

9 **Abstract**

10 The topography of ocean worlds is often used to infer ice shell thicknesses by assuming
11 topography is compensated by a basal root. We systematically test the stability of isostatically
12 compensated topography in ice shells. At short horizontal wavelengths, lithospheric strength can
13 support surface topography, while at long wavelengths, buoyancy forces can support topography
14 at the surface and base of the ice shell over geologic time scales. These behaviors are also seen
15 for Airy isostasy in terrestrial worlds. Contrastingly at intermediate scales, the mechanically
16 weak lower ice shell can inhibit the transfer of buoyancy forces to the surface. Factors such as
17 surface temperature can alter the contribution of lithospheric strength, decreasing the stability of
18 a compensating root. This nuanced understanding of icy shell lithospheres provides crucial
19 insights for interpreting surface features and inferring underlying ice shell thickness, with
20 substantial relevance for upcoming space missions to the Jovian system.

21 **Plain Language Summary**

22 Ocean moons have topography on their surfaces that can be used to estimate the thickness of the
23 capping ice shells. In contrast to rocky worlds, the nature of ice shells prevents traditional
24 approaches from being applied. In this study, we use geophysical models to simulate how
25 topography evolves over geologic time. For small features, the stiffness of the ice crust plays a
26 significant role in supporting topography. For larger features, buoyancy forces, like how an
27 iceberg floats, keep topography stable over long periods. In the lower shell near the underlying
28 ocean, the ice is near its melting temperature, which makes it much weaker than the ice nearer
29 the surface. This weakness can prevent the buoyancy forces from reaching the surface for
30 intermediate scale features. Understanding the dynamics of Airy isostasy in ocean worlds can
31 help interpret data collected from space missions, and further our understanding of these icy
32 satellites.

33 **1 Introduction**

34 Topography on terrestrial bodies is generally supported by a combination of factors.
35 These include variations in crustal thickness, known as Airy isostasy (Airy, 1855), variations in
36 density, referred to as Pratt isostasy (Pratt, 1855), and the strength of the lithosphere (Turcotte et
37 al., 1981). The extent to which lithospheric strength contributes to topographic support depends
38 on the wavelength of the topography (e.g., Turcotte & Schubert, 2014). For instance, at short
39 wavelengths, the lithosphere can fully support the topographic load, analogous to a bookshelf
40 supporting a single book. However, at sufficiently long wavelengths, the rigidity of the
41 lithosphere is reduced, similar to many books on a sagging bookshelf. In this case, the
42 topography is compensated by isostasy.

43 The concept of surface topography being compensated by an opposing basal topography
44 (i.e., Airy isostasy) is simple and appealing. It enables interpretation of gravity and topography
45 data collected by spacecraft using only an *a priori* knowledge of density. Consequently,

46 researchers have widely used this concept to estimate variations in the thickness of icy shells on
47 ocean worlds within the Jovian and Saturnian systems.

48 These icy satellites display a diverse array of topographic features at various horizontal
49 wavelengths. In addition to its use with gravity measurements, Airy isostasy has been used to
50 estimate ice shell thickness from observations of surface features and their respective
51 topographies. Shell thickness estimates are commonly derived from long-wavelength topography
52 (e.g., Čadek et al., 2019b; Nimmo et al., 2011; Schenk & McKinnon, 2009), and is often used as
53 an explanation for unexpected topography observations. For example, the unexpectedly large
54 amplitude of Titan's long-wavelength topography may be explained, in part, by isostatic
55 compensation (Nimmo & Bills, 2010). Estimates based on long-wavelength variations shell
56 thickness assume that the shell is in a constant state of equilibrium. Effectively, ice flow at the
57 base of the ice shell, driven by horizontal pressure differences, is faster than the rate of relaxation
58 resulting in a constant compensation (e.g., Ojakangas & Stevenson, 1989; Stevenson, 2000).

59 Short-wavelength features are strewn across icy, ocean worlds. Europa's surface is most
60 notably characterized by intersecting ridges that can extend for more than 1000 km with a relief
61 of ~200 m (e.g., Pappalardo et al., 1999; Prockter & Patterson, 2009). Additionally, dark bands
62 on Europa can rise up to 150 m above the surrounding terrain and span several tens of kilometers
63 across (Nimmo et al., 2003b). Although linear features are ubiquitous on Europa, there are also
64 small pits and uplifts with average diameters of around 5 km. Singer et al. (2021) employed a
65 basic Airy isostasy model to estimate a minimum shell thickness for Europa of about 3-8 km,
66 based on the size of these small pits. However, if sufficiently thick, the ice shell likely has the
67 rigidity to support short-wavelength features (e.g., Nimmo et al., 2003a).

68 Ice shell thickness estimates have also been derived from chaos terrain on Europa.
69 Williams & Greeley (1998) estimated a minimum thickness of just 0.2-3 km at Conamara Chaos,
70 based on the interpretation that the blocks of the chaos terrain were floating in a liquid at the time
71 of formation. In addition to the short-wavelength topography that is the icy blocks sitting in a
72 matrix, chaos terrain can have their own longer wavelength topographic signature suggesting a
73 compensated state (e.g., Schmidt et al., 2011).

74 Recently, efforts have been made to refine Airy isostasy for use on ice shells, specifically
75 at low spherical harmonic degrees (see Beuthe 2021a, 2021b for an extensive review).

76 Hemingway and Matsuyama (2017) argued that the traditional form of Airy isostasy, in which
77 columns of equal width contain equal mass, is not applicable when using spherical geometry as it
78 causes lateral pressure gradients. The authors proposed an approach that assumes continuous
79 pressure along a solid crust-mantle boundary. Čadek et al. (2019a) assessed the accuracy of these
80 two approaches, and an “equal stress approach” proposed by Dahlen (1982), to a numerical
81 solution of viscous flow in the crust. The authors concluded that the equal pressure approach
82 may lead to inaccurate estimates of shell thickness for spherical harmonic degrees $l \leq 10$ when
83 assuming a constant viscosity. When viscosity of the ice was temperature dependent, all models
84 lost accuracy with increasing spherical harmonic degrees.

85 A few researchers have presented models that aimed to explain the support mechanism
86 for observed features on the surface of icy, ocean worlds such as the plateaus on Titan
87 (Schurmeier et al., 2016) and the megadome on Ganymede (Kay et al., 2018). For both instances,
88 buoyancy forces were not transferred to the surface, which caused the topography at the surface
89 and at the base of the shell to relax away. The megadome on Ganymede was found to be stable
90 over geologic times only when Pratt isostasy was the assumed mechanism of support. On Titan,
91 the plateaus could not be sustained via Airy isostasy, which implies that these features are
92 unlikely to be the result of crustal thickening.

93 Inherent to the nature of icy shells is the mechanical weakness near the ice-ocean
94 interface. In principle, a material near its melting point (i.e., the lower ice shell) should be
95 limited in its ability to transfer buoyancy stress to support surface topography. This characteristic
96 injects a level of complexity into the understanding of icy shell topographies and the mechanisms
97 that give rise to their support. In this paper, we explore the mechanics of buoyantly supported
98 topography from short wavelength to hemispheric wavelength scales. With upcoming spacecraft
99 missions such as NASA’s Europa Clipper, understanding the dynamics of ice shells is crucial for
100 providing a foundation for interpreting the trove of data that could profoundly alter our
101 knowledge of icy, ocean worlds.

102 **2 Methods**

103 2.1 Finite element simulations

104 We use the commercially available Hexagon Marc finite element package to test
105 systematically for buoyant support of topography in icy shells. The software is well vetted for

106 investigations of lithospheres of icy shells (e.g., Dampitz & Dombard, 2011; Dombard &
107 McKinnon, 2002, 2006a; Kay & Dombard, 2018). The code simulates Maxwell viscoelasticity,
108 capturing the behavior of geologic materials at both short and long timescales, (and can include
109 plasticity as a continuum approximation for brittle failure at large stresses). We test for buoyant
110 support of topography for wavelengths $\lambda = 10$ km up to hemispherical scales (i.e., spherical
111 harmonic degree $l = 2$), increasing logarithmically. Although applicable for all icy-ocean worlds,
112 we use Europa to guide selection of model parameters. The full finite element simulation is
113 comprised of two steps. A thermal simulation is run to determine the temperature structure, and
114 the results of the thermal simulation are used in the mechanical simulation.

115 2.2 Mesh and boundary conditions

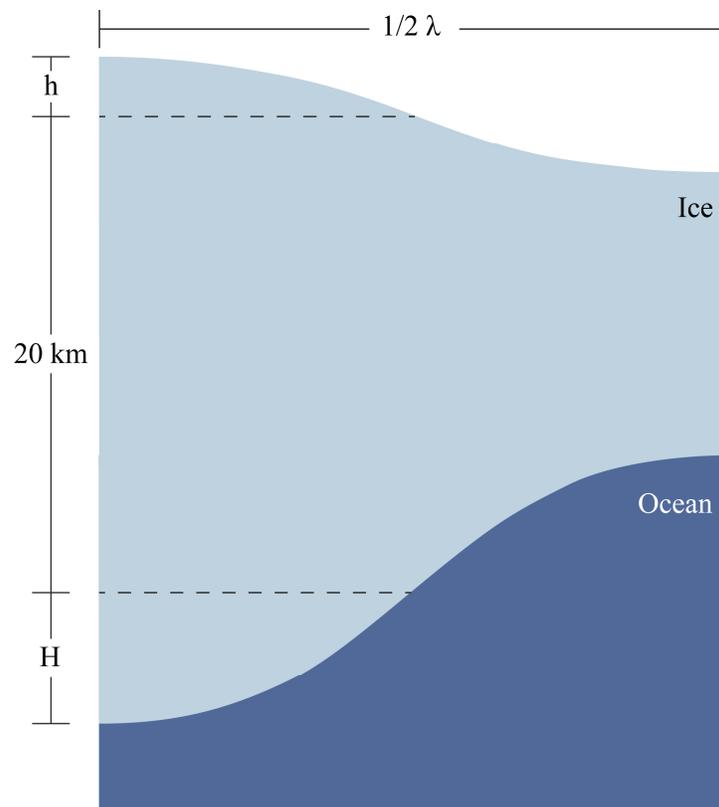
116 We utilize planar half space meshes split into two layers: an outer ice shell, ostensibly 20
117 km thick (e.g., Billings & Kattenhorn, 2005; Hussmann et al., 2002), and an ocean with a
118 boundary far enough away to not affect the solution (Figure 1). For short wavelengths, the ocean
119 mesh layer is ~ 2.5 times the wavelength. For long wavelengths ($l \geq 6$), the thickness of the ocean
120 layer is reduced to < 2.5 times the wavelength, because of computational limits for large meshes.
121 The planar meshes we implement do not include support from membrane stresses, which
122 becomes appreciable when the horizontal scale of a comparable spherical harmonic degree is
123 roughly the same as the square root of the planetary radius divided by the lithospheric thickness
124 (Turcotte, 1981). For Europa, this corresponds to a spherical harmonic degree of ~ 16 (~ 600 km).
125 As will be discussed in subsequent sections, the additional support provided by membrane stress
126 simply adds to the already sufficient support to maintain topography at those wavelengths.

127 Elements in our mesh are uniformly distributed in the ice shell and concentrated towards
128 the ice-ocean boundary in the ocean layer, while remaining below a 4:1 aspect ratio. Mesh
129 resolution is variable depending on the scale of the wavelength. At the smallest scales, the ice
130 portion of the mesh has, at minimum, 400 finite elements (40 in depth and 10 in width). At the
131 largest scales, this ratio is reversed to keep within the 4:1 aspect ratio, with 400 elements in
132 width and 10 in depth. Free-slip conditions are applied to the symmetry boundaries at the sides
133 of the mesh, restricting normal displacements, and the bottom of the mesh is locked horizontally
134 and vertically. On the surface, we place sinusoidal topography with an amplitude h of 100 m for
135 all wavelengths, as wavelength controls buoyant support rather than topographic amplitude (see

136 Turcotte & Schubert, 2014). We simulate two scenarios for initial topography at the base of the
 137 ice shell: 1) a flat basal surface (i.e., no initial compensating topography), and 2) a root in
 138 isostatic equilibrium. The maximum amplitude of the isostatic root H is determined by the
 139 density contrast between the water ice and ocean where

$$140 \quad H = \frac{\rho_i h}{\rho_w - \rho_i}. \quad (1)$$

141 In this equation, the density of the liquid water ρ_w is assumed to be 1000 kg/m^3 , while the density
 142 of the ice shell ρ_i is 920 kg/m^3 . Gravitational body forces are applied uniformly to the mesh with
 143 an acceleration equal to 1.315 m/s^2 .



144

145 **Figure 1.** Schematic of basic model setup using an ice shell assuming an initial state of isostasy.
 146 The amplitude h is 100 m for all models regardless of wavelength resulting in an amplitude for
 147 basal topography H of 1150 m.

148 2.3 Material and thermal properties

149 The elastic parameters for ice include a Young's modulus of 9.32 GPa and a Poisson ratio
 150 of 0.325 (Gammon et al., 1983). However, in simulations, applying a gravitational load to a
 151 compressible material leads to deviatoric stresses from self-compaction. Following Dombard and

152 McKinnon (2006b), we make the material nearly incompressible by adjusting the Poisson ratio to
 153 approach 0.5, while also reducing the Young's modulus to 7.83 GPa as to maintain the flexural
 154 rigidity of the ice. The viscosity structure of the ice layer has the greatest influence on relaxation
 155 rates (Dombard & McKinnon, 2006a), and when incompressibility is approximated, the variation
 156 is negligible.

157 Additionally, we follow the ductile creep laws from Goldsby and Kohlstedt (2001).
 158 Several mechanisms affect the viscous flow of ice below the brittle-ductile transition. There are
 159 three different dislocation (dis) creep mechanisms with temperature dependent contributions
 160 (Durham & Stern, 2001), diffusion (diff) creep; and dislocation in an easy-slip (ES) system and
 161 grain boundary sliding (GBS) acting as rate limiters. Thus, the total strain rate is

$$162 \quad \dot{\epsilon}_{total} = \dot{\epsilon}_{dis} + \dot{\epsilon}_{diff} + \left(\frac{1}{\dot{\epsilon}_{ES}} + \frac{1}{\dot{\epsilon}_{GBS}} \right)^{-1}. \quad (2)$$

163 Each of these strain rates can take the general form

$$164 \quad \dot{\epsilon} = A \left(\frac{1}{d} \right)^m \sigma' e^{-\left(\frac{Q}{RT} \right)}, \quad (3)$$

165 where $\dot{\epsilon}$ is the equivalent strain rate, A is a material dependent pre-exponential constant
 166 normalized for uniaxial deformation, m is a grain size index relating to the grain size d , Q is the
 167 activation energy, R is the universal gas constant, and T is absolute temperature. We assume a
 168 grain size of 1 mm (e.g., Dombard & McKinnon, 2006a). The full creep parameters are listed in
 169 Durham et al. (2010) and references therein.

170 The thermal conductivity of water ice is temperature dependent, given by $651/T$
 171 (Petrenko & Whitworth, 1999). In our steady state thermal simulation, we lock the surface
 172 temperature at 100 K and the ice-ocean interface temperature at the melting point of ice, 273 K
 173 ($q \approx 32 \text{ mW/m}^3$). Heat flows on Europa might be higher, instigating convection near the base of
 174 the shell (e.g., McKinnon, 1999; Tobie et al., 2003). Nonetheless, for our purposes, the situation
 175 is mechanically identical: a structurally weak base overlaid by a relatively thinner lithosphere
 176 (see below).

177 The water below the ice shell lacks material strength and therefore does not provide
 178 resistance; rather, it provides a buoyant restoring force. The viscosity is so low compared to the
 179 lithospheric ice in the upper shell that it simply needs to accommodate space for the deforming

180 ice at least as fast as that ice is deforming. Therefore, we set the material properties of the water
181 to be the same as the overlying ice layer, with the exception of the density contrast. We also test
182 for the effects of surface temperature, which have been shown to be a controlling factor in creep
183 rates (e.g., Dampitz & Dombard., 2011), and the effects for a reduction in the melting
184 temperature of ice due to the presence of an antifreeze such as ammonia (e.g., Spohn & Schubert,
185 2003). To test for effects of buoyancy, we run the simulations without the density contrast,
186 precluding isostasy. Variations in model parameters are shown in Supporting Information Table
187 S1.

188 We run the simulations for 3 Gyr, a time more than sufficient to capture any appreciable
189 flow in the lower ice shell. Time steps are controlled by the minimum Maxwell time, which is
190 dependent upon, in part, the viscosity (Turcotte & Schubert, 2014). We limit minimum viscosity
191 in the mesh to 10^{18} Pa s, keeping run times reasonable, and confirm that our results are not
192 sensitive to viscosity cutoffs below this value.

193 Simulations are performed under full large-strain formulation, which includes a second-
194 order term to strain-displacement relationships (e.g., Ranalli, 1995), and a continual geometric
195 update. Although strains are typically small, the large strain scheme is essential for the geometric
196 update, as without it, stresses would not decay as topography relaxes (e.g., Dombard &
197 McKinnon, 2006a). In addition, we apply constant dilatation across the elements to eliminate
198 potential numerical errors caused by simulating a nearly incompressible material.

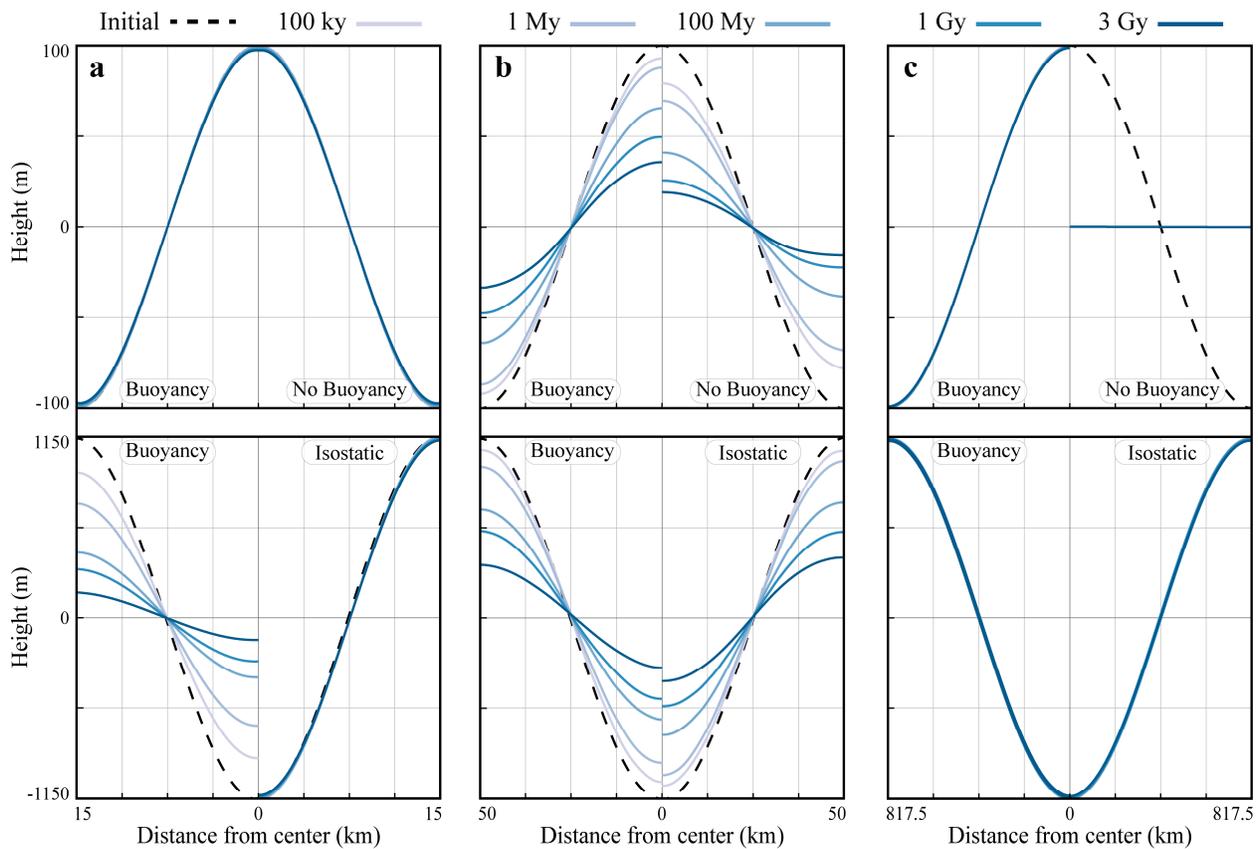
199 **3 Results**

200 The evolution of topography can be described best by three general wavelength
201 categories: short, intermediate, and long. Figure 2 shows the simulation results for the evolution
202 of topography at the surface and base of the ice shell for three representative wavelengths within
203 these categories.

204 At short wavelengths, the surface topography remains stable over the simulation time
205 regardless of buoyancy. In contrast, the basal topography relaxes away, losing 991 m of its initial
206 amplitude. For intermediate wavelengths, the surface topography relaxes in both the buoyancy
207 and no buoyancy simulations. However, the degree of relaxation is substantially greater when the
208 density contrast is not present. For instance, for a wavelength of 100 km, the surface loses
209 approximately 65% of its elevation over the simulation time, compared to an 85% loss for the

210 no-buoyancy scenario (Figure 2c). The root topography at intermediate wavelengths also relaxes
 211 when the density contrast is present. Moreover, it relaxes at a rate out of isostatic equilibrium
 212 with the relaxation of the surface topography (Figure 2d), resulting from the shared contribution
 213 of both lithospheric support and buoyancy (Figure 3a).

214 In the case of long wavelengths, without the density contrast, the surface completely
 215 relaxes away in less than 100 kyr. Conversely with the density contrast, the surface and the root
 216 are stable over geologic times.

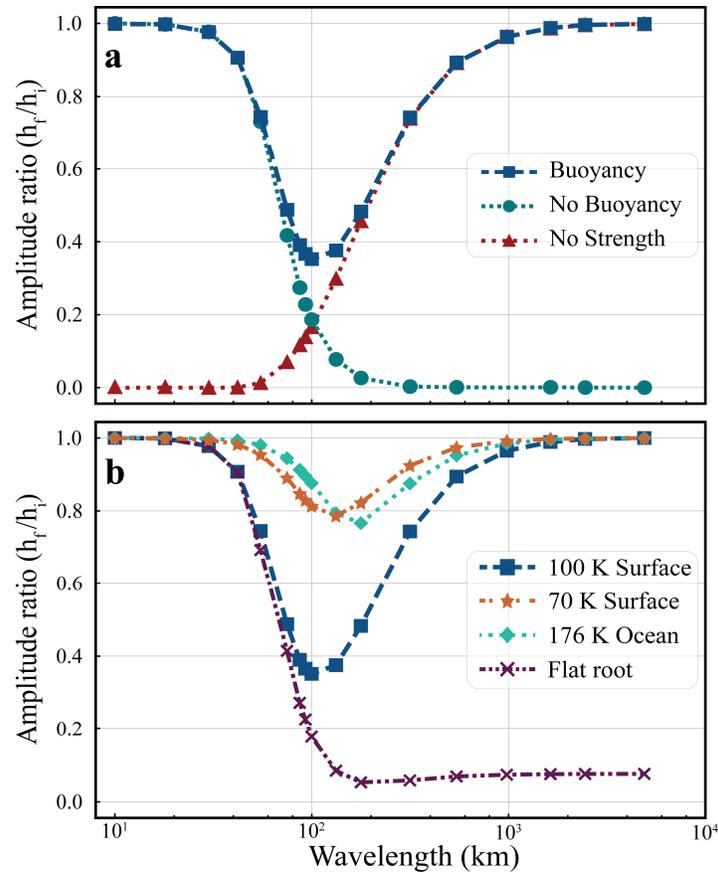


217

218 **Figure 2.** Simulation results for the surface topography (top row) and root topography (bottom
219 row) for wavelengths of a) 30 km, b) 100 km, and c) 1635 km (spherical harmonic degree $l \approx 6$).
220 The initial topography (black dashed line) is the same for each model, sinusoidal with an
221 amplitude of 100 m for the surface topography and 1150 m for the root topography. For the
222 surface topography, the left side of $x = 0$ are simulations with an ice-ocean density contrast (i.e.,
223 buoyancy), and the right side of $x = 0$ are the simulations without the density contrast (i.e., no
224 buoyancy). For the root topography, the left side of $x = 0$ are the simulations with the density
225 contrast (i.e., buoyancy), and the right side of $x = 0$ are the expected isostatic root topography
226 given by equation (1) assuming the surface of the buoyancy simulations (top row, left side of x -
227 axes). Because the buoyancy simulations are set up to be in isostatic equilibrium, the results
228 should be static if the precepts of Airy isostasy hold.

229 Simulations with a surface temperature of 100 K show that the lithosphere supports the
230 topographic load up to wavelengths of ~ 30 km (Figure 3). Lowering the surface temperature
231 increases the wavelengths at which lithospheric strength can effectively contribute to
232 topographic support. A decreased surface temperature also reduces the degree of relaxation
233 occurring at intermediate wavelengths. For example at 100 km, the simulated surface topography
234 with temperatures of 100 K relax ~ 65 m, compared to ~ 15 m when the surface temperature is 70
235 K. A similar effect is observed when the melting temperature (i.e., the temperature at the ice-
236 ocean interface) is reduced to 176 K (Figure 3b). Under these conditions, the contribution of
237 lithospheric support is greater at longer wavelengths, and the maximum topographic relaxation is
238 reduced.

239 At wavelengths < 80 km, the simulations with the base of the ice shell initially flat is
240 nearly identical to that with the simulations without the density contrast. This finding further
241 underscores that lithospheric strength is the primary mechanism of topographic support at short
242 wavelengths. However, at wavelengths greater than 100 km, without an initial compensated root,
243 the surface topography relaxes to just a few meters over the simulated time. At the longest
244 wavelengths, a root forms that is in isostatic equilibrium with the reduced surface topography.



245

246 **Figure 3.** Ratio of initial to final topographic amplitude for simulations across simulated
 247 wavelengths for a) simulations with and without buoyancy and b) variations in the assumed
 248 initial state of the temperature at the surface and base of the ice shell. Because the simulations
 249 (except the “Flat root” ones) are set up to be in isostatic equilibrium, the ratio should be pegged
 250 to a value of 1 if the precepts of Airy isostasy hold; i.e., Airy isostasy fails when the amplitude
 251 ratio is < 1 .

252 4 Discussion

253 The prevalent assumption that Airy isostasy can be utilized for estimating ice shell
 254 thickness does not hold true except at the longest wavelengths. A significant oversight in this
 255 assumption lies in the positioning of the lithosphere relative to the ice-ocean boundary. On Earth,
 256 the lithospheric boundary is situated beneath the density contrast of the crust-mantle boundary.
 257 Thus, when a surface load deforms the lithosphere, both the crust and the upper mantle will
 258 respond to the deformation (e.g., Barrell, 1914; Daly, 1940). However, in the case of ice shells,
 259 the lithosphere is embedded within the ice itself, because the ice in the lower portion of the shell
 260 is approaching its melting point and is therefore weak over geologic time scales. (A convective
 261 portion in an ice shell that passes a higher heat flux would only exasperate this effect.)

262 Our findings also illustrate that the lithosphere of ice shells can support topographic loads
263 at short wavelengths. The short-wavelength scenario shows surface topography to be stable with
264 and without buoyancy, while the initially isostatic basal topography relaxes away (Figure 2a),
265 indicating that contribution of buoyancy supporting the load is insignificant and that any root
266 sticking into the ocean would flow away rapidly. The length at which lithospheric support begins
267 to decrease is contingent on the surface temperature, and potentially, the presence of antifreeze in
268 the ocean. These findings agree with Damptz and Dombard (2011), who highlighted the
269 significant role of surface temperature in controlling creep at the base of the lithosphere for the
270 moons of Jupiter and Saturn. Due to viscous creep, the lithosphere is not static. Stresses relax at
271 the base of the lithosphere, allowing the lithosphere to thin with time. Surface temperature, and
272 to a lesser degree heat flow and grain size, control creep rates.

273 It is important to note that while the surface topography at both short and long
274 wavelengths can be stable over geologic times, the basal topography exhibits distinctly different
275 behaviors at varying spatial scales. At short wavelengths, basal topography relaxes away
276 indicating topographic features at these scales are not compensated. Conversely, at long
277 wavelengths, topography at the base of the ice shell stays in isostatic equilibrium with the surface
278 topography. This finding can be attributed to the relative size of the vertical flow region and the
279 lateral extent of the topography. For long wavelengths, the vertical extent of the area where
280 material can flow laterally and redistribute is small compared to the horizontal extent (cf.
281 Stevenson, 2000). This imbalance results in a highly channelized flow that inhibits the overall
282 relaxation process observed at shorter wavelengths.

283 Moreover, when the wavelength of a load is small compared to the planetary radius, the
284 stresses caused by the body's curvature can be ignored. However, when curvature is introduced,
285 horizontal membrane stresses can contribute to the support of the surface load (Turcotte, 1981).
286 In the context of this study, membrane stresses are appreciable at approximately spherical
287 harmonic degree $l = 16$, assuming the lithosphere is ~ 6 km thick. This degree corresponds to a
288 wavelength of ~ 600 km. At this scale, topography loses just 10% of its initial height when
289 surface temperature is 100 K, and this relaxation is further reduced for colder surface
290 temperatures (Figure 3). Consequently, the inclusion of membrane stresses would reinforce our
291 findings regarding the stability of long wavelength topography.

292 To sum up, there are two competing effects for the application of Airy isostasy in an ice
293 shell over an ocean, one of which does not exist in traditional applications of Airy isostasy in
294 terrestrial worlds. First, the lithospheric strength of icy shells is a sufficient support mechanism
295 for topography at short wavelengths, without needing help from buoyancy. This finding is
296 evidenced by the “No buoyancy” curve in Figure 3a, and it mirrors the lithospheric behavior of
297 rocky planets. In contrast, the “No strength” curve in Figure 3a, which does not come from
298 simulations but from subtracting the “Buoyancy” and “No buoyancy” curves and thus mimics a
299 scenario where any lithosphere would have no strength, stands in contrast to the expected
300 behavior in a terrestrial world, which would be locked at 1 at all scales. These two curves – “No
301 buoyancy” and “No strength” – together determine the final topographic support.

302 To understand this behavior, consider the effect of pressing one’s finger into a seat
303 cushion. The cushion diffuses the stresses introduced by the force of the finger, and the opposite
304 side of the cushion is negligibly deformed, if at all. A sufficiently wide scale load, however,
305 would feel the hard seat underneath, even when pressed into the cushion at the same force per
306 unit area. Translating to ocean worlds, the seat cushion inhibits buoyant support at short
307 wavelengths, though the lithosphere can provide the support, while concurrent, the basal
308 topography can relax away without affecting the surface. At long wavelengths, the seat cushion
309 is too thin for buoyancy to be inhibited, so topography can be supported, even if the lithosphere
310 provides no support. Simultaneously at these scales, highly channelized flow precludes
311 relaxation of the basal topography. At intermediate scales, neither the seat cushion or the
312 lithosphere are fully effective, and topography cannot be supported over geologic timescales by a
313 crustal root alone, and would need another mechanism of support (e.g., Pratt). How these curves
314 add, and thus the size of the intermediate zone, is dependent on local conditions, as suggested by
315 Figure 3b. For the case of Europa explored here, this wavelength range spans over an order of
316 magnitude (from < 100 km to > 1000 km), with substantial lack of support for equatorial regions
317 where surface temperatures are higher.

318 **4 Conclusion**

319 Our results yield a more nuanced understanding of the wavelength dependence of icy
320 shell evolution, offering insights that could enhance the interpretation of topography and gravity
321 data from the icy, ocean worlds. While surface topography of short-wavelength features persists

322 over geologic time, compensated roots relax away quickly. Consequently, shell thickness
323 estimates derived from short-wavelength features will not accurately represent the local
324 thickness. In contrast, long-wavelength topography could be supported by shell thickness
325 variations, such as on Titan (Nimmo and Bills, 2010). Surface topography at intermediate scales
326 is unlikely to be isostatically compensated via buoyant support.

327 The practical implications of this more complex behavior has already been revealed at
328 other icy worlds. The isolated plateaus on Titan appear to exist in the short wavelength regime,
329 being supported by the strength of Titan's lithosphere with any root relaxing away quickly,
330 suggesting top-down construction of the plateaus (Schurmeier et al., 2016). Conversely, the
331 megadome on Ganymede appears to be firmly in the intermediate regime, unable to be supported
332 by either lithospheric strength of a buoyant crustal root and implicating Pratt isostasy (Kay et al.,
333 2018).

334 Such complexities and limitations underscore the challenges inherent in applying
335 traditional isostatic models to icy shells and highlight the importance of continuous model
336 refinement. Furthermore, the importance of a nuanced understanding of the dynamics of icy
337 shells cannot be overstated, especially in the context of future space missions.

338 **Acknowledgments**

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340 subcontract 1656601.

341

342 **Open Research**

343 The Marc software is available from <https://hexagon.com/products/marc>; academic pricing is
344 available. Supporting information Table S2 shows the elevations of the surface and root
345 topographies, calculated from displacements, for each simulation type and wavelength. A csv file
346 of Table S2 is available at <https://doi.org/10.5281/zenodo.8277281> (Tucker & Dombard, 2023).

347

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