy number, it can be transformed into interval-valued intuitionistic fuzzy number: $o_{jl}^k = [\mu_{jl}^k, 1 - \nu_{jl}^k]$.

Based on the above processing, the uncertain target situational information in complex dynamic environment can be reasonably represented for subsequent target dynamic threat assessment. The specific threat assessment model and procedure are described in Section 3.

2.3 Construction of multi-weapon and multi-target allocation model

In this work, we assumed that all missions that were assigned would be finished simultaneously. This assumption could be divided into several assumptions, which are listed below.

Assumption 1: Assume that there is no time consumption for a UAV when conducting assignment. It means that every UAV could start and finish assignment simultaneously.

Assumption 2: Assume that the MWMTA problem would only be solved once and all assign operations would be started right after the allocation solved.

Assumption 3: Each UAV can use any number of weapon resource it carries to attack a target. Every weapon must be assigned to targets and each weapon can only attack one target.

Assumption 4: The damage probability between the k^{th} missile to the j^{th} target is already known and is labeled as p_{kj} . The threat value of the j^{th} target against our i^{th} UAV is also calculated beforehand through the threat assessment.

Denote x_{kj} as the decision variable of the k^{th} weapon assigned to the j^{th} target. When $x_{kj} = 1$, represent the k^{th} weapon assigned to the j^{th} target. When $x_{kj} = 0$, then means no assign. Based on the principle of the minimum threat of enemy UAVs to our UAVs and the maximum operational efficiency of weapon resources, the global utility function is established:

$$\begin{cases} \min F = \sum_{j=1}^n \sum_{i=1}^m \left[S_{ij} \prod_{k=1}^q (1 - p_{kj})^{x_{kj}} \right] \\ \max E = \sum_{j=1}^n \left[1 - \prod_{k=1}^q (1 - p_{kj})^{x_{kj}} \right] \end{cases} \quad (3)$$

Where F denotes the minimum threat value of enemy residual targets, and E denotes the destruction of enemy UAV target with the greatest probability.

The constraint condition is

$$s.t. \begin{cases} \sum_{j=1}^n x_{kj} \leq 1 \\ \sum_{k=1}^q x_{kj} \leq q_j \\ \sum_{k=1}^q \sum_{j=1}^n x_{kj} \leq q \end{cases} \quad (4)$$

Where, constraint 1 denotes that a weapon can only be assigned to a UAV target individually. Constraint 2 denotes that up to q_j weapons can be assigned to attack j^{th} target. Constrains 3 indicates that the max number of weapons can be used is q .

The multi-objective optimization problem is simplified into a single-objective optimization problem by linear weighting method. The penalty function method is used to deal with the constraints. Therefore, the improved objective function model is

$$\begin{aligned} \min G(x) = & \theta_1 \sum_{j=1}^n \sum_{i=1}^m \left[S_{ij} \prod_{k=1}^q (1 - p_{kj})^{x_{kj}} \right] + \theta_2 \left(n - \sum_{j=1}^n \left[1 - \prod_{k=1}^q (1 - p_{kj})^{x_{kj}} \right] \right) \\ & + M \cdot \sum_{k=1}^q \left[\min(0, 1 - \sum_{j=1}^n x_{kj}) \right]^2 \end{aligned} \quad (5)$$

Where M is a penalty factor of the penalty function, it is mainly used to punish the error that one weapon resource attacks more than one target at the same time in the solution space.

3 Interval-valued intuitionistic fuzzy Multi-Attribute Decision Making-based dynamic threat assessment

3.1 Timing weighting model

The situation information of air combat will change dynamically with time. The result of threat assessment is most affected by the situation information of air combat at the current moment. The closer the situation data is to the current moment, the more important it is. However, only relying on the data at the current moment for assessment and ignoring the implicit influence of historical information will lead to the narrow limitation of the assessment results, and the degree of rationality will be greatly reduced. Therefore, it is necessary to deeply analyze the relationship between air combat situation and threat assessment at multiple consecutive moments. A time series weight calculation model based on normal cumulative distribution is established, and the cumulative distribution function algorithm of normal distribution is used to analyze the time weight sequence. The normal cumulative distribution function as

$$F_{t_k}(\mu_K, \sigma_K) = \frac{1}{\sqrt{2\pi}\sigma_K} \int_{-\infty}^k \exp\left(-\frac{(t-\mu_K)^2}{2\sigma_K^2}\right) dt, k=1,2,\dots,K; t>0 \quad (6)$$

The special function based on the error function is expressed as

$$\phi(z) = \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{z-\mu_K}{\sqrt{2}\sigma_K}\right) \right] \quad (7)$$

Where K is the number of continuous moments. μ_K denotes the mean value of the set K and σ_K denotes the std value of the set K , they refer to

$$\mu_K = \frac{1+K}{2} \quad (8)$$

$$\sigma_K = \sqrt{\frac{1}{K} \sum_{k=1}^K (k - \mu_K)^2} \quad (9)$$

Based on the cumulative distribution function of normal distribution, the weight of time series is calculated.

$$\eta(t_k) = \frac{F_{t_k}(\mu_K, \sigma_K)}{\sum_{k=1}^p F_{t_k}(\mu_K, \sigma_K)} = \frac{\int_0^k \exp\left(-\frac{(t-\mu_K)^2}{2\sigma_K^2}\right) dt}{\sum_{k=1}^p \int_0^k \exp\left(-\frac{(t-\mu_K)^2}{2\sigma_K^2}\right) dt} \quad (10)$$

Where, $\eta(t_k)$ is the weight of t_k .

3.2 Threat factor weight optimization model

The AHP method based on subjective expert experience and the entropy method based on objective data characteristics, they reflect the weight of target threat attributes from different aspects. Therefore, we expect to find an optimal threat factor weighting method to combine the above two weighting methods. Assume that the weight obtained by AHP is $\hat{w} = [\hat{w}_1, \hat{w}_2, \dots, \hat{w}_l, \dots, \hat{w}_s]^T$, $l \in \{1, 2, \dots, s\}$. The weight obtained by entropy method is $\tilde{w} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_l, \dots, \tilde{w}_s]^T$, $l \in \{1, 2, \dots, s\}$. The objective function of threat factor weight optimization model is

$$\min w = \sum_{l=1}^s \left[\alpha (w_l - \tilde{w}_l)^2 + (1-\alpha) (w_l - \hat{w}_l)^2 \right] \quad (11)$$

The constraint condition is

$$s.t. \sum_{l=1}^s w_l = 1, \quad w_l > 0 \quad (12)$$

Where α is the preference coefficient, when the expert experience is more accurate and reliable, take a larger value, otherwise take a smaller value. In this article, take $\alpha = 0.5$.

3.3 Procedure of the dynamic threat assessment

In summary, the decision-making process of threat assessment based on dynamic intuitionistic fuzzy multi-attribute decision making is shown in Figure 2.

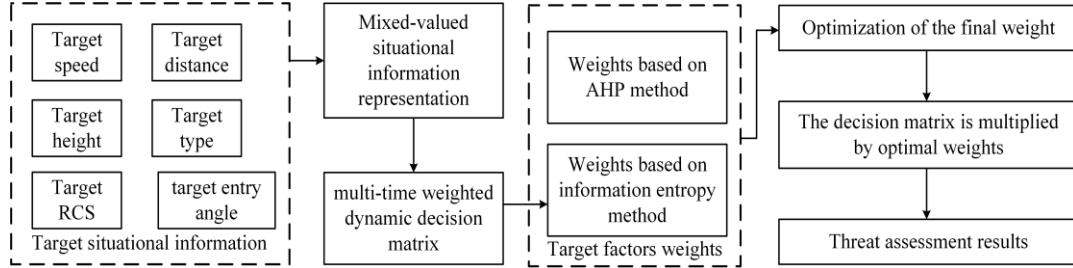


Figure 2 Flow of dynamic threat assessment

The specific steps are as follows:

Step 1: Based on the mixed situational information processing method in Section 2.2, the threat assessment model is constructed, and the decision matrix of target situational information at the moment is established as $R_{t_k} = [o_{jl}^k]_{n \times s}$.

Step 2: According to equation (6) to equation (10), combined with the time series weight model, the multi-time weighted dynamic decision matrix is constructed by integrating the multi-time target situational information.

$$R_i = (o_{jl})_{n \times s} = \begin{bmatrix} o_{11} & o_{12} & \dots & o_{1s} \\ o_{21} & o_{22} & \dots & o_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ o_{n1} & o_{n2} & \dots & o_{ns} \end{bmatrix} \quad (13)$$

Step 3: Since the entropy method needs accurate value to calculate the objective weight, the continuous ordered weighted average operator method is used to transform the interval-valued intuitionistic fuzzy number to accurate value according to follow equations

$$h_{jl} = \int_0^1 \frac{d\rho(y)}{dy} [1 - v_{jl} - y(1 - v_{jl} - \mu_{jl})] dy \quad (14)$$

Where $\rho(y)$ is a monotone increasing function in $[0, 1]$. Generally, $\rho(y) = y^t, t > 0$. So as to

$$h_{jl}^t = \frac{1 - v_{jl} + t \cdot \mu_{jl}}{1 + t} \quad (15)$$

Where t is inversely proportional to the degree of risk aversion of decision makers.

Step 4: Calculate the weight of threat factors based on AHP method as $\hat{w} = [\hat{w}_1, \hat{w}_2, \dots, \hat{w}_s]^T$. Calculate the weight of threat factors based on entropy method as $\tilde{w} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_s]^T$.

Step 5: According to equation (11) to equation (12), calculate the optimal weight of threat factors based on threat factor weight optimization model as $W = [w_1, w_2, \dots, w_s]^T$.

Step 6: Calculate the threat assessment value of the enemy UAVs to our i^{th} UAV according to follow equation

$$V_i = \tilde{R}W = \begin{bmatrix} o_{11} & o_{12} & \dots & o_{1s} \\ o_{21} & o_{22} & \dots & o_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ o_{n1} & o_{n2} & \dots & o_{ns} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_s \end{bmatrix} \quad (16)$$

Step 7: The above threat assessment process is carried out for all UAVs, and the comprehensive threat assessment matrix of the enemy UAVs to our UAVs can be obtained.

$$V = (S_{ij})_{m \times n} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1} & S_{m2} & \cdots & S_{mn} \end{bmatrix} \quad (17)$$

Where S_{ij} denotes the threat assessment value of the j^{th} enemy UAV to i^{th} our UAV.

4.VNS-BPSO algorithm for MWMTA problem

The MWMTA problem is a typical nonlinear combinatorial optimization problem. With the increase of the types and quantities of weapons and targets, the number of solutions will increase exponentially. The intelligent optimization algorithm can find the optimal solution quickly and accurately for the case of large scale and complex constraints. In this section, VBS-IBPSO optimization algorithm is proposed to realize coordinated target assignment under Complex dynamic environment.

4.1 Concept of BPSO

Binary Particle Swarm Optimization (BPSO) is proposed by J. Kennedy and R.C. Eberhart in the year of 1997, make the PSO algorithm can solve the discrete combinatorial optimization problem^[38].

Using the $q \times n$ variables in the weapon target assignment matrix as the solution space, the dimension of it is d , $d = q \times n$. Denote X_{id} as one origin solution with the initial velocity V_{id} , X_{id} is binary encoded, as shown in figure 3.

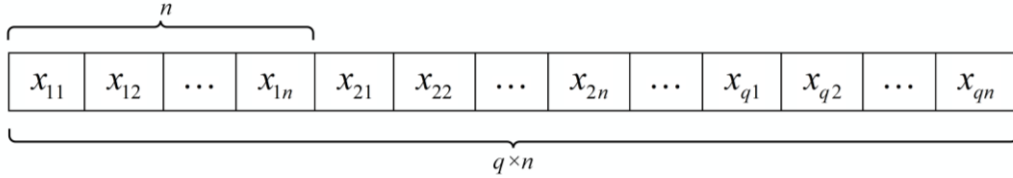


Figure 3 binary codes of X_{id}

The update rule for the i^{th} particle is expressed as follows:

$$V_{id} = V_{id} + \phi_1 \cdot rand_1 \cdot (pbest_{id} - X_{id}) + \phi_2 \cdot rand_2 \cdot (gbest_{id} - X_{id}) \quad (18)$$

$$S(V_{id}) = (1 + \exp(-V_{id}))^{-1} \quad (19)$$

$$X_{id} = \begin{cases} 1 & rand \leq S(V_{id}) \\ 0 & rand > S(V_{id}) \end{cases} \quad (20)$$

Where $pbest_{id}$ denotes individual optimal particle position, $gbest_{id}$ denotes global optimal particle position. ϕ_1 and ϕ_2 are the coefficient of particle learning from $pbest_{id}$ and $gbest_{id}$. S is the sigmoid function^[39] and $rand$ is a random number between 0 to 1.

The rule in equation (19) to (20) transform the summation relationship between velocity and position into a mapping relationship. This means the greater the speed, the higher the probability of the position to take 1.

4.2 The improved BPSO

In the BPSO algorithm, each iteration of the particle is mainly to change its binary sequence. The concept of probability of the bit changing is proposed in Ref. [38], assume that a bit of binary codes is 0, then the probability it changes into 1 is $S(V_{id})$. Identically, if it is 1 originally then the probability it changes into 0 is $1 - S(V_{id})$. The probability of the bit changing is:

$$p(\Delta) = S(V_{id})(1 - S(V_{id})) \quad (21)$$

Substitute equation (19) into equation (21):

$$p(\Delta) = \frac{1}{1 + \exp(-V_{id})} - \left(\frac{1}{1 + \exp(-V_{id})} \right)^2 \quad (22)$$

According to equation (21), the correlation between the particle speed V_{id} and $p(\Delta)$ is shown as Figure 4.

It can be seen from the figure that when the V_{id} is 0, the bit change rate is the largest, and the maximum value is 0.25. In the BPSO algorithm, the update of particles is related to the individual optimal position and the global optimal position. When the particle velocity approaches 0, the probability of bit changing is 0.25, it means when the particle reaches the optimal point, it still has 25% probability of jumping. Therefore, although the BPSO algorithm has a strong global search ability, it cannot converge to the global optimal position, the randomness of the BPSO becomes stronger with the iteration of the algorithm and the local search ability becomes weaker with the iteration.

Considering the intrinsic logical relationship of the update rules in the PSO algorithm and drawing on the probability-based mapping rules in BPSO. Here we propose an improved BPSO update strategy:

$$\begin{cases} V_{id} = V_{id} + \phi_1 \cdot rand_1 \cdot (pbest_{id} - X_{id}) + \phi_2 \cdot rand_2 \cdot (gbest_{id} - X_{id}) \\ S(V_{id}) = \begin{cases} -1 + \frac{2}{1 + \exp(-V_{id})}, V_{id} > 0 \\ 1 - \frac{2}{1 + \exp(-V_{id})}, V_{id} \leq 0 \end{cases} \\ X_{id} = X_{id} + Trans(V_{id}) \end{cases} \quad (23)$$

Where the $Trans$ function is as follows:

$$Trans(V_{id}) = \begin{cases} 1 & \text{if } rand \leq S(V_{id}) \& V_{id} > 0 \\ 0 & \text{if } rand \leq S(V_{id}) \& V_{id} \leq 0 \\ X_{id} & \text{if } rand > S(V_{id}) \end{cases} \quad (24)$$

The difference between IBPSO and BPSO is the modification of function $Trans$ and $S(V_{id})$. The correlation between the particle speed V_{id} and $p(\Delta)$ after the change of function is shown as Figure 5. The probability of bit changing tend to 0 when the particle velocity tends to 0. Moreover, when the particle velocity is positive, the binary bit value can only be changed to 1. Otherwise, the binary bit value can only be changed to 0. This method makes it easier for the particle swarm to approach the global optimal particle, and improves the local search ability of the BPSO algorithm.

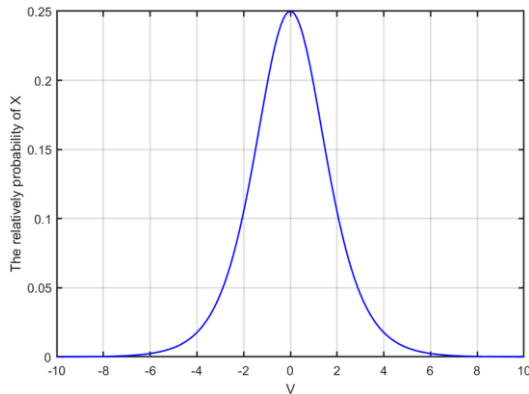


Figure 4 Bit change rate of BPSO

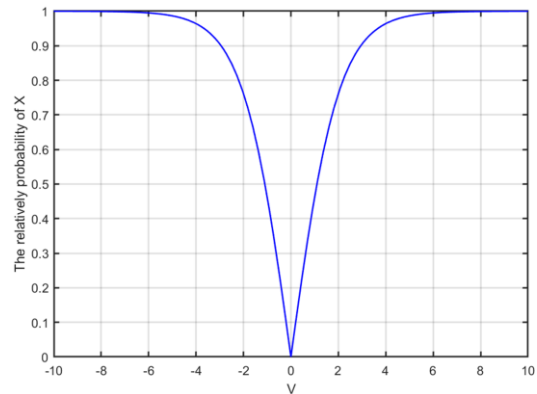


Figure 5 Bit change rate of IBPSO

4.3 Variable Neighborhood Search Operator

The basic principle of the variable neighborhood search algorithm (VNS) is to obtain a wider search range by changing the neighborhood structure of multiple historical solutions within a local range[40]. That is, in the case of the same initial solution, the algorithm can expand a wider search space and has a more superior ability to jump out of the ' premature trap '. Therefore, On the basis of IBPSO, the variable neighborhood search operator is introduced to further improve its local search ability. The specific flow is shown in Figure 6.

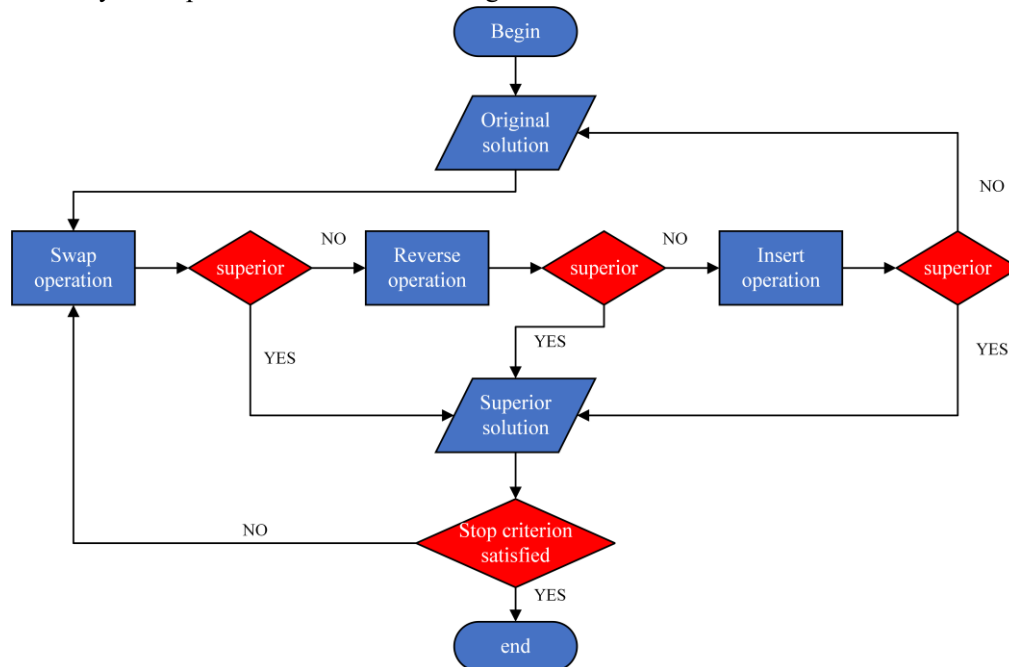


Figure 6 Flow of the variable neighborhood search

The core of VNS is the design of neighborhood search operation. In this section, three different neighborhood operations are designed for MWMTA, as follows:

1. Swap operation

Suppose that in the MWMTA problem, the individual optimal solution of the current particle has been found by the IBPSO algorithm, and the values of the first and second positions in the solution space are swapped by the swap operation to obtain its neighborhood solution by arbitrarily choosing two positions in the solution space. The specific operation is shown in the Figure 7.

2. Reverse operation

Suppose the current particle individual optimal solution has been obtained, arbitrarily choose two positions in the solution space and, reverse all values between the first position and the first position by the reversal operation to reverse the ordering. The specific operation is shown in the Figure 8.

3. Insert operation

If the value of the former is smaller than the latter, the value of the former is inserted after the latter. Conversely, the value at the latter position is inserted after the former to obtain its neighborhood solution. The specific operation is shown in the Figure 9.

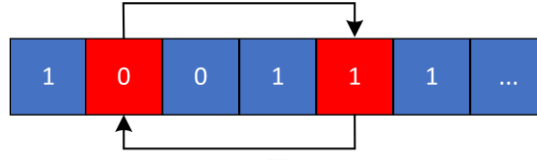


Figure 7 Swap operation

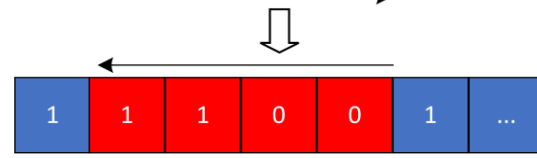


Figure 8 Reverse operation

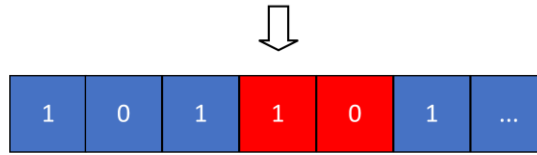
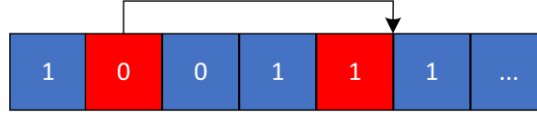


Figure 9 Insert operation

4.4 Implementation of VNS-IBPSO

The pseudocode of VBS-IBPSO is shown in the Table 3. The hyperparameters that need to be set in advance include particle swarms, $popsiz$, maximum number of IBPSO iterations, $maxiter$, learning coefficient factors ϕ_1 and ϕ_2 , maximum number of VND iterations k_{max} .

Table 3 The pseudocode of VBS-IBPSO

Procedure VNS-IBPSO

Initialize the hyperparameters

For each particle i

Generate the position X_{id} and velocity V_{id}

Calculate its fitness

Set $pbest_{id} = X_{id}$

End for

$gbest_{id} = \arg \max_i Fit(X_{id})$

While $iter \leq Maxiter$

For $i = 1$ to $popsiz$

Update the velocity and position of particle i

Perform constraint processing on particles that do not satisfy the constraint

Calculate its new fitness

The new particle is X'_{id}

If $Fit(X'_{id}) > Fit(pbest_{id})$

$pbest_{id} = X'_{id}$

```

If  $Fit(X'_{id}) > Fit(gbest_{id})$ 
     $gbest_{id} = X'_{id}$ 
    For  $k=1$  to  $k_{max}$ 
        Perform VNS operator on  $pbest_{id}$  to get the local optimal value
        The new particle is  $X''_{id}$ 
        If  $Fit(X''_{id}) > Fit(pbest_{id})$ 
             $pbest_{id} = X''_{id}$ 
        If  $Fit(X'_{id}) > Fit(gbest_{id})$ 
             $gbest_{id} = X'_{id}$ 
     $iter = iter + 1$ 
End while
Record and print process data
End procedure

```

The function Fit is the fitness function according to equation (3). The main steps of IBPSO are explained as follows:

Step 1: Initialize the particle swarms including its population, maximum number of iterations, speed range of particle, and learning coefficient. Then calculate the fitness value of each particle and record $pbest_{id}$ and $gbest_{id}$.

Step 2: Update the particle swarms according to equation (23) and equation (24). Perform constraint processing on particles that do not satisfy the constraint and calculate the fitness value of new particle.

Step 3: Compare the updated fitness value of the particle with the historical optimal fitness value of $pbest_{id}$. If the former is better than the latter, update the $pbest_{id}$ and further compare its value with the value of $gbest_{id}$. After comparison, determine whether the current particle has fully updated; if the update is complete, go to step 4, otherwise update the next particle.

Step 4: Denote $k=1$, Perform VNS operation on $pbest_{id}$ and obtain the neighborhood solution X''_{id} . Perform constraint processing on it and calculate updated fitness value.

Step5: Compare the updated fitness value of the particle with the historical optimal fitness value of $pbest_{id}$. If the former is better than the latter, update the $pbest_{id}$ and further compare its value with the value of $gbest_{id}$. After comparison, determine whether the current particle has fully updated; if the update is complete, then continue to search within the local search range of the next neighborhood solution until $k=k_{max}$.

Step 6: If the number of iterations reaches its maximum value, then return $gbest_{id}$ and exit the algorithm. Otherwise, the iterative process of updating in the next round is started again from the first particle.

5 Simulation results and analysis

5.1 Simulation of target threat assessment

Assume that after the target search and tracking identification phase, it is known that the enemy UAV cluster consists of 6 UAVs, marked as $T = \{T_1, T_2, T_3, T_4, T_5, T_6\}$. Our side is composed of 4 UAVs, marked as $T = \{\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4\}$. Take Γ_1 as an example, the target situational information of t_1 , t_2 , t_3 moments are obtained as multi-attribute decision information for target threat assessment. Based on the interval-valued intuitionistic fuzzy number for its representation, as shown in the Table 4 to Table 5.

Table 4 target situation index at t_1

Target	Evaluation Indicators					
	Speed/(m • s ⁻¹)	Height/(m)	Distance/(m)	Entry Angle/(°)	RCS/(m ²)	Type
1	[0.38,0.41]	350	3320	[0.43,0.64]	0.16	[0.90,0.95]
2	[0.39,0.42]	344	1720	[0.37,0.58]	0.15	[0.90,0.95]
3	[0.38,0.43]	310	2955	[0.27,0.50]	0.13	[0.90,0.95]
4	[0.38,0.43]	400	3600	[0.16,0.36]	0.02	[0.25,0.8]
5	[0.39,0.43]	305	3200	[0.32,0.70]	0.05	[0.5,0.75]
6	[0.39,0.43]	440	4800	[0.16,0.64]	0.07	[0.75,0.90]

Table 5 target situation index at t_2

Target	Evaluation Indicators					
	Speed/(m • s ⁻¹)	Height/(m)	Distance/(m)	Entry Angle/(°)	RCS/(m ²)	Type
1	[0.38,0.41]	362	3270	[0.43,0.64]	0.16	[0.90,0.95]
2	[0.39,0.42]	355	1805	[0.36,0.57]	0.15	[0.90,0.95]
3	[0.38,0.43]	330	3005	[0.27,0.50]	0.13	[0.90,0.95]
4	[0.38,0.43]	405	3705	[0.16,0.36]	0.02	[0.25,0.8]
5	[0.39,0.43]	295	3200	[0.32,0.70]	0.05	[0.5,0.75]
6	[0.39,0.43]	420	4800	[0.17,0.63]	0.07	[0.75,0.90]

Table 6 target situation index at t_3

Target	Evaluation Indicators					
	Speed/(m • s ⁻¹)	Height/(m)	Distance/(m)	Entry Angle/(°)	RCS/(m ²)	Type
1	[0.39,0.42]	345	3100	[0.43,0.64]	0.16	[0.90,0.95]
2	[0.39,0.42]	350	2000	[0.37,0.58]	0.15	[0.90,0.95]
3	[0.38,0.43]	320	3200	[0.27,0.50]	0.13	[0.90,0.95]
4	[0.36,0.45]	415	3840	[0.15,0.37]	0.04	[0.25,0.8]
5	[0.39,0.43]	305	3210	[0.32,0.70]	0.08	[0.5,0.75]
6	[0.37,0.45]	435	4500	[0.16,0.64]	0.05	[0.75,0.90]

According to equation (6) to equation (10), the time series weights are calculated:

$$\eta(t_1) = 0.070, \eta(t_2) = 0.333, \eta(t_3) = 0.597$$

According to equation (11) to equation (12), the optimal threat factor weights are obtained:

$$W = [0.174 \quad 0.033 \quad 0.126 \quad 0.326 \quad 0.208 \quad 0.133]^T$$

According to equation (15), multi-Moment Weighted Dynamic Decision Matrix are determined:

$$R_1 = \begin{bmatrix} 0.183 & 0.167 & 0.165 & 0.209 & 0.286 & 0.195 \\ 0.187 & 0.164 & 0.091 & 0.185 & 0.232 & 0.195 \\ 0.125 & 0.152 & 0.152 & 0.150 & 0.232 & 0.195 \\ 0.125 & 0.187 & 0.187 & 0.101 & 0.036 & 0.110 \\ 0.190 & 0.136 & 0.162 & 0.199 & 0.089 & 0.132 \\ 0.190 & 0.194 & 0.243 & 0.156 & 0.125 & 0.173 \end{bmatrix}$$

According to equation (16), the combined threat value of enemy UAVs to Γ_1 is determined as

$$V_1 = [0.488 \quad 0.426 \quad 0.400 \quad 0.278 \quad 0.342 \quad 0.414]$$

The above threat assessment process is carried out for each of our UAVs, the comprehensive threat assessment value of enemy UAVs to our UAVs is obtained. The specific values are shown in Table 6.

5.2 Simulation of MWMTA

After completing the threat assessment of the enemy UAV targets, the multi-weapon multi-target assignment phase is entered. Assume that each UAV carries 2 weapons and the number of

weapon resources to attack the same target is at most 2. Using the VNS-IBPSO algorithm to solve the MWMTA problem, the number of initial particle swarms is set to 500, the initial times are set to 200, learning coefficients are set to 0.8 and 0.9, the number of VNS operations is set to 30. The target threat of 6 enemy UAVs to our 4 UAVs and the damage probability of our 8 weapons to 6 enemy UAVs are given in Tables 7 and Table 8.

Table 7 The comprehensive threat assessment matrix of enemy UAVs to our UAVs

Targets	Target threat					
	1	2	3	4	5	6
1	0.488	0.426	0.400	0.278	0.342	0.414
2	0.254	0.203	0.252	0.601	0.275	0.482
3	0.195	0.341	0.235	0.371	0.164	0.335
4	0.614	0.109	0.631	0.484	0.292	0.195

Table 8 The damage probability of our weapons to the enemy UAVs

Weapons	Hit rate					
	1	2	3	4	5	6
1	0.29	0.92	0.23	0.89	0.14	0.72
2	0.49	0.82	0.41	0.10	0.51	0.37
3	0.33	0.46	0.39	0.90	0.43	0.30
4	0.12	0.38	0.52	0.26	0.71	0.41
5	0.22	0.15	0.44	0.29	0.21	0.13
6	0.61	0.95	0.76	0.32	0.24	0.19
7	0.44	0.56	0.16	0.22	0.88	0.17
8	0.81	0.42	0.35	0.43	0.77	0.90

To verify the rationality of the algorithm, the VNS-IBPSO was compared with the origin BPSO, the improved BPSO, hybrid genetic algorithm[34], the curve of the optimal fitness value, the expected value of operational effectiveness and the expected value of residual target threat are given in Figure 10 to 12.

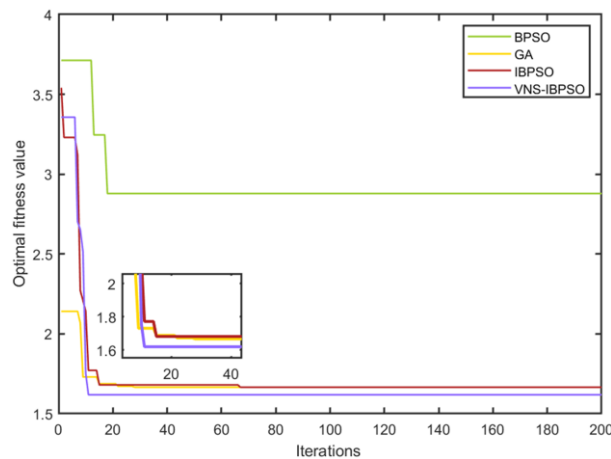


Figure 10 The expected value of the optimal fitness

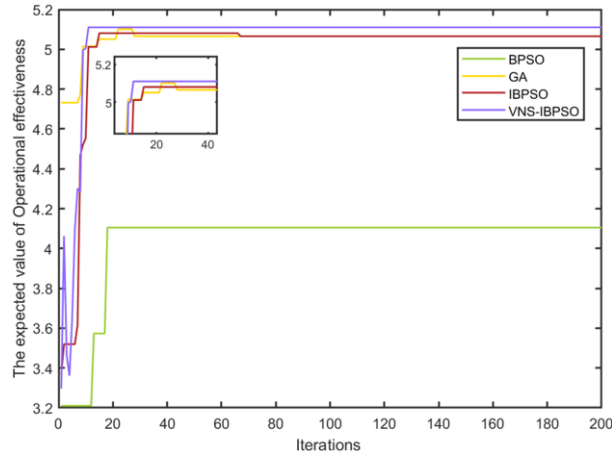


Figure 11 The expected value of operational effectiveness

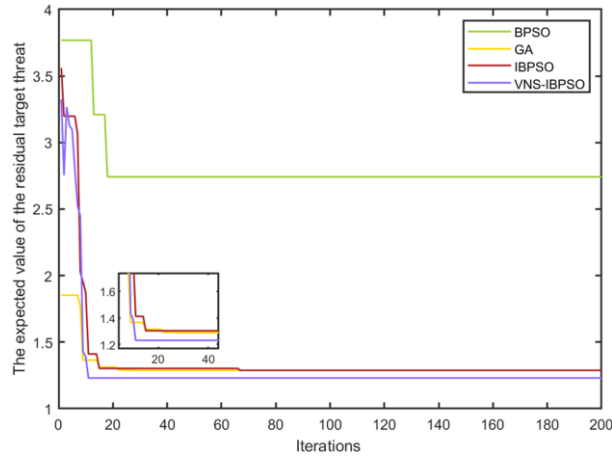


Figure 12 The expected value of residual target threat

In our experiment, the expected value of operational effectiveness is required to be as large as possible and the expected value of residual target threat is required to be as small as possible. It can be seen that the VNS-IBPSO algorithm is the first to reach convergence among the four algorithms and achieves optimal results on both fronts. After the 11th iterations, the VNS-IBPSO algorithm calculates the optimal fitness value is 1.6186, the optimal value of the residual target threat expectation is 1.2306 and the optimal value of operational effectiveness expectation is 5.1104. The weapon target assignment matrix obtained from the solution is:

$$U_{q \times n} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be seen that the weapon target assignment scheme is as follows: weapon 8 attacks target 1, weapon 2 attacks target 2, weapon 5 and 6 attack target 3, weapon 3 attacks target 4, weapon 7 attacks target 5, and weapon 1 and 4 attack target 6.

5.3 Comparative analysis

For further analysis, 100 Monte Carlo simulation experiments were conducted for each of the four algorithms to solve this MWMTA problem. The variation curves of the optimal fitness values and the statistical analysis of the algorithm performance are obtained, as shown in Figure 13 to Figure 14 and Table 9.

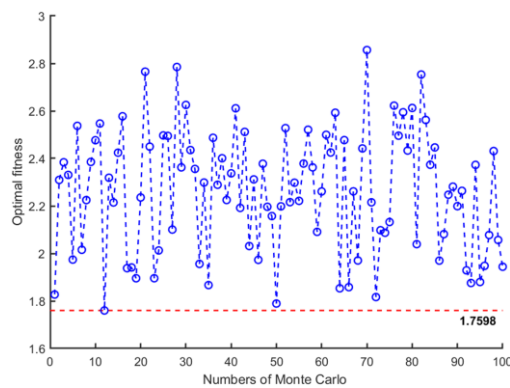


Figure 13 BPSO algorithm results

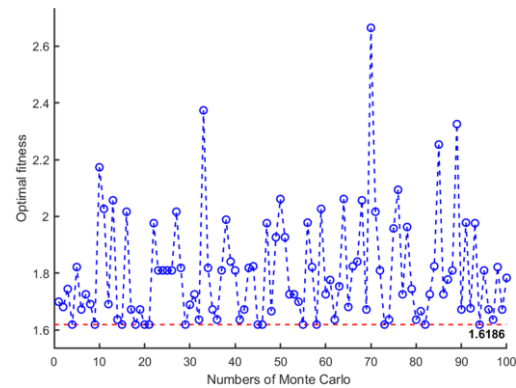


Figure 14 IBPSO algorithm results

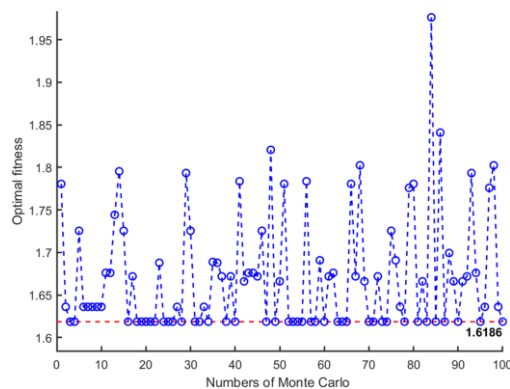


Figure 15 Hybrid GA algorithm results

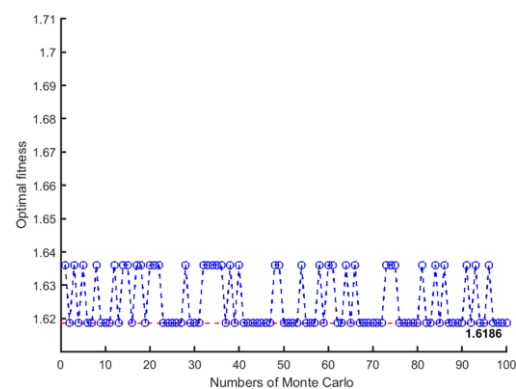


Figure 16 VNS-IBPSO algorithm results

Table 9 Algorithm performance statistics analysis

Algorithm	The best value	The worst value	Average	Variance	Average convergence time /s
BPSO	2.5598	3.6563	3.0625	0.0659	5.52
IBPSO	1.6186	2.6649	1.8025	0.0363	6.86
Hybrid-GA	1.6186	1.9763	1.6717	0.0047	4.66
VNS-IBPSO	1.6186	1.6361	1.6251	0.0001	4.08

The performance comparison of the four algorithms for 100 simulations is as follows:

(1) In terms of the solution quality of the algorithms, it can be seen that the optimal fitness values obtained by VNS-IBPSO for solving the MWMTA problem are the same as those obtained by Hybrid-GA and IBPSO algorithms, which also indicates the validity of the solution results. The difference is that the mean and variance of the VNS-IBSO solution results are much smaller than those of the other three algorithms, indicating that the solution obtained by using VNS-IBPSO is the most stable.

(2) In terms of the convergence rate of the algorithm, it can be seen from Figure 13 to 16 and Table 8 that the convergence rate of the VNS-IBPSO algorithm is significantly better than the other three algorithms, which is because the VNS-IBPSO algorithm takes into account the balance of global search and local search ability during each round of iteration, which effectively suppresses the occurrence of immature convergence and improves the solution effectiveness of the algorithm.

(3) In terms of the time performance of the algorithm, it can be seen that VNS-IBPSO is able to find the optimal feasible solution in the shortest time when solving the same problem because the solution quality and convergence rate of VNS-IBPSO are better than the other three algorithms.

The simulations are implemented in a MATLAB environment, and the main configuration of the computer is Win 10, Intel Core i9-11900H, 2.50GHz CPU and 16 GB RAM.

6 Conclusion

In the context of multi-UAVs cooperative air combat, this study investigates the problem of target threat assessment and MWMTA in a complex dynamic environment. In target threat assessment, a representation method based on interval-valued intuitionistic fuzzy number is proposed for the uncertainty and incompleteness of target situational information. A time-series weight generation model is proposed to solve the problem of dynamically matching attribute parameters and weights for multi-moment information fusion. Furthermore, an optimal weighting method is proposed. This method organically integrates the weights obtained by the AHP method and the entropy method. In the MWMTA problem, the global utility function is constructed by minimization of the threat to our UAVs and maximization of the operational effectiveness of our weapons, a target assignment model is established. Then, a VNS-IBPSO algorithm is proposed that combines an improved BPSO update strategy and a VNS operator. This algorithm overcomes the shortcomings of the original BPSO algorithm in terms of poor local search capability and a tendency to premature convergence. Finally, a complete execution process for target threat assessment and target assignment is given. The experimental results justify the decision scheme.

Data Availability Statements:

Data available on request from the authors.

Reference:

- [1] L. Fu, F. Xie, D. Wang, and G. Meng, "The overview for UAV Air-Combat Decision method," in *The 26th Chinese Control and Decision Conference (2014 CCDC)*, May 2014, pp. 3380–3384. doi: 10.1109/CCDC.2014.6852760.
- [2] "Air-Combat Strategy Using Approximate Dynamic Programming | Journal of Guidance, Control, and Dynamics." <https://arc.aiaa.org/doi/abs/10.2514/1.46815> (accessed Jun. 27, 2023).
- [3] H. Jiaxin, Z. Yanwei, H. Huan, Z. Ming, and L. Xi, "AHP-Based Interactive Target Threat Level Assessment and Visualization Method," in *International Conference on Guidance, Navigation and Control*, Springer, 2022, pp. 456–467.
- [4] G. Yin, S. Zhou, and W. Zhang, "A threat assessment algorithm based on AHP and principal components analysis," *Procedia Engineering*, vol. 15, pp. 4590–4596, 2011.
- [5] K. Zhang, P. P. Liu, K. Li, W. R. Kong, and J. D. Zhang, "Multi-target threat assessment in air combat based on AHP and FVIKOR," in *2017 IEEE International Conference on Unmanned Systems (ICUS)*, IEEE, 2017, pp. 523–526.
- [6] E. Azimirad and J. Haddadnia, "Target threat assessment using fuzzy sets theory," *International Journal of Advances in Intelligent Informatics*, vol. 1, no. 2, pp. 57–74, 2015.
- [7] Y. Xu, Y. Wang, and X. Miu, "Multi-attribute decision making method for air target threat evaluation based on intuitionistic fuzzy sets," *Journal of Systems Engineering and Electronics*, vol. 23, no. 6, pp. 891–897, Dec. 2012, doi: 10.1109/JSEE.2012.00109.

- [8] Y. Gao, D. Li, and H. Zhong, "A novel target threat assessment method based on three-way decisions under intuitionistic fuzzy multi-attribute decision making environment," *Engineering Applications of Artificial Intelligence*, 2020.
- [9] Y. Yin, R. Zhang, and Q. Su, "Threat assessment of aerial targets based on improved GRA-TOPSIS method and three-way decisions," *Mathematical Biosciences and Engineering*, vol. 20, no. 7, pp. 13250–13266, 2023.
- [10] Q. Changwen and H. You, "A method of threat assessment using multiple attribute decision making," in *6th International Conference on Signal Processing, 2002.*, IEEE, 2002, pp. 1091–1095.
- [11] J. Li and Z. Hao, "A Quantum Probabilistic Linguistic Term Framework to Multi-attribute Decision-Making for Battlefield Situation Assessment," *Int. J. Fuzzy Syst.*, vol. 24, no. 1, pp. 495–507, Feb. 2022, doi: 10.1007/s40815-021-01151-5.
- [12] Z. Qiang, H. Junhua, F. Jinfu, and L. An, "Air multi-target threat assessment method based on improved GGIFSS," *Journal of Intelligent & Fuzzy Systems*, vol. 36, no. 5, pp. 4127–4139, 2019.
- [13] Y. Wang, Y. Sun, J.-Y. Li, and S.-T. Xia, "Air defense threat assessment based on dynamic Bayesian network," in *2012 International Conference on Systems and Informatics (ICSAI2012)*, May 2012, pp. 721–724. doi: 10.1109/ICSAI.2012.6223112.
- [14] Z. Kun, K. Weiren, L. Peipei, S. Jiao, L. Yu, and Z. Jie, "Assessment and sequencing of air target threat based on intuitionistic fuzzy entropy and dynamic VIKOR," *Journal of Systems Engineering and Electronics*, vol. 29, no. 2, pp. 305–310, Apr. 2018, doi: 10.21629/JSEE.2018.02.11.
- [15] Z. Zhang, J. Jiang, J. Wu, and X. Zhu, "Efficient and optimal penetration path planning for stealth unmanned aerial vehicle using minimal radar cross-section tactics and modified A-Star algorithm," *ISA Transactions*, vol. 134, pp. 42–57, Mar. 2023, doi: 10.1016/j.isatra.2022.07.032.
- [16] A. Kline, D. Ahner, and R. Hill, "The Weapon-Target Assignment Problem," *Computers & Operations Research*, vol. 105, pp. 226–236, May 2019, doi: 10.1016/j.cor.2018.10.015.
- [17] R. M. Soland, "Optimal defensive missile allocation: A discrete min-max problem," *Operations Research*, vol. 21, no. 2, pp. 590–596, 1973.
- [18] A. G. Kline, D. K. Ahner, and B. J. Lunday, "Real-time heuristic algorithms for the static weapon target assignment problem," *Journal of Heuristics*, vol. 25, pp. 377–397, 2019.
- [19] J. Yan, W. Daobo, B. Tingting, and Y. Zongyuan, "Multi-UAV objective assignment using Hungarian fusion genetic algorithm," *IEEE Access*, vol. 10, pp. 43013–43021, 2022.
- [20] G. A. Mills-Tettey, A. Stentz, and M. B. Dias, "The dynamic hungarian algorithm for the assignment problem with changing costs," *Robotics Institute, Pittsburgh, PA, Tech. Rep. CMU-RI-TR-07-27*, 2007.
- [21] Z. J. G. H. C. Tong, "Weapon-target Assignment Based on Adaptable Hungarian Algorithm," *Acta Armamentarii*, vol. 42, no. 6, p. 1339, Jun. 2021, doi: 10.3969/j.issn.1000-1093.2021.06.025.
- [22] P. Zhao, J. Wang, and L. Kong, "Decentralized Algorithms for Weapon-Target Assignment in Swarming Combat System," *Mathematical Problems in Engineering*, vol. 2019, p. e8425403, Aug. 2019, doi: 10.1155/2019/8425403.

- [23] B. Xin, J. Chen, Z. Peng, L. Dou, and J. Zhang, "An efficient rule-based constructive heuristic to solve dynamic weapon-target assignment problem," *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, vol. 41, no. 3, pp. 598–606, 2010.
- [24] M. Ni, Z. Yu, F. Ma, and X. Wu, "A lagrange relaxation method for solving weapon-target assignment problem," *Mathematical Problems in Engineering*, vol. 2011, 2011.
- [25] B. Xin, J. Chen, J. Zhang, L. Dou, and Z. Peng, "Efficient decision makings for dynamic weapon-target assignment by virtual permutation and tabu search heuristics," *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 40, no. 6, pp. 649–662, 2010.
- [26] J. Jang, H. G. Yoon, J. C. Kim, and C. O. Kim, "Adaptive weapon-to-target assignment model based on the real-time prediction of hit probability," *IEEE Access*, vol. 7, pp. 72210–72220, 2019.
- [27] A. Shojaeifard, A. N. Amroudi, A. Mansoori, and M. Erfanian, "Projection recurrent neural network model: a new strategy to solve weapon-target assignment problem," *Neural Processing Letters*, vol. 50, pp. 3045–3057, 2019.
- [28] D. Gibbons, C.-C. Lim, and P. Shi, "Deep learning for bipartite assignment problems," in *2019 IEEE International Conference on Systems, Man and Cybernetics (SMC)*, IEEE, 2019, pp. 2318–2325.
- [29] L. Wu, X.-T. Yu, and F.-X. Lu, "Constraint Optimization GA and Its Application to Constrained WTA Problem," in *2015 7th International Conference on Intelligent Human-Machine Systems and Cybernetics*, Aug. 2015, pp. 141–144. doi: 10.1109/IHMSC.2015.60.
- [30] H. Zhai, W. Wang, Q. Li, and W. Zhang, "Weapon-Target Assignment Based on Improved PSO Algorithm," in *2021 33rd Chinese Control and Decision Conference (CCDC)*, May 2021, pp. 6320–6325. doi: 10.1109/CCDC52312.2021.9601736.
- [31] X. Liu, J. Liang, D.-Y. Liu, R. Chen, and S.-M. Yuan, "Weapon-target assignment in unreliable peer-to-peer architecture based on adapted artificial bee colony algorithm," *Front. Comput. Sci.*, vol. 16, no. 1, p. 161103, Oct. 2021, doi: 10.1007/s11704-021-0395-8.
- [32] Y. Li, Y. Kou, Z. Li, A. Xu, and Y. Chang, "A Modified Pareto Ant Colony Optimization Approach to Solve Biobjective Weapon-Target Assignment Problem," *International Journal of Aerospace Engineering*, vol. 2017, p. 1746124, Mar. 2017, doi: 10.1155/2017/1746124.
- [33] M. Nan, Y. Zhu, L. Kang, T. Wang, and X. Zhou, "A Modified RL-IGWO Algorithm for Dynamic Weapon-Target Assignment in Frigate Defending UAV Swarms," *Electronics*, vol. 11, no. 11, Art. no. 11, Jan. 2022, doi: 10.3390/electronics11111796.
- [34] Y. Zhao, Y. Chen, Z. Zhen, and J. Jiang, "Multi-weapon multi-target assignment based on hybrid genetic algorithm in uncertain environment," *International Journal of Advanced Robotic Systems*, vol. 17, no. 2, p. 172988142090592, Mar. 2020, doi: 10.1177/1729881420905922.
- [35] Z. Zhen, L. Wen, B. Wang, Z. Hu, and D. Zhang, "Improved contract network protocol algorithm based cooperative target allocation of heterogeneous UAV swarm," *Aerospace Science and Technology*, vol. 119, p. 107054, Dec. 2021, doi: 10.1016/j.ast.2021.107054.
- [36] D. Xing, Z. Zhen, and H. Gong, "Offense–defense confrontation decision making for dynamic UAV swarm versus UAV swarm," *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, vol. 233, no. 15, pp. 5689–5702, Dec. 2019, doi: 10.1177/0954410019853982.

- [37] Y. Song, Q. Xi, X. Xing, and B. Yang, "Multi-UAV Cooperative Multi-Target Allocation Method based on Differential Evolutionary Algorithm," in *2020 39th Chinese Control Conference (CCC)*, Jul. 2020, pp. 1655–1660. doi: 10.23919/CCC50068.2020.9189332.
- [38] J. Kennedy and R. C. Eberhart, "A discrete binary version of the particle swarm algorithm," in *1997 IEEE International conference on systems, man, and cybernetics. Computational cybernetics and simulation*, IEEE, 1997, pp. 4104–4108.
- [39] M. T. Tommiska, "Efficient digital implementation of the sigmoid function for reprogrammable logic," *IEE Proceedings - Computers and Digital Techniques*, vol. 150, no. 6, pp. 403–411, Nov. 2003, doi: 10.1049/ip-cdt:20030965.
- [40] P. Hansen, N. Mladenović, R. Todosijević, and S. Hanafi, "Variable neighborhood search: basics and variants," *EURO Journal on Computational Optimization*, vol. 5, no. 3, pp. 423–454, Sep. 2017, doi: 10.1007/s13675-016-0075-x.