

RESEARCH ARTICLE

Output-feedback H_∞ control for IT-2 T-S fuzzy system with hybrid attacks via switching-like adaptive event-triggered mechanism

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Abstract

In this article, the switching-like adaptive event-triggered dynamic output feedback H_∞ control for the interval type-2 (IT-2) Takagi-Sugeno (T-S) fuzzy system over networks with hybrid attacks and Markovian packet losses is investigated. The switching-like adaptive event-triggered mechanism (SAETM) is involved with the characteristic of dynamically adjusting trigger threshold according to the change of the system states and appropriately selecting the event-triggered scheme in the light of the packet loss process is creatively proposed. The hybrid attack which is consisted of deception attack and aperiodic Denial-of-Service (DoS) attack is considered, and the deception attack is expressed as the Bernoulli random variable, while the aperiodic DoS attack is characterized by the frequency and duration in view of the average dwell-time (ADT) technique. By applying the packet loss dependent multiple Lyapunov function technique and iterative method, the sufficient condition is derived to address the stability of NCS, and the existence condition is established to guarantee the prescribed performance of the H_∞ controller. Finally, two simulation examples are applied to show the availability of the provided algorithm.

KEYWORDS:

Switching-like adaptive event-triggered mechanism, the hybrid attacks, IT-2 T-S fuzzy system, dynamic output feedback, H_∞ performance.

1 | INTRODUCTION

Networked control systems (NCSs) are extensively used in industrial control field due to the benefits on low budget expenditure, convenient maintenance and installations, and a large number of literatures can be found in studying the NCSs^{1,2,3,4}. It should be noticed that the NCSs are increasingly intricate due to the uncertainty and nonlinearity which becomes a hot research topic in the last decades. Among the existing approaches for addressing the problem of nonlinear NCSs with uncertainty^{5,6,7,8}, Takagi-Sugeno (T-S) fuzzy model demonstrates great advantage, since it takes a sequence of local linear submodels to represent the global nonlinear system and the fruitful of theories of linear system can be directly utilized. For instance,^{9,10} investigated the stability and controller design for the fuzzy nonlinear systems sufficiently based on the T-S fuzzy model, and the time delay system approach and the switch system approach were introduced, respectively. Remarkably, the aforementioned results of dealing with nonlinear NCSs in view of type-1 T-S fuzzy model are incapable of describing the uncertainties existing in the nonlinear system. Hence, the interval type-2 (IT2) T-S fuzzy model which possesses the outstanding property to model nonlinear NCSs with uncertainties by the momentous upper and lower membership functions is further studied, see e.g.,^{11,12}.

Recently, the event-triggered mechanism (ETM) is addressed to improve the efficiency of resource utilization under limited network bandwidth in NCSs, where a settled threshold is given in advance to determine that to send or not to send the sampled signal at each sampling instant, see^{13,14,15,16}. The traditional ETM based on settled threshold is unable to adjust itself to cater the system changes and may cause the incongruous utilization of communication resources. Hence, the adaptive event-triggered mechanism (AETM) with variable threshold has been investigated in^{17,18,19} to save more communication resources. However, it is worth pointing out that the above works are under the common assumption that all triggered signals are successfully transferred to the sequential nodes. Unfortunately, most communication networks are vulnerable to the packet losses due to the network congestion, which may destroy the predetermined trigger condition and even result in poor control performance and instability of the system. Therefore, the AETM is no longer practicable again and should be redesigned by explicitly considering the packet loss process. This paper proposes a switching-like adaptive event-triggered mechanism (SAETM) for nonlinear NCSs expressed by IT2 T-S fuzzy model, in which the AETM means that the trigger threshold is adaptively updated along with the change of the system states and the switching-like implies that the appropriate AETM is actively selected depending on whether or not the packet loss occurs, which is regarded as the first motivation of the present study.

Another research focus is the cyber attack, which may give rise to extremely negative effects on NCS due to the distortion of the transmitted signal. Denial-of-Service (DoS) attack and deception attack are two major cyber attacks. The former is to interrupt transmission process between the components of system, see, e.g.,^{20,21,22}.²⁰ proposed a novel collaborative design framework for event-triggered control for NCS involved with unexpected DoS attack. In²¹, the security issue of cyber-physical system under DoS attack was investigated, and the stability condition dependent on the quantity of consecutive attacks was derived by constructing the nested switching model. However, the above studies are all focused on periodic DoS attack, that is, the duration of every DoS attack happens within regular time interval. Hence, a much more universal aperiodic DoS attack was proposed in^{23,24} and the results were applied to DC microgrids and the distributed cooperative control, respectively. The latter deception attack discussed in^{25,26,27} generally have the feature of injecting fraudulent information into the data obtaining from communication network.²⁵ proposed a distributed algorithm for realizing consensus of multiagent network with deception attack.²⁶ concerned about the cloud-aided active suspension plant secure issue with memory-based ETM and deception attack. Nevertheless, many published researches were concentrated on single attack, while the practical plant would be exposed to various cyber attacks, it claims the attention to hybrid attack which has been discussed in^{28,29}. In²⁸, the security control problem for multiagent system considering hybrid attack including periodic DoS attack and replay attack together was presented.²⁹ addressed the multiarea power system dynamic ETM load frequency control with the periodic DoS attack and deception attack. However, few studies have focused on the hybrid attack with aperiodic DoS attack and deception attack simultaneously, not to mention that the IT2 T-S fuzzy system under SAETM, which is identified as the second motivation of the present study.

Significantly, the literatures mentioned above almost all concentrated on the scenario that the states are able to be accurately acquired, which, in practice, for most practical complex systems, the states are usually cannot be readily obtainable. Two types of output-feedback control approaches for designing and analyzing NCSs with unmeasurable state are investigated in recent years. The first type is the output-feedback control based on state observer, and quite a lot works have been done, see e.g.,^{30,31,32,33}. An observer-based dynamic event-triggered strategy for NCSs was addressed in³⁰ and the power-constrained DoS attack which aimed at impeding the network communication occasionally was considered.³¹ provided an event-triggered observer-based controller for multiagent systems with DoS attack. Similar contributions have been done in^{32,33}. The second type of output-feedback control with less conservative by online updating all of the controller parameters, is dynamic output-feedback control, see, e.g.,^{34,35,36,37}.³⁴ designed a dynamic output-feedback controller collaborative with the AETM for a sires of nonlinear plants subjected to actuator faults and the control dilemma of constructing an expected controller in a mode-dependent framework was discussed. Based on the dynamic output-feedback controller,³⁵ investigated a novel discrete ETM for the synchronization of delayed neural networks. Similar works can be found in^{36,37}. It is undoubtedly that some very nice research works have been provided on the output-feedback control for NCSs. However, to the best of the authors' knowledges, extremely rare literature addressed the dynamic output-feedback control relating to the SAETM for IT2 T-S fuzzy system with hybrid attacks and packet losses, which further motivates the study in this paper.

In this paper, we study the switching-like adaptive event-triggered dynamic output-feedback H_∞ control for IT2 T-S fuzzy system with hybrid attacks and Markovian packet losses. The main contributions are listed below.

1) Compared with the traditional ETM^{13,36}, the creatively proposed SAETM strategy not only dynamically adjusts the trigger threshold by the change of system states, but also fully considers the properties of the packet loss process, which significantly improves the transmission efficiency.

2) The hybrid attack involving deception attack and aperiodic DoS attack is considered, which is more practical than that of the single type of aperiodic DoS attack or deception attack.

3) By considering the SAETM and packet loss and hybrid cyber attack in an unified framework, the desired H_∞ performance is ensured based upon the packet loss dependent multiple Lyapunov function technique and iterative method.

Notations: In this paper, $\text{Prob}\{\cdot\}$ is the possibility of events occurring. $\mathbb{E}\{\delta\}$ implies the expectation of stochastic variable respected to δ . λ_{\max} and λ_{\min} are the maximum and minimum eigenvalues of the matrix. And \mathcal{N}^+ represents the positive integer.

2 | PROBLEM FORMULATION

Plant rule i : IF $f_1(x(k))$ is M_1^i , $f_2(x(k))$ is M_2^i and \dots and $f_\ell(x(k))$ is M_ℓ^i , THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + D_{1i} \omega(k) \\ z(k) &= C_{1i} x(k) + D_{2i} \omega(k) \\ y(k) &= C_{2i} x(k) \end{aligned} \quad (1)$$

where $f_1(x(k)), f_2(x(k)), \dots, f_\ell(x(k))$ are the premise variables, ℓ implies the total amount of inference rules, i implies the i th fuzzy inference rule, and $M_v^i, i \in \{1, \dots, \ell\}, v \in \{1, \dots, \ell\}$ denotes the fuzzy set of rule i ; $x \in \mathfrak{R}^{n_x}, u \in \mathfrak{R}^{n_u}$ are the unmeasurable state and the input of the system, respectively; $z \in \mathfrak{R}^{n_z}$ is the controlled output and $y \in \mathfrak{R}^{n_y}$ is the measured output, and $\omega \in \mathfrak{R}^{n_\omega}$ is the external disturbance. In addition, $A_i, B_i, C_{1i}, C_{2i}, D_{1i}, D_{2i}$ are pre-given constant matrices. The firing strength of the i th rule is denoted by the interval sets as follows:

$$h_i(x(k)) = [\underline{h}_i(x(k)), \bar{h}_i(x(k))] = \left[\prod_{v=1}^{\sigma} \underline{\mu}_{M_v^i}(f_v(x(k))), \prod_{v=1}^{\sigma} \bar{\mu}_{M_v^i}(f_v(x(k))) \right]$$

Then, the above system is inferred as follows:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{\ell} h_i(x(k)) [A_i x(k) + B_i u(k) + D_{1i} \omega(k)] \\ z(k) &= \sum_{i=1}^{\ell} h_i(x(k)) [C_{1i} x(k) + D_{2i} \omega(k)] \\ y(k) &= \sum_{i=1}^{\ell} h_i(x(k)) [C_{2i} x(k)] \end{aligned} \quad (2)$$

where $\underline{h}_i(x(k)) \geq 0$ and $\bar{h}_i(x(k)) \geq 0$ denote the lower and upper grades of membership, respectively. $\underline{\mu}_{M_v^i}(f_v(x(k))) \in [0, 1]$ and $\bar{\mu}_{M_v^i}(f_v(x(k))) \in [0, 1]$ respectively are the lower and upper membership functions. Furthermore, $h_i(x(k)) = \underline{h}_i(x(k)) \underline{\zeta}_i(x(k)) + \bar{h}_i(x(k)) \bar{\zeta}_i(x(k))$, $h_i(x(k)) \in [0, 1]$ and $\sum_{i=1}^{\ell} h_i(x(k)) = 1$. $\underline{\zeta}_i(x(k))$ and $\bar{\zeta}_i(x(k))$ are weighting functions and satisfy $\underline{\zeta}_i(x(k)) + \bar{\zeta}_i(x(k)) = 1$.

3 | COMMUNICATION NETWORKED DESCRIPTION

The detailed description to illustrate the workflow of the studied NCS in Figure.1 is presented as follows:

The sampled signal $y(k)$ is judged by a switching-like adaptive event-triggered mechanism (SAETM) and the triggered signal $y(t_m)$ is suffered from the packet loss due to the characteristic of the communication network. The acknowledge character (ACK) technique is employed to detect whether data packet $y(t_m)$ is successfully delivered to the buffer. In view of the signal $\bar{y}(k)$ acquired from the buffer, the controller calculates $\hat{u}(k)$ and transmits it into the communication network. Assume that $\hat{u}(k)$ is threatened by the hybrid cyber attack including aperiodic DoS attack and deception attack. The SAETM, packet loss as well as hybrid cyber attack are described as follows.

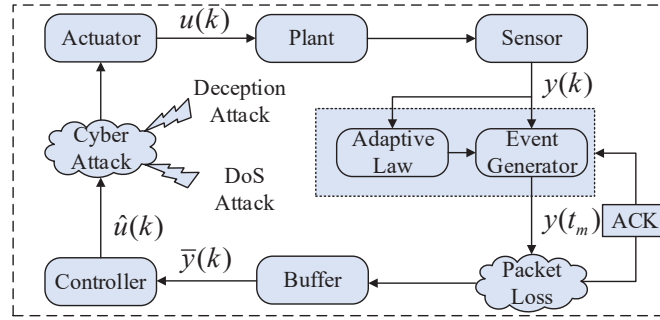


Figure 1 Structure chart of the NCS.

(1) Description of the SAETM.

Define $y(t_m)$ as the latest triggered signal, and $\tau(k) = y(t_m) - y(k)$ as the transmission error. The SAETM in (3) is designed to determine the next triggered instant

$$t_{m+1} = \begin{cases} \inf_k \{k > t_m, \|\tau(k)\|_{\Xi}^2 \geq \varepsilon_1(k) \|\mathbf{y}(t_m)\|_{\Xi}^2\}, \text{ACK} = 1 \\ \inf_k \{k > t_m, \|\tau(k)\|_{\Xi}^2 \geq \varepsilon_0(k) \|\mathbf{y}(t_m)\|_{\Xi}^2\}, \text{ACK} = 0 \end{cases} \quad (3)$$

where ACK signal is introduced to reveal that the triggered signal is successfully transmitted or not. When $\text{ACK} = 1$, $y(t_m)$ has been successfully transmitted, the adaptive event-triggered threshold is set as $\varepsilon_1(k)$; otherwise, when $\text{ACK} = 0$, the packet loss occurs, the adaptive event-triggered threshold is switched to $\varepsilon_0(k)$. Moreover, the adaptive laws are

$$\begin{cases} \Delta \varepsilon_1(k) = \left(\frac{1}{\varepsilon_1(k)} - \mathcal{J}_1\right) \tau(k)^T \Xi \tau(k) \\ \Delta \varepsilon_0(k) = \left(\frac{1}{\varepsilon_0(k)} - \mathcal{J}_0\right) \tau(k)^T \Xi \tau(k) \end{cases} \quad (4)$$

Remark 1. In view of the adaptive law in (4) depending on the transmission error $\tau(k)$, the threshold $\varepsilon_1(k)$ and $\varepsilon_0(k)$ are adjusted dynamically. When $\tau(k) \rightarrow 0$, $\Delta \varepsilon_1(k) = 0$ and $\Delta \varepsilon_0(k) = 0$, thus, both $\varepsilon_1(k)$ and $\varepsilon_0(k)$ approaches to a fixed value. Moreover, if $\mathcal{J}_1 = \frac{1}{\varepsilon_1(0)}$ and $\mathcal{J}_0 = \frac{1}{\varepsilon_0(0)}$, we can derive that $\Delta \varepsilon_1(k) = 0$ and $\Delta \varepsilon_0(k) = 0$.

From the adaptive laws and the trigger conditions above, we can obtain that

$$\begin{cases} y(t_{m+1}) = y(k), (3) \text{ is satisfied.} \\ y(t_m) \text{ holds, } (3) \text{ is not satisfied.} \end{cases} \quad (5)$$

(2) Description of the Markovian packet loss.

We introduce the Markovian process to represent the packet loss. $\beta(k) = 1$ corresponds to that the signal has been successfully transmitted, while $\beta(k) = 0$ refers to that the packet loss occurs. Subsequently, the transition matrix is $\mathfrak{F} = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix}$, wherein $p = \text{Prob}(\beta(k+1) = 0 | \beta(k) = 1)$ and $q = \text{Prob}(\beta(k+1) = 1 | \beta(k) = 0)$ are the probabilities for failure and recovery, respectively. Thus, the controller input is

$$\bar{y}(k) = \beta(k)y(t_m) + (1 - \beta(k))\xi \bar{y}(k-1) \quad (6)$$

where ξ is the forgetting factor.

(3) Description of the hybrid cyber attack.

On the one hand, in the shared communication network, the controller output may be tampered by malicious opponents, which called deception attack, and expressed as

$$\bar{u}(k) = \hat{u}(k) + \theta(k)\phi(k) \quad (7)$$

where $\phi(k) = f(\hat{u}(k)) - \hat{u}(k)$ is the malicious signal sent by the opponent and $f(\hat{u}(k))$ implies the deception attack signal. The deception attack probability is expressed by a Bernoulli random variable $\theta(k)$, wherein $\theta(k) = 1$ means the deception attack

occurs while $\theta(k) = 0$ indicates the absence of deception attack, and it satisfies

$$\begin{cases} \text{Prob}\{\theta(k) = 1\} = \mathbb{E}\{\theta(k)\} = \bar{\theta} \\ \text{Prob}\{\theta(k) = 0\} = 1 - \bar{\theta} \end{cases}$$

Assumption 1. The deception attack signal $f(\hat{u}(k))$ satisfies the following condition for given matrix F

$$\|f(\hat{u}(k))\|_2 \leq \|F\hat{u}(k)\|_2.$$

On the other hand, the DoS attacks to interrupt the transmission between the components of system may also cause disastrous consequences in the open communication network. And, the amount of DoS attacks that the NCS can susceptible need to be determined. Thus, suitable conditions are imposed on both DoS frequency and duration. Define $\Pi_f[\psi, k]$ and $\Pi_d[\psi, k]$ respectively as the amount of DoS attacks transition between on/off and the duration within the interval of $[\psi, k]$. The assumptions about DoS attack are presented as follows.

Assumption 2. (Dos frequency): For given constants $\pi > 0$ and $\sigma > 0$, the following condition holds

$$\Pi_f[\psi, k] \leq \sigma + \frac{k - \psi}{\pi}.$$

Assumption 3. (Dos duration): For given constants $T > 0$ and $\epsilon > 0$, the following condition holds

$$\Pi_d[\psi, k] \leq \epsilon + \frac{k - \psi}{T}.$$

Assume that $[I_n^{\text{on}}, I_n^{\text{off}}]$, $n \in \mathcal{N}^+$ is the active interval for the aperiodic DoS attack. Similarly, the variable $\eta(k)$ is invoked to characterize the DoS attacks. $\eta(k) = 1$ refers to the DoS attacks occurs and $\eta(k) = 0$ indicates the absence of DoS attacks. Therefore, we can get

$$u(k) = (1 - \eta(k))\bar{u}(k) = (1 - \eta(k))(\hat{u}(k) + \theta(k)\phi(k)) \quad (8)$$

where

$$\eta(k) = \begin{cases} 1, k \in [I_n^{\text{on}}, I_n^{\text{off}}] \\ 0, \text{others} \end{cases}$$

The output feedback fuzzy controller is designed for the NCS. The controller for IT2 T-S fuzzy NCS with \wp rules is formulated as below.

Controller rule j : IF $p_1(x(k))$ is L_1^j , $p_2(x(k))$ is L_2^j and \dots and $p_{\mathfrak{S}}(x(k))$ is $L_{\mathfrak{S}}^j$. THEN,

$$\begin{aligned} \hat{x}(k+1) &= \hat{A}_j \hat{x}(k) + \hat{B}_j \bar{y}(k) \\ \hat{u}(k) &= \hat{C}_j \hat{x}(k) \end{aligned}$$

where $p_1(x(k)), p_2(x(k)), \dots, p_{\mathfrak{S}}(x(k))$ are the premise variables, \wp is the total amount of inference rules, j is the j th fuzzy inference rule and L_i^j , $j \in \{1, \dots, \wp\}$, $i \in \{1, \dots, \mathfrak{S}\}$ refers to the fuzzy set of Rule j ; $\hat{x} \in \mathfrak{R}^{n_x}$ denotes the controller state; $\{\hat{A}_j, \hat{B}_j, \hat{C}_j\}$ are matrices that need to be confirmed. The firing strength of the j th rule expressed by the interval sets is shown as follows:

$$\vartheta_j(x(k)) = [\underline{\vartheta}_j(x(k)), \bar{\vartheta}_j(x(k))] = \left[\prod_{i=1}^{\mathfrak{S}} \underline{\varrho}_{L_i^j}(p_i(x(k))), \prod_{i=1}^{\mathfrak{S}} \bar{\varrho}_{L_i^j}(p_i(x(k))) \right]$$

where $\underline{\vartheta}_j(x(k))$ and $\bar{\vartheta}_j(x(k))$ indicate the lower and upper grades of membership, respectively. $\underline{\varrho}_{L_i^j}(p_i(x(k)))$ and $\bar{\varrho}_{L_i^j}(p_i(x(k)))$ respectively denote the lower and upper membership functions. Consider

$$\vartheta_j(x(k)) = \frac{\underline{\vartheta}_j(x(k))\underline{\alpha}_j(x(k)) + \bar{\vartheta}_j(x(k))\bar{\alpha}_j(x(k))}{\sum_{j=1}^{\wp} (\underline{\vartheta}_j(x(k))\underline{\alpha}_j(x(k)) + \bar{\vartheta}_j(x(k))\bar{\alpha}_j(x(k)))}$$

where $\vartheta_j(x(k)) \in [0, 1]$ and $\sum_{j=1}^{\wp} \vartheta_j(x(k)) = 1$. Similarly, $\underline{\alpha}_j(x(k))$ and $\bar{\alpha}_j(x(k))$ are nonlinear functions and meet $\underline{\alpha}_j(x(k)) + \bar{\alpha}_j(x(k)) = 1$.

It is easy to obtain that the IT2 T-S fuzzy controller

$$\begin{aligned}\hat{x}(k+1) &= \sum_{j=1}^{\wp} \vartheta_j(\chi(k)) [\hat{A}_j \hat{x}(k) + \hat{B}_j \bar{y}(k)] \\ \hat{u}(k) &= \sum_{j=1}^{\wp} \vartheta_j(\chi(k)) [\hat{C}_j \hat{x}(k)]\end{aligned}\quad (9)$$

It is noted that the abbreviation h_i, ϑ_j are utilized to represent $h_i(\chi(k))$ and $\vartheta_j(\chi(k))$.

By combining (2), (6), (8) and (9), by denoting $\tilde{x}(k) = [x(k)^T \hat{x}(k)^T]^T$, defining $\tilde{\theta}(k) = \theta(k) - \bar{\theta}$, $\mathbb{E}\{\tilde{\theta}(k)\} = 0$, $\mathbb{E}\{\tilde{\theta}(k)\tilde{\theta}(k)\} = \bar{\theta}(1 - \bar{\theta}) = b^2$, we can get the closed-loop control system as below:

$$\tilde{x}(k+1) = \begin{cases} \sum_{i=1}^{\wp} \sum_{j=1}^{\wp} h_i \vartheta_j \left[\Upsilon_{ij}^1 \tilde{x}(k) + \Upsilon_{ij}^2 \bar{y}(k-1) + \Upsilon_{ij}^3 \omega(k) + \Upsilon_{ij}^4 f(\hat{u}(k)) + \Upsilon_{ij}^5 \tau(k) + \tilde{\theta}(k) \left(\Upsilon_{ij}^6 \tilde{x}(k) + \Upsilon_{ij}^7 f(\hat{u}(k)) \right) \right], & k \in [I_n^{\text{on}}, I_n^{\text{off}}) \\ \sum_{i=1}^{\wp} \sum_{j=1}^{\wp} h_i \vartheta_j \left[\mathcal{H}_{ij}^1 \tilde{x}(k) + \Upsilon_{ij}^2 \bar{y}(k-1) + \Upsilon_{ij}^3 \omega(k) + \Upsilon_{ij}^5 \tau(k) \right], & \text{others} \end{cases}\quad (10)$$

where

$$\begin{aligned}\Upsilon_{ij}^1 &= \begin{bmatrix} A_i & (1-\bar{\theta})B_i\hat{C}_j \\ \beta(k)\hat{B}_jC_{2i} & \hat{A}_j \end{bmatrix}, \Upsilon_{ij}^2 = \begin{bmatrix} 0 \\ (1-\beta(k))\hat{B}_j\xi \end{bmatrix}, \Upsilon_{ij}^3 = \begin{bmatrix} D_{1i} \\ 0 \end{bmatrix}, \Upsilon_{ij}^4 = \begin{bmatrix} \bar{\theta}B_i \\ 0 \end{bmatrix}, \\ \Upsilon_{ij}^5 &= \begin{bmatrix} 0 \\ \beta(k)\hat{B}_j \end{bmatrix}, \Upsilon_{ij}^6 = \begin{bmatrix} 0 & -B_i\hat{C}_j \\ 0 & 0 \end{bmatrix}, \Upsilon_{ij}^7 = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \mathcal{H}_{ij}^1 = \begin{bmatrix} A_i & 0 \\ \beta(k)\hat{B}_jC_{2i} & \hat{A}_j \end{bmatrix}.\end{aligned}$$

Before proceeding, some definitions are given:

Definition 1. The closed-loop control system is stochastically stable if there exists such condition,

$$\mathbb{E} \left\{ \sum_{k=0}^{\infty} \|\tilde{x}(k)\|_2 \right\} < \infty.$$

Definition 2. The closed-loop control system is stochastically stable with H_{∞} performance level \mathfrak{d} if there are:

- 1) The closed-loop control system is stochastically stable with $\omega(k) = 0$.
- 2) For any $\omega(k) \neq 0$ under zero initial condition, we have

$$\sum_{k=0}^{\infty} \mathbb{E} z^T(k) z(k) \leq \mathfrak{d}^2 \sum_{k=0}^{\infty} \mathbb{E} \omega^T(k) \omega(k).$$

4 | MAIN RESULT

In this section, the sufficient condition is derived to ensure the stability of NCS, and the existence condition of IT2 T-S fuzzy controller is established to guarantee the prescribed H_{∞} performance by applying the packet loss dependent multiple Lyapunov functions technique and iterative method.

4.1 | Stability analysis of the NCS

Theorem 1. For the IT2 T-S fuzzy system (1), $\bar{\theta}, \gamma$ are prescribed scalars, the closed-loop control system (10) is stochastically stable if there exist positive definite matrices $P_{\beta(k)}^m, \bar{P}_{\beta(k)}^m, Q_{\beta(k)}^m, \bar{Q}_{\beta(k)}^m, \bar{W}_m, \bar{G}_m, (m=1,2)$ and matrices $\{\hat{A}_j, \hat{B}_j, \hat{C}_j\}$ satisfying

$\Omega_{\beta(k)}^1 < \chi \Omega_{\beta(k)}^2$, $\Omega_{\beta(k)}^2 < \chi \Omega_{\beta(k)}^1$, $\tilde{\varphi}_3 \chi^{1/\pi} < 1$, $0 < \frac{\ln \tilde{\varphi}_2 - \ln \tilde{\varphi}_3}{-\ln \tilde{\varphi}_1 + \ln \tilde{\varphi}_3} \leq T - 1$ and the following inequalities:

$$\begin{bmatrix} \Theta_{ij}^1 & * \\ \Theta_{ij}^2 & \Theta_{ij}^3 \end{bmatrix} \geq 0 \quad (11)$$

$$\begin{bmatrix} B_{ij}^1 & * \\ B_{ij}^2 & B_{ij}^3 \end{bmatrix} \geq 0 \quad (12)$$

$$\begin{bmatrix} D_{ij}^1 & * \\ D_{ij}^2 & \Theta_{ij}^3 \end{bmatrix} \geq 0 \quad (13)$$

$$\begin{bmatrix} B_{ij}^1 & * \\ \Theta_{ij}^2 & B_{ij}^3 \end{bmatrix} \geq 0 \quad (14)$$

$$\begin{bmatrix} \mathcal{E}_{ij}^1 & * \\ \mathcal{E}_{ij}^2 & \mathcal{E}_{ij}^3 \end{bmatrix} \geq 0 \quad (15)$$

$$\begin{bmatrix} H_{ij}^1 & * \\ H_{ij}^2 & H_{ij}^3 \end{bmatrix} \geq 0 \quad (16)$$

$$\begin{bmatrix} J_{ij}^1 & * \\ J_{ij}^2 & J_{ij}^3 \end{bmatrix} \geq 0 \quad (17)$$

$$\begin{bmatrix} H_{ij}^1 & * \\ \mathcal{K}_{ij}^2 & H_{ij}^3 \end{bmatrix} \geq 0 \quad (18)$$

where

$$\begin{aligned} \bar{W}_m &= \begin{bmatrix} \bar{R}_m & * \\ -\bar{R}_m & \bar{T}_m \end{bmatrix}, P_{\beta(k)}^m = \begin{bmatrix} M_{\beta(k)}^m & * \\ -M_{\beta(k)}^m & N_{\beta(k)}^m \end{bmatrix}, \bar{P}_{\beta(k)}^m = \begin{bmatrix} \bar{M}_{\beta(k)}^m & * \\ -\bar{M}_{\beta(k)}^m & \bar{N}_{\beta(k)}^m \end{bmatrix}, \\ \Theta_{ij}^1 &= \text{diag} \{ (1 - \varphi_1)P_1^1, (1 - \varphi_1)Q_1^1, \gamma^2 I, \bar{\theta} I, \mathcal{J}_1 \Xi \}, \Theta_{ij}^3 = \text{diag} \{ \bar{W}_1, \bar{W}_1, \bar{G}_1, I, I, \Xi \}, \\ B_{ij}^1 &= \text{diag} \{ \bar{W}_1, \bar{G}_1 \}, B_{ij}^3 = \text{diag} \{ \bar{P}_0^1, \bar{Q}_0^1, \bar{P}_1^1, \bar{Q}_1^1 \}, D_{ij}^1 = \text{diag} \{ (1 - \varphi_1)P_0^1, (1 - \varphi_1)Q_0^1, \gamma^2 I, \bar{\theta} I, \mathcal{J}_0 \Xi \}, \\ \mathcal{E}_{ij}^1 &= \text{diag} \{ (1 + \varphi_2)P_1^2, (1 + \varphi_2)Q_1^2, \gamma^2 I, \mathcal{J}_1 \Xi \}, \mathcal{E}_{ij}^3 = \text{diag} \{ \bar{W}_2, \bar{G}_2, I, \Xi \}, \mathcal{H}_{ij}^1 = \text{diag} \{ \bar{W}_2, \bar{G}_2 \}, \\ \mathcal{H}_{ij}^3 &= \text{diag} \{ \bar{P}_0^2, \bar{Q}_0^2, \bar{P}_1^2, \bar{Q}_1^2 \}, \mathcal{J}_{ij}^3 = \text{diag} \{ \bar{W}_2, \bar{G}_2, I, \Xi \}, \mathcal{J}_{ij}^1 = \text{diag} \{ (1 + \varphi_2)P_0^2, (1 + \varphi_2)Q_0^2, \gamma^2 I, \mathcal{J}_0 \Xi \}, \\ \mathcal{E}_{ij}^2 &= \begin{bmatrix} A_i & 0 & 0 & D_{1i} & 0 \\ \hat{B}_j C_{2i} & \hat{A}_j & 0 & 0 & \hat{B}_j \\ C_{2i} & 0 & 0 & 0 & I \\ C_{1i} & 0 & 0 & D_{2i} & 0 \\ C_{2i} & 0 & 0 & 0 & I \end{bmatrix}, \mathcal{J}_{ij}^2 = \begin{bmatrix} A_i & 0 & 0 & D_{1i} & 0 \\ 0 & \hat{A}_j & \xi \hat{B}_j & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 \\ C_{1i} & 0 & 0 & D_{2i} & 0 \\ C_{2i} & 0 & 0 & 0 & I \end{bmatrix}, \\ \Theta_{ij}^2 &= \begin{bmatrix} A_i & (1 - \bar{\theta})B_i \hat{C}_j & 0 & D_{1i} & \bar{\theta} B_i & 0 \\ \hat{B}_j C_{2i} & \hat{A}_j & 0 & 0 & 0 & \hat{B}_j \\ 0 & -b B_i \hat{C}_j & 0 & 0 & b B_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_{2i} & 0 & 0 & 0 & 0 & I \\ C_{1i} & 0 & 0 & D_{2i} & 0 & 0 \\ 0 & \bar{\theta}^{\frac{1}{2}} F \hat{C}_j & 0 & 0 & 0 & 0 \\ C_{2i} & 0 & 0 & 0 & 0 & I \end{bmatrix}, D_{ij}^2 = \begin{bmatrix} A_i & (1 - \bar{\theta})B_i \hat{C}_j & 0 & D_{1i} & \bar{\theta} B_i & 0 \\ 0 & \hat{A}_j & \xi \hat{B}_j & 0 & 0 & 0 \\ 0 & -b B_i \hat{C}_j & 0 & 0 & b B_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi & 0 & 0 & 0 \\ C_{1i} & 0 & 0 & D_{2i} & 0 & 0 \\ 0 & \bar{\theta}^{\frac{1}{2}} F \hat{C}_j & 0 & 0 & 0 & 0 \\ C_{2i} & 0 & 0 & 0 & 0 & I \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{ij}^2 &= \begin{bmatrix} p^{\frac{1}{2}} \bar{R}_1 & * & * \\ -p^{\frac{1}{2}} \bar{R}_1 & p^{\frac{1}{2}} \bar{T}_1 & * \\ 0 & 0 & p^{\frac{1}{2}} \bar{G}_1 \\ (1-p)^{\frac{1}{2}} \bar{R}_1 & * & * \\ -(1-p)^{\frac{1}{2}} \bar{R}_1 & (1-p)^{\frac{1}{2}} \bar{T}_1 & * \\ 0 & 0 & (1-p)^{\frac{1}{2}} \bar{G}_1 \end{bmatrix}, \mathcal{O}_{ij}^2 = \begin{bmatrix} q^{\frac{1}{2}} \bar{R}_1 & * & * \\ -q^{\frac{1}{2}} \bar{R}_1 & q^{\frac{1}{2}} \bar{T}_1 & * \\ 0 & 0 & q^{\frac{1}{2}} \bar{G}_1 \\ (1-q)^{\frac{1}{2}} \bar{R}_1 & * & * \\ -(1-q)^{\frac{1}{2}} \bar{R}_1 & (1-q)^{\frac{1}{2}} \bar{T}_1 & * \\ 0 & 0 & (1-q)^{\frac{1}{2}} \bar{G}_1 \end{bmatrix}, \\
\mathcal{H}_{ij}^2 &= \begin{bmatrix} p^{\frac{1}{2}} \bar{R}_2 & * & * \\ -p^{\frac{1}{2}} \bar{R}_2 & p^{\frac{1}{2}} \bar{T}_2 & * \\ 0 & 0 & p^{\frac{1}{2}} \bar{G}_2 \\ (1-p)^{\frac{1}{2}} \bar{R}_2 & * & * \\ -(1-p)^{\frac{1}{2}} \bar{R}_2 & (1-p)^{\frac{1}{2}} \bar{T}_2 & * \\ 0 & 0 & (1-p)^{\frac{1}{2}} \bar{G}_2 \end{bmatrix}, \mathcal{K}_{ij}^2 = \begin{bmatrix} q^{\frac{1}{2}} \bar{R}_2 & * & * \\ -q^{\frac{1}{2}} \bar{R}_2 & q^{\frac{1}{2}} \bar{T}_2 & * \\ 0 & 0 & q^{\frac{1}{2}} \bar{G}_2 \\ (1-q)^{\frac{1}{2}} \bar{R}_2 & * & * \\ -(1-q)^{\frac{1}{2}} \bar{R}_2 & (1-q)^{\frac{1}{2}} \bar{T}_2 & * \\ 0 & 0 & (1-q)^{\frac{1}{2}} \bar{G}_2 \end{bmatrix}.
\end{aligned}$$

Proof. Firstly, we consider the situation that in the absence of DoS attack. The following Lyapunov function corresponding to no DoS attack is defined

$$V_1(k) = \tilde{z}(k)^T \Omega_{\beta(k)}^1 \tilde{z}(k) + \beta(k) \varepsilon_1(k) + (1 - \beta(k)) \varepsilon_0(k)$$

where $\tilde{z}(k) = [\tilde{x}(k)^T \ \bar{y}(k-1)^T]^T$, $\Omega_{\beta(k)}^1 = \text{diag}\{P_{\beta(k)}^1, Q_{\beta(k)}^1\}$. Subsequently, it is assumed that $P_{\beta(k)}^1$ and its inverse $P_{\beta(k)}^{1^{-1}} = \bar{P}_{\beta(k)}^1$ satisfy

$$P_{\beta(k)}^1 = \begin{bmatrix} M_{\beta(k)}^1 & * \\ -M_{\beta(k)}^1 & N_{\beta(k)}^1 \end{bmatrix}, \bar{P}_{\beta(k)}^1 = \begin{bmatrix} \bar{M}_{\beta(k)}^1 & * \\ -\bar{M}_{\beta(k)}^1 & \bar{N}_{\beta(k)}^1 \end{bmatrix} \quad (19)$$

We can get

$$\begin{aligned}
\mathbb{E}\{\Delta V_1(k)\} &= \mathbb{E}\{V_1(k+1) - V_1(k)\} \\
&= \mathbb{E}\left\{ \hat{z}(k)^T \left(\mathcal{R}_1^T P_{\beta(k+1)}^1 \mathcal{R}_1 + b^2 \mathcal{R}_2^T P_{\beta(k+1)}^1 \mathcal{R}_2 \right) \hat{z}(k) \right. \\
&\quad + \bar{y}(k)^T Q_{\beta(k+1)}^1 \bar{y}(k) - \tilde{x}(k)^T P_{\beta(k)}^1 \tilde{x}(k) \\
&\quad - \bar{y}(k-1)^T Q_{\beta(k)}^1 \bar{y}(k-1) \\
&\quad \left. + \beta(k) \Delta \varepsilon_1(k) + (1 - \beta(k)) \Delta \varepsilon_0(k) \right\} \quad (20)
\end{aligned}$$

where

$$\begin{aligned}
\hat{z}(k) &= [\tilde{x}(k)^T \ \bar{y}(k-1)^T \ \omega(k)^T \ f(\hat{u}(k))^T \ \tau(k)^T]^T, \\
Y^s &= \sum_{i=1}^{\wp} \sum_{j=1}^{\wp} h_i \vartheta_j Y_{ij}^s, (s = 1, \dots, 7), \\
\mathcal{R}_1 &= [Y^1{}^T \ Y^2{}^T \ Y^3{}^T \ Y^4{}^T \ Y^5{}^T]^T, \mathcal{R}_2 = [Y^6{}^T \ 0 \ 0 \ Y^7{}^T \ 0]^T.
\end{aligned}$$

According to the adaptive law, we can get

$$\begin{aligned}
\Delta \varepsilon_1(k) &\leq y(t_m)^T \Xi y(t_m) - \mathcal{J}_1 \tau(k)^T \Xi \tau(k), \text{ACK} = 1 \\
\Delta \varepsilon_0(k) &\leq y(t_m)^T \Xi y(t_m) - \mathcal{J}_0 \tau(k)^T \Xi \tau(k), \text{ACK} = 0
\end{aligned} \quad (21)$$

Besides, the following inequality can be obtained by applying Assumption III.1.:

$$\bar{\theta} \hat{u}(k)^T F^T F \hat{u}(k) - \bar{\theta} f(\hat{u}(k))^T f(\hat{u}(k)) \geq 0 \quad (22)$$

Combing the above inequalities (20)-(22), it yields

$$\begin{aligned}
& \mathbb{E} \{ \Delta V_1(k) + \varphi_1 V_1(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \} \\
& \leq \mathbb{E} \{ \Delta V_1(k) + \varphi_1 V_1(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \\
& \quad + \bar{\theta} \hat{u}(k)^T F^T F \hat{u}(k) - \bar{\theta} f(\hat{u}(k))^T f(\hat{u}(k)) \\
& \quad + (y(k) + \tau(k))^T \Xi (y(k) + \tau(k)) \\
& \quad - \beta(k) \mathcal{J}_1 \tau(k)^T \Xi \tau(k) - (1 - \beta(k)) \mathcal{J}_0 \tau(k)^T \Xi \tau(k) \}
\end{aligned} \tag{23}$$

For $\beta(k) = 1$, in consideration of the failure probability, that is considering $1 \rightarrow 1$ and $1 \rightarrow 0$ simultaneously. And by invoking a slack matrix $\Phi_1 = \{W_1, G_1\}$, we can get

$$\begin{aligned}
& \mathbb{E} \{ \Delta V_1(k) + \varphi_1 V_1(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \} \\
& \leq \mathbb{E} \left\{ -\hat{z}(k)^T \sum_{i=1}^{\wp} \sum_{j=1}^{\wp} h_i \vartheta_j (\Theta_{ij}^1) \hat{z}(k) \right. \\
& \quad + [\hat{z}(k)^T (\mathcal{R}_1^T W_1 \mathcal{R}_1 + b^2 \mathcal{R}_2^T W_1 \mathcal{R}_2) \hat{z}(k) \\
& \quad + \bar{y}(k)^T G_1 \bar{y}(k) + z(k)^T z(k) + \bar{\theta} \hat{u}(k)^T F^T F \hat{u}(k) \\
& \quad \left. + (y(k) + \tau(k))^T \Xi (y(k) + \tau(k)) \right\}
\end{aligned} \tag{24}$$

$$p\Omega_0^1 + (1-p)\Omega_1^1 \leq \Phi_1 \tag{25}$$

Thus, when $\beta(k) = 1$, the inequality (23) can be guaranteed if the above constraints hold. Using the Schur complement, (11),(12) can be obtained.

For $\beta(k) = 0$, the proof procedure is similar to $\beta(k) = 1$.

$$\begin{aligned}
& \mathbb{E} \{ \Delta V_1(k) + \varphi_1 V_1(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \} \\
& \leq \mathbb{E} \left\{ -\hat{z}(k)^T \sum_{i=1}^{\wp} \sum_{j=1}^{\wp} h_i \vartheta_j (\mathcal{D}_{ij}^1) \hat{z}(k) \right. \\
& \quad + [\hat{z}(k)^T (\mathcal{R}_1^T W_1 \mathcal{R}_1 + b^2 \mathcal{R}_2^T W_1 \mathcal{R}_2) \hat{z}(k) \\
& \quad + \bar{y}(k)^T G_1 \bar{y}(k) + z(k)^T z(k) + \bar{\theta} \hat{u}(k)^T F^T F \hat{u}(k) \\
& \quad \left. + (y(k) + \tau(k))^T \Xi (y(k) + \tau(k)) \right\}
\end{aligned} \tag{26}$$

$$q\Omega_0^1 + (1-q)\Omega_1^1 \leq \Phi_1 \tag{27}$$

Therefore, by utilizing Schur complement on the above conditions, (23) can be ensured by (13) and (14) when $\beta(k) = 0$.

Next, we consider the situation that DoS attack occurs. We define the following Lyapunov function corresponding to DoS attacks.

$$V_2(k) = \tilde{z}(k)^T \Omega_{\beta(k)}^2 \tilde{z}(k) + \beta(k) \epsilon_1(k) + (1 - \beta(k)) \epsilon_0(k)$$

where $\Omega_{\beta(k)}^2 = \text{diag}\{P_{\beta(k)}^2, Q_{\beta(k)}^2\}$. The slack matrix $\Phi_2 = \{W_2, G_2\}$. Similarly, we suppose $P_{\beta(k)}^2$ and its inverse $P_{\beta(k)}^{2^{-1}} = \bar{P}_{\beta(k)}^2$ satisfy

$$P_{\beta(k)}^2 = \begin{bmatrix} M_{\beta(k)}^2 & * \\ -M_{\beta(k)}^2 & N_{\beta(k)}^2 \end{bmatrix}, \bar{P}_{\beta(k)}^2 = \begin{bmatrix} \bar{M}_{\beta(k)}^2 & * \\ -\bar{M}_{\beta(k)}^2 & \bar{N}_{\beta(k)}^2 \end{bmatrix} \tag{28}$$

In view of the theory of switching system and the existence of DoS attacks, we can get

$$\begin{aligned}
& \mathbb{E} \{ \Delta V_2(k) - \varphi_2 V_2(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \} \\
& \leq \mathbb{E} \{ \Delta V_2(k) - \varphi_2 V_2(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k) \\
& \quad + (y(k) + \tau(k))^T \Xi (y(k) + \tau(k)) - \beta(k) \mathcal{J}_1 \tau(k)^T \Xi \tau(k) \\
& \quad - (1 - \beta(k)) \mathcal{J}_0 \tau(k)^T \Xi \tau(k) \} \leq 0
\end{aligned}$$

and

$$p\Omega_0^2 + (1-p)\Omega_1^2 \leq \Phi_2.$$

Similarly, according to the Schur complement, considering the situation of $\beta(k) = 1$ and $\beta(k) = 0$, (15), (16), (17), (18) can be obtained.

According to the constraints (11)-(18), we can get the following two constraints:

$$\begin{aligned} \mathbb{E}\{\Delta V_1(k) + \varphi_1 V_1(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k)\} &\leq 0 \\ \mathbb{E}\{\Delta V_2(k) - \varphi_2 V_2(k) + z(k)^T z(k) - \gamma^2 \omega(k)^T \omega(k)\} &\leq 0 \end{aligned} \quad (29)$$

Thus, from the condition $\Omega_{\beta(k)}^1 < \chi \Omega_{\beta(k)}^2$, $\Omega_{\beta(k)}^2 < \chi \Omega_{\beta(k)}^1$, we can infer $\mathbb{E}V_1(k) < \chi \mathbb{E}V_2(k)$, $\mathbb{E}V_2(k) < \chi \mathbb{E}V_1(k)$.

Define $V(k) = V_1(k)$ corresponding to the absence of DoS attack and $V(k) = V_2(k)$ refers to the existence of DoS attack, the relationship between $\mathbb{E}V(k)$ and $V(0)$ can be formulated.

It is worth noting that the situation under Dos attacks is $k \in [\mathcal{I}_n^{\text{on}}, \mathcal{I}_n^{\text{off}})$, then, according to the iterative technique, it can be obtained that

$$\begin{aligned} \mathbb{E}V(k) &\leq \tilde{\varphi}_1 \mathbb{E}V_1(k-1) + \aleph(k) \\ &\leq \tilde{\varphi}_1 [\tilde{\varphi}_1 \mathbb{E}V_1(k-2) - z(k-2)^T z(k-2) + \gamma^2 \omega(k-2)^T \omega(k-2)] + \aleph(k) \\ &\dots \\ &\leq \tilde{\varphi}_1^{k-\mathcal{I}_n^{\text{on}}} \mathbb{E}V_1(\mathcal{I}_n^{\text{on}}) - \tilde{\varphi}_1^{k-\mathcal{I}_n^{\text{on}}-1} [z(\mathcal{I}_n^{\text{on}})^T z(\mathcal{I}_n^{\text{on}}) \\ &\quad - \gamma^2 \omega(\mathcal{I}_n^{\text{on}})^T \omega(\mathcal{I}_n^{\text{on}})] - \dots + \aleph(k) \\ &\leq \tilde{\varphi}_1^{k-\mathcal{I}_n^{\text{on}}} \chi \mathbb{E}V_2(\mathcal{I}_n^{\text{on}}) - \tilde{\varphi}_1^{k-\mathcal{I}_n^{\text{on}}-1} [z(\mathcal{I}_n^{\text{on}})^T z(\mathcal{I}_n^{\text{on}}) \\ &\quad - \gamma^2 \omega(\mathcal{I}_n^{\text{on}})^T \omega(\mathcal{I}_n^{\text{on}})] - \dots + \aleph(k) \\ &\leq \tilde{\varphi}_1^{k-\mathcal{I}_n^{\text{on}}} \chi \tilde{\varphi}_2 [\mathbb{E}V_2(\mathcal{I}_n^{\text{on}}-1) - z(\mathcal{I}_n^{\text{on}}-1)^T z(\mathcal{I}_n^{\text{on}}-1) \\ &\quad + \gamma^2 \omega(\mathcal{I}_n^{\text{on}}-1)^T \omega(\mathcal{I}_n^{\text{on}}-1)] \\ &\quad - \tilde{\varphi}_1^{k-\mathcal{I}_n^{\text{on}}-1} [z(\mathcal{I}_n^{\text{on}})^T z(\mathcal{I}_n^{\text{on}}) - \gamma^2 \omega(\mathcal{I}_n^{\text{on}})^T \omega(\mathcal{I}_n^{\text{on}})] - \dots + \aleph(k) \\ &\dots \\ &\leq \chi^{\Pi_f[0,k]} \tilde{\varphi}_1^{k-\Pi_d[0,k]} \tilde{\varphi}_2^{\Pi_d[0,k]} V(0) \\ &\quad - \sum_{\psi=0}^{k-1} \chi^{\Pi_f[\psi,k-1]} \tilde{\varphi}_1^{k-1-\psi-\Pi_d[\psi,k-1]} \tilde{\varphi}_2^{\Pi_d[\psi,k-1]} [z(\psi)^T z(\psi) - \gamma^2 \omega(\psi)^T \omega(\psi)] \end{aligned} \quad (30)$$

where $\tilde{\varphi}_1 = 1 - \varphi_1$, $\tilde{\varphi}_2 = 1 + \varphi_2$, $\aleph(k) = -z(k-1)^T z(k-1) + \gamma^2 \omega(k-1)^T \omega(k-1)$.

When the DoS attack occurs, we can get the same result as (30), which is omitted here.

Next, in view of the iterative result derived in (30), H_∞ performance is discussed further.

Firstly, when $\omega(\psi) = 0$, combing with the duration and the frequency given in Assumption III.2. and Assumption III.3., we can get

$$\begin{aligned} \mathbb{E}V(k) &\leq \chi^{\Pi_f[0,k]} \tilde{\varphi}_1^{k-\Pi_d[0,k]} \tilde{\varphi}_2^{\Pi_d[0,k]} V(0) \\ &= e^{\Pi_f[0,k] \ln \chi + (k-\Pi_d[0,k]) \ln \tilde{\varphi}_1 + \Pi_d[0,k] \ln \tilde{\varphi}_2} V(0) \\ &\leq e^{(\sigma + \frac{k}{\pi}) \ln \chi + \ln \tilde{\varphi}_1 (k - \epsilon - \frac{k}{T}) + \ln \tilde{\varphi}_2 (\epsilon + \frac{k}{T})} V(0) \\ &= e^{(\sigma + \frac{k}{\pi}) \ln \chi + \epsilon (\ln \tilde{\varphi}_2 - \ln \tilde{\varphi}_1) + k \left[\ln \tilde{\varphi}_1 \frac{T-1}{T} + \ln \tilde{\varphi}_2 \frac{1}{T} \right]} V(0). \end{aligned}$$

Combing the condition that $0 < \frac{\ln \tilde{\varphi}_2 - \ln \tilde{\varphi}_3}{-\ln \tilde{\varphi}_1 + \ln \tilde{\varphi}_3} \leq T-1$ implies that $\ln \tilde{\varphi}_2 \frac{1}{T} + \frac{(T-1) \ln \tilde{\varphi}_1}{T} \leq \ln \tilde{\varphi}_3$, and the following can be got

$$\begin{aligned} \mathbb{E}V(k) &\leq e^{(\sigma + \frac{k}{\pi}) \ln \chi + \epsilon (\ln \tilde{\varphi}_2 - \ln \tilde{\varphi}_1) + k \ln \tilde{\varphi}_3} V(0) \\ &\leq \frac{\tilde{\varphi}_2^\epsilon \chi^\sigma}{\tilde{\varphi}_1^\epsilon} (\tilde{\varphi}_3 \chi^{1/\pi})^k V(0). \end{aligned}$$

Set $\delta \mathbb{E}||\tilde{z}(k)||^2 \leq \mathbb{E}V(k)$, $V(0) \leq c||\tilde{z}(0)||^2$. Thus,

$$\mathbb{E}||\tilde{z}(k)||^2 \leq \frac{\delta}{c} \frac{\tilde{\varphi}_2^\epsilon \chi^\sigma}{\tilde{\varphi}_1^\epsilon} (\tilde{\varphi}_3 \chi^{1/\pi})^k ||\tilde{z}(0)||^2 \quad (31)$$

where

$$\begin{aligned} \delta &= \min\{\min(\lambda_{\min}(\Omega_{\beta(k)}^1), \min(\lambda_{\min}(\Omega_{\beta(k)}^2))\}, \\ c &= \max\{\max(\lambda_{\max}(\Omega_{\beta(k)}^1), \max(\lambda_{\max}(\Omega_{\beta(k)}^2))\}. \end{aligned}$$

Secondly, when $\omega(\psi) \neq 0$, under the zero initial condition and (30),

$$\begin{aligned} \mathbb{E}V(k) &\leq - \sum_{\psi=0}^{k-1} \chi^{\Pi_f[\psi, k-1]} \tilde{\varphi}_1^{k-1-\psi-\Pi_d[\psi, k-1]} \tilde{\varphi}_2^{\Pi_d[\psi, k-1]} \\ &\quad [z(\psi)^T z(\psi) - \gamma^2 \omega(\psi)^T \omega(\psi)]. \end{aligned}$$

Considering $\mathbb{E}V(k) > 0$ and substituting $\check{k} = k - 1$ by $k - 1$, we have

$$\begin{aligned} \sum_{\psi=0}^{\check{k}} e^{\ln \chi \Pi_f[\psi, \check{k}] + \ln \tilde{\varphi}_1(\check{k} - \psi - \Pi_d[\psi, \check{k}]) + \ln \tilde{\varphi}_2(\Pi_d[\psi, \check{k}])} \mathbb{E}z(\psi)^T z(\psi) &\leq \\ \gamma^2 \sum_{\psi=0}^{\check{k}} e^{\ln \chi \Pi_f[\psi, \check{k}] + \ln \tilde{\varphi}_1(\check{k} - \psi - \Pi_d[\psi, \check{k}]) + \ln \tilde{\varphi}_2(\Pi_d[\psi, \check{k}])} \mathbb{E}\omega(\psi)^T \omega(\psi). \end{aligned}$$

Owing to $\Pi_d[\psi, \check{k}] > 0$, $\Pi_f[\psi, \check{k}] > 0$, $\tilde{\varphi}_1 < 1$, $\tilde{\varphi}_2 \geq 1$, $\chi \geq 1$, we have

$$\begin{aligned} \sum_{\psi=0}^{\check{k}} e^{\ln \chi \Pi_f[\psi, \check{k}] + \ln \tilde{\varphi}_1(\check{k} - \psi - \Pi_d[\psi, \check{k}]) + \ln \tilde{\varphi}_2(\Pi_d[\psi, \check{k}])} \mathbb{E}z(\psi)^T z(\psi) \\ \geq \sum_{\psi=0}^{\check{k}} \tilde{\varphi}_1^{\check{k}-\psi} \mathbb{E}z(\psi)^T z(\psi) \end{aligned}$$

and

$$\begin{aligned} \gamma^2 \sum_{\psi=0}^{\check{k}} e^{(\sigma + \frac{\check{k}}{\pi}) \ln \chi + \ln \tilde{\varphi}_1(\check{k} - \psi - \frac{\check{k}}{\pi}) + \ln \tilde{\varphi}_2(\frac{\check{k}}{\pi})} \mathbb{E}\omega(\psi)^T \omega(\psi) \\ \leq \gamma^2 \sum_{\psi=0}^{\check{k}} e^{\epsilon(\ln \tilde{\varphi}_2 - \ln \tilde{\varphi}_1) + \sigma \ln \chi + (\check{k} - \psi)(\ln \tilde{\varphi}_3 + \frac{\ln \chi}{\pi})} \mathbb{E}\omega(\psi)^T \omega(\psi) \\ \leq \gamma^2 \frac{\tilde{\varphi}_2^\epsilon \chi^\sigma}{\tilde{\varphi}_1^\epsilon} \sum_{\psi=0}^{\check{k}} (\tilde{\varphi}_3 \chi^{1/\pi})^{(\check{k}-\psi)} \mathbb{E}\omega(\psi)^T \omega(\psi). \end{aligned}$$

Combing the above inequalities, we can obtain

$$\sum_{\psi=0}^{\check{k}} \tilde{\varphi}_1^{\check{k}-\psi} \mathbb{E}z(\psi)^T z(\psi) \leq \gamma^2 \frac{\tilde{\varphi}_2^\epsilon \chi^\sigma}{\tilde{\varphi}_1^\epsilon} \sum_{\psi=0}^{\check{k}} (\tilde{\varphi}_3 \chi^{1/\pi})^{(\check{k}-\psi)} \mathbb{E}\omega(\psi)^T \omega(\psi).$$

Summing \check{k} from 0 to ∞ , it holds

$$\sum_{\check{k}=0}^{\infty} \sum_{\psi=0}^{\check{k}} \tilde{\varphi}_1^{\check{k}-\psi} \mathbb{E}z(\psi)^T z(\psi) \leq \gamma^2 \frac{\tilde{\varphi}_2^\epsilon \chi^\sigma}{\tilde{\varphi}_1^\epsilon} \sum_{\check{k}=0}^{\infty} \sum_{\psi=0}^{\check{k}} (\tilde{\varphi}_3 \chi^{1/\pi})^{(\check{k}-\psi)} \mathbb{E}\omega(\psi)^T \omega(\psi).$$

Exchanging the summation order with the fact $\tilde{\varphi}_1 < 1$, $\tilde{\varphi}_3 \chi^{1/\pi} < 1$, we have

$$\sum_{k=0}^{\infty} \mathbb{E}z^T(k)z(k) \leq \mathfrak{d}^2 \sum_{k=0}^{\infty} \mathbb{E}\omega^T(k)\omega(k),$$

where $\mathfrak{d}^2 = \gamma^2 \frac{1-\tilde{\varphi}_1}{1-\tilde{\varphi}_3 \chi^{1/\pi}} \frac{\tilde{\varphi}_2^\epsilon \chi^\sigma}{\tilde{\varphi}_1^\epsilon}$. □

4.2 | Controller design for NCS

Theorem 2. For the IT2 T-S fuzzy system (1), $\bar{\theta}, \gamma$ are prescribed scalars, the closed-loop control system (10) is stochastically stable with an expected H_∞ performance $\mathfrak{d} = \gamma \sqrt{\frac{1-\bar{\varphi}_1}{1-\bar{\varphi}_3} \frac{\bar{\varphi}_2^c \chi^\sigma}{\bar{\varphi}_1^c}}$ if there exist positive definite matrices $P_l^m, \bar{P}_l^m, Q_l^m, \bar{Q}_l^m, (l = 0, 1), (m = 1, 2), \bar{W}_1, \bar{W}_2, \bar{G}_1, \bar{G}_2$ and matrices $\{\hat{A}_j, \hat{B}_j, \hat{C}_j\}$ satisfying (11)-(18) and the following inequalities:

$$\begin{bmatrix} P_l^m & I \\ I & \bar{P}_l^m \end{bmatrix} \geq 0, \begin{bmatrix} Q_l^m & I \\ I & \bar{Q}_l^m \end{bmatrix} \geq 0, \begin{bmatrix} \Xi & I \\ I & \bar{\Xi} \end{bmatrix} \geq 0 \quad (32)$$

Proof. Since (11)-(18) are not strict linear matrix inequalities (LMIs), the cone complementarity linearization (CCL) algorithm can be utilized to deal with the non-convex optimization problem. For convenience, we apply $\beta(k) = I (l = 0, 1)$ and $\eta(k) = m (m = 1, 2)$ in the presentation.

Hence, (11)-(18) can be guaranteed by $P_l^m \bar{P}_l^m = I, Q_l^m \bar{Q}_l^m = I, \Xi \bar{\Xi} = I$.

Consequently, the non-convex optimization problem is tackled successfully. \square

Further, the optimization problem of the controller design can be formulated as below

$$\begin{aligned} \mathbf{P1} : \min \text{tr} \left\{ \sum_{l=0}^1 \sum_{m=1}^2 (P_l^m \bar{P}_l^m + Q_l^m \bar{Q}_l^m) + \Xi \bar{\Xi} \right\} \\ \text{s.t. } (11) - (18), (32). \end{aligned} \quad (33)$$

The completed Algorithm is shown in Figure.2

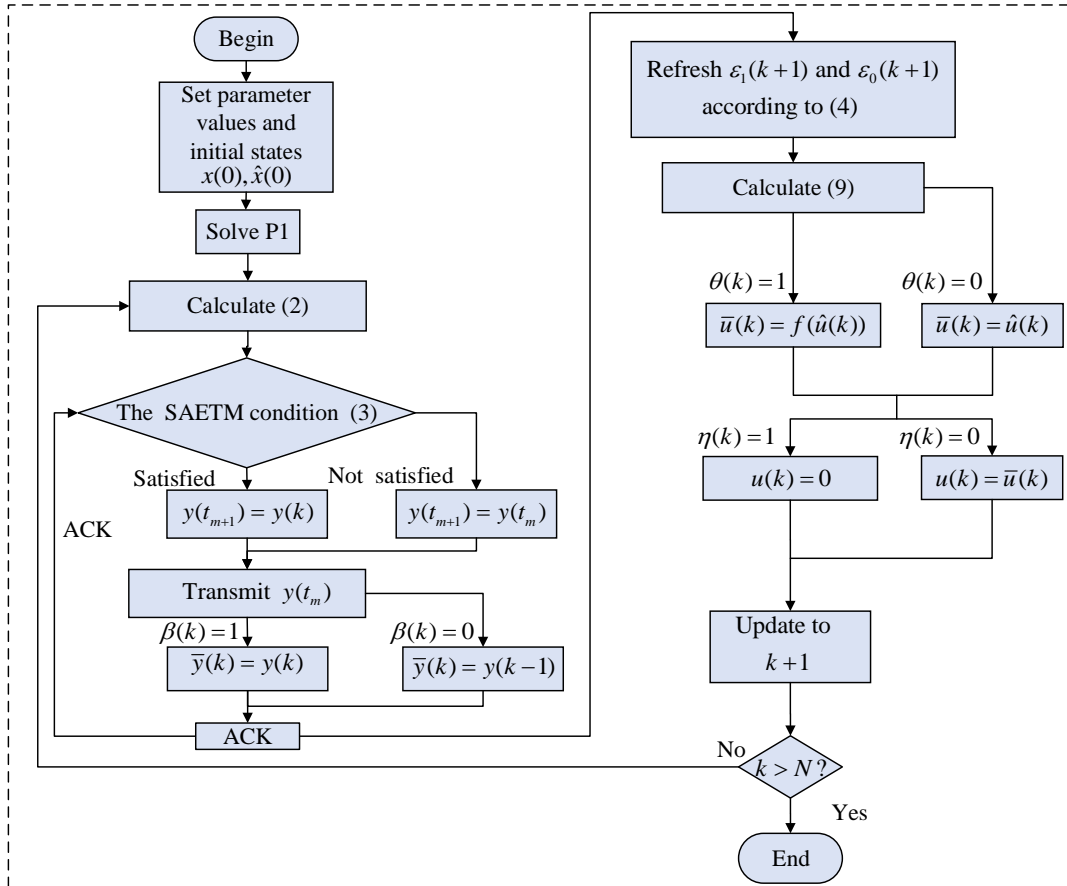


Figure 2 Algorithm procedure in this paper.

5 | SIMULATION EXAMPLE

Two examples are presented to verify the validity of the proposed approaches in this section.

Example 1: A mass-spring-damping system depicted in Figure. 3 is utilized. In view of the Newton's Law, we can acquire $m\ddot{d} + F_f + F_s = u$, wherein u and m are the control input and the mass, respectively. $F_f = e\dot{d}$ and $F_s = o(1 + \kappa^2 d^2)d$ imply the friction force and the hardening spring force, respectively, wherein $e > 0$, both o and κ are constant. Furthermore, $m\ddot{d} + e\dot{d} + od + o\kappa^2 d^3 = u$ can be formulated, wherein d represents the displacement from the reference point. Set $x(k) = [x_1(k) \ x_2(k)]^T = [d \ \dot{d}]^T$ and $\hat{f} = (-o - o\kappa^2 x_1^2(k))/m$, and let $o = 8\text{N/m}$, $e = 2\text{N} \cdot \text{m/s}$, $\kappa = 0.3\text{m}^{-1}$ and $m = 1\text{kg}$. The value ranges of system membership functions are $x_1(k) \in [-2, 2]$ and $o \in [5, 8]$, then, when $o = 5$, $x_1(k) = 0$ and $\hat{f}_{\max} = -5$; when $o = 8$, $x_1^2(k) = 4$ and $\hat{f}_{\min} = -10.88$. Since the membership function satisfies $h_1(x(k)) + h_2(x(k)) = 1$, then $\hat{f} = h_1(x(k))\hat{f}_{\min} + h_2(x(k))\hat{f}_{\max}$. It can be deduced that $h_1(x(k)) = (-\hat{f} + \hat{f}_{\max})/(\hat{f}_{\max} - \hat{f}_{\min})$ and $h_2(x(k)) = (\hat{f} - \hat{f}_{\min})/(\hat{f}_{\max} - \hat{f}_{\min})$.

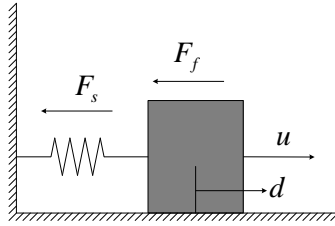


Figure 3 Mass-spring-damping system.

We select $x(k) = x_1(k)$. Then, the system membership functions can be represented by the scalar o . $\underline{h}_1(x_1(k)) = (-\hat{f} + \hat{f}_{\max})/(\hat{f}_{\max} - \hat{f}_{\min})$ and $\bar{h}_2(x_1(k)) = (\hat{f} - \hat{f}_{\min})/(\hat{f}_{\max} - \hat{f}_{\min})$ with $o = 5$. $\bar{h}_1(x_1(k)) = (-\hat{f} + \hat{f}_{\max})/(\hat{f}_{\max} - \hat{f}_{\min})$ and $\underline{h}_2(x_1(k)) = (\hat{f} - \hat{f}_{\min})/(\hat{f}_{\max} - \hat{f}_{\min})$ with $o = 8$.

The controller membership functions are $\underline{\vartheta}_1(x_1(k)) = e^{-(x_1^2(k))/0.5}$, $\bar{\vartheta}_2(x_1(k)) = 1 - e^{-(x_1^2(k))/0.5}$, and $\bar{\vartheta}_1(x_1(k)) = e^{-x_1^2(k)}$, $\underline{\vartheta}_2(x_1(k)) = 1 - e^{-x_1^2(k)}$. Define the weighting the function $\zeta_1(x_1(k)) = \underline{\alpha}_1(x_1(k)) = \sin^2(x_1(k))$, $\bar{\zeta}_1(x_1(k)) = \bar{\alpha}_1(x_1(k)) = 1 - \sin^2(x_1(k))$ and $\zeta_2(x_1(k)) = \underline{\alpha}_2(x_1(k)) = \cos^2(x_1(k))$, $\bar{\zeta}_2(x_1(k)) = \bar{\alpha}_2(x_1(k)) = 1 - \cos^2(x_1(k))$.

The membership functions regarding the system and controller are shown in Figure. 4.

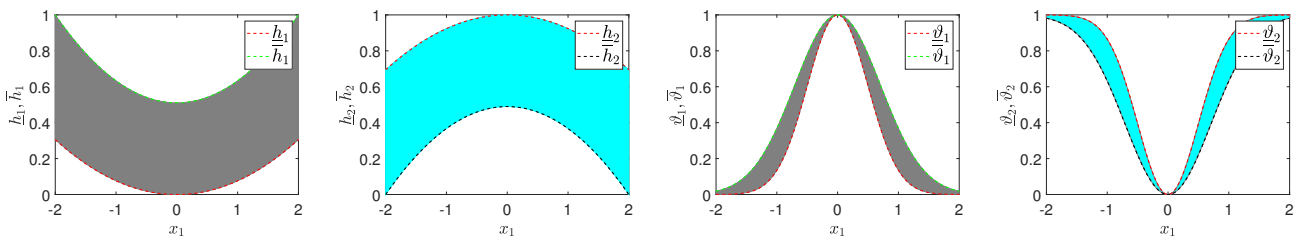


Figure 4 The membership functions regarding the system and controller.

Then, the matrices of the above actual system are inferred as below

$$A_1 = \begin{bmatrix} 0 & 1 \\ \hat{f}_{\min} & -\frac{c}{m} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ \hat{f}_{\max} & -\frac{c}{m} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}.$$

Set sampling period $T_s = 0.5s$, by applying the method of Eulerian discretization, the following matrices can be acquired

$$A_1 = \begin{bmatrix} 0.1925 & 0.1930 \\ -2.0994 & -0.1935 \end{bmatrix}, A_2 = \begin{bmatrix} 0.3711 & 0.2222 \\ -1.7779 & -0.0734 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0742 \\ 0.1930 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0786 \\ 0.2222 \end{bmatrix}.$$

Other matrices are

$$C_{11} = \begin{bmatrix} -0.0621 & 0.7448 \end{bmatrix}, C_{12} = \begin{bmatrix} 0.7045 & 0.7086 \end{bmatrix}, C_{21} = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_{22} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ D_{11} = \begin{bmatrix} 0.0015; 0.0018 \end{bmatrix}, D_{12} = \begin{bmatrix} -0.0329; -0.0326 \end{bmatrix}, D_{21} = 0.01, D_{22} = -0.03.$$

Assume that the bounded disturbance $\omega(k) = 0.1\sin(\pi k)$ and the deception attack signal $f(\hat{u}(k)) = \tanh(0.15\hat{u}(k))$, $F = 0.15$. Choose the initial states $\hat{x}_0 = [1.8 \ -1.5]^T$, $x_0 = [0.8 \ 0.7]^T$. The parameters to describe the frequency and duration of the DoS attack are $\sigma = 5$, $\epsilon = 5$, $\pi = 7$ and $T = 5$. The parameters used in the iterative process are $\chi = 3$, $\tilde{\varphi}_1 = 0.7$, $\tilde{\varphi}_2 = 1.2$ and $\tilde{\varphi}_3 = 0.8$. The parameters to characterize the SAETM are $\mathcal{J}_1 = 5$, $\epsilon_1(k) = 0.5$, $\mathcal{J}_0 = 11$, $\epsilon_0(k) = 0.1$. The parameter γ is chose as 0.2. Table 1 shows the data transmission status with transition probability matrix $\mathfrak{F} = \begin{bmatrix} 0.44 & 0.56 \\ 0.15 & 0.85 \end{bmatrix}$. The forgetting factor $\xi = 0.1$.

Table 1 Data transmission status

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

In Figure. 5, the state responses of the system and controller are presented. And Figure. 6 depicts the input of the plant. Figure. 7 plots the release time and interval, and the filled circles show that the transference of the triggered data is successful.

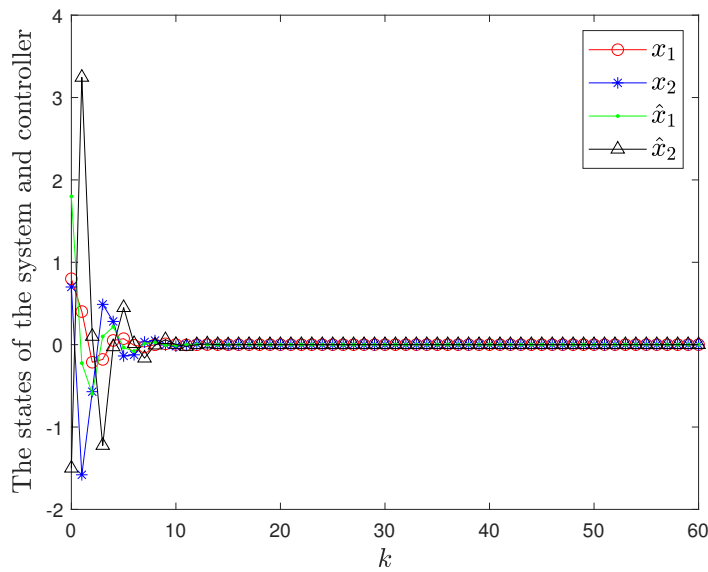


Figure 5 The system states and controller states.

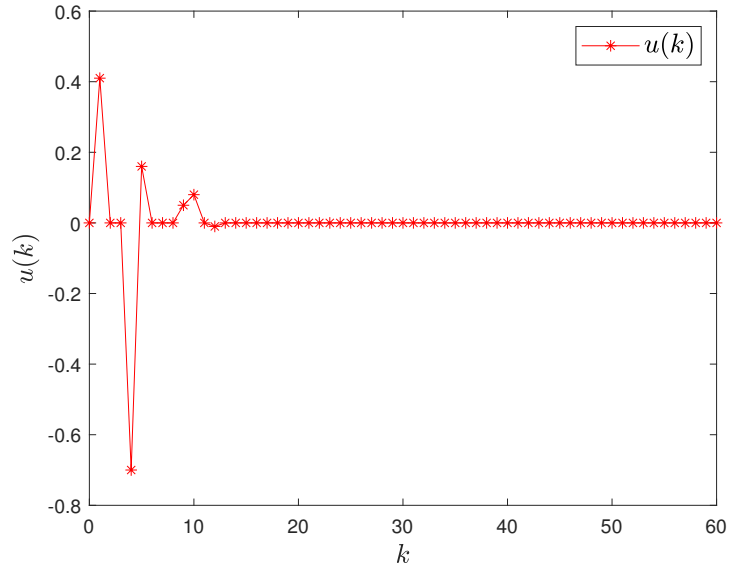


Figure 6 The control input.

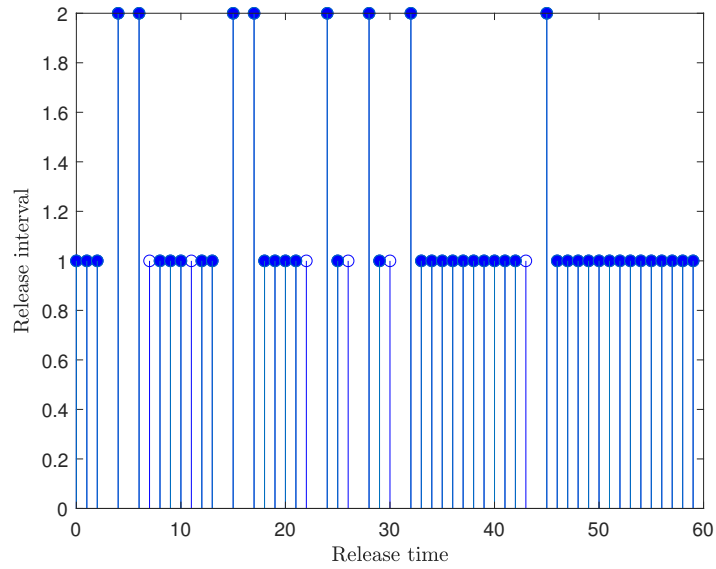


Figure 7 The release time and release interval.

Example 2: Consider a continuous stirred tank reactor (CSTR) system investigated in³⁸. Select the sampling time $T_s = 0.05$ min, we can get the matrices as follows

$$\begin{aligned}
 A_1 &= \begin{bmatrix} 0.8227 & -0.0017 \\ 6.1233 & 0.9367 \end{bmatrix}, A_2 = \begin{bmatrix} 0.9654 & -0.0018 \\ -0.6759 & 0.9433 \end{bmatrix}, A_3 = \begin{bmatrix} 0.8895 & -0.0029 \\ 2.9447 & 0.9968 \end{bmatrix}, A_4 = \begin{bmatrix} 0.8930 & -0.0006 \\ 2.7738 & 0.8864 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} -0.0001 \\ 0.1014 \end{bmatrix}, B_2 = \begin{bmatrix} -0.0001 \\ 0.1016 \end{bmatrix}, B_3 = \begin{bmatrix} -0.0002 \\ 0.1045 \end{bmatrix}, B_4 = \begin{bmatrix} -0.000034 \\ 0.0986 \end{bmatrix}, \\
 C_{11} &= C_{12} = C_{13} = C_{14} = \begin{bmatrix} 0.2 & 0.6 \end{bmatrix}, C_{21} = C_{22} = C_{23} = C_{24} = \begin{bmatrix} -0.1 & 0.2 \end{bmatrix}, \\
 D_1 &= D_2 = D_3 = D_4 = \begin{bmatrix} 0.0022 \\ 0.0564 \end{bmatrix}, E_1 = E_2 = E_3 = E_4 = 0.004.
 \end{aligned}$$

Moreover

$$\begin{aligned}
 \bar{\partial}_1(x(k)) &= 7.2 \times 10^{10} \times \tilde{\phi}_1 - \phi_1, \\
 \underline{\partial}_1(x(k)) &= 6.6 \times 10^{10} \times \tilde{\phi}_1 - \phi_1, \\
 \bar{\partial}_2(x(k)) &= 3.6 \times 10^{10} \left[\tilde{\phi}_1 - \exp\left(\frac{-8750}{350}\right) \right] \frac{1}{x(k)} - \phi_2, \\
 \underline{\partial}_2(x(k)) &= 3.3 \times 10^{10} \left[\tilde{\phi}_1 - \exp\left(\frac{-8750}{350}\right) \right] \frac{1}{x(k)} - \phi_2, \\
 \underline{h}_1 &= \frac{1}{2} \frac{\underline{\partial}_1(x(k)) - \underline{\partial}_1(-10)}{\underline{\partial}_1(10) - \underline{\partial}_1(-10)}, \bar{h}_1 = \frac{1}{2} \frac{\bar{\partial}_1(x(k)) - \bar{\partial}_1(-10)}{\bar{\partial}_1(10) - \bar{\partial}_1(-10)}, \\
 \underline{h}_2 &= \frac{1}{2} \frac{\underline{\partial}_1(10) - \bar{\partial}_1(x(k))}{\underline{\partial}_1(10) - \bar{\partial}_1(-10)}, \bar{h}_2 = \frac{1}{2} \frac{\underline{\partial}_1(10) - \underline{\partial}_1(x(k))}{\underline{\partial}_1(10) - \bar{\partial}_1(-10)}, \\
 \underline{h}_3 &= \frac{1}{2} \frac{\underline{\partial}_2(x(k)) - \underline{\partial}_2(-10)}{\underline{\partial}_2(10) - \underline{\partial}_2(-10)}, \bar{h}_3 = \frac{1}{2} \frac{\bar{\partial}_2(x(k)) - \bar{\partial}_2(-10)}{\bar{\partial}_2(10) - \bar{\partial}_2(-10)}, \\
 \underline{h}_4 &= \frac{1}{2} \frac{\underline{\partial}_2(10) - \bar{\partial}_2(x(k))}{\underline{\partial}_2(10) - \bar{\partial}_2(-10)}, \bar{h}_4 = \frac{1}{2} \frac{\underline{\partial}_2(10) - \underline{\partial}_2(x(k))}{\underline{\partial}_2(10) - \bar{\partial}_2(-10)},
 \end{aligned}$$

where

$$\begin{aligned}
 \phi_1 &= (\underline{\phi}_1 + \bar{\phi}_1)/2, \tilde{\phi}_1 = \exp\left(\frac{-8750}{x(k)+350}\right), \\
 \underline{\phi}_1 &= 6.6 \times 10^{10} \exp\left(\frac{-8750}{-10+350}\right), \\
 \bar{\phi}_1 &= 7.2 \times 10^{10} \exp\left(\frac{-8750}{10+350}\right), \phi_2 = (\underline{\phi}_2 + \bar{\phi}_2)/2, \\
 \underline{\phi}_2 &= 3.3 \times 10^{10} \left[\exp\left(\frac{-8750}{-10+350}\right) - \exp\left(\frac{-8750}{350}\right) \right] \times \frac{1}{(-10)}, \\
 \bar{\phi}_2 &= 3.6 \times 10^{10} \left[\exp\left(\frac{-8750}{10+350}\right) - \exp\left(\frac{-8750}{350}\right) \right] \times \frac{1}{10}.
 \end{aligned}$$

Define the weighting functions $\underline{\zeta}_1(x(k)) = \underline{\zeta}_3(x(k)) = \cos^2(x(k))$, $\bar{\zeta}_1(x(k)) = \bar{\zeta}_3(x(k)) = 1 - \cos^2(x(k))$, $\underline{\zeta}_2(x(k)) = \underline{\zeta}_4(x(k)) = \sin^2(x(k))$, $\bar{\zeta}_2(x(k)) = \bar{\zeta}_4(x(k)) = 1 - \sin^2(x(k))$. The controller membership functions are: $\underline{\vartheta}_1(x(k)) = \underline{w}_3(x(k)) = e^{-x^2(k)/0.5}$, $\bar{\vartheta}_1(x(k)) = \bar{w}_3(x(k)) = e^{-x^2(k)}$, $\underline{\vartheta}_2(x(k)) = \underline{w}_4(x(k)) = 1 - e^{-x^2(k)}$, $\bar{\vartheta}_2(x(k)) = \bar{w}_4(x(k)) = 1 - e^{-x^2(k)/0.5}$. It is assumed that the nonlinear weighting functions are: $\underline{\alpha}_1(x(k)) = \underline{\alpha}_3(x(k)) = \sin^2(x(k))$, $\bar{\alpha}_1(x(k)) = \bar{\alpha}_3(x(k)) = 1 - \sin^2(x(k))$, $\underline{\alpha}_2(x(k)) = \underline{\alpha}_4(x(k)) = \cos^2(x(k))$, $\bar{\alpha}_2(x(k)) = \bar{\alpha}_4(x(k)) = 1 - \cos^2(x(k))$. In which $x(k) = x_2(k)$.

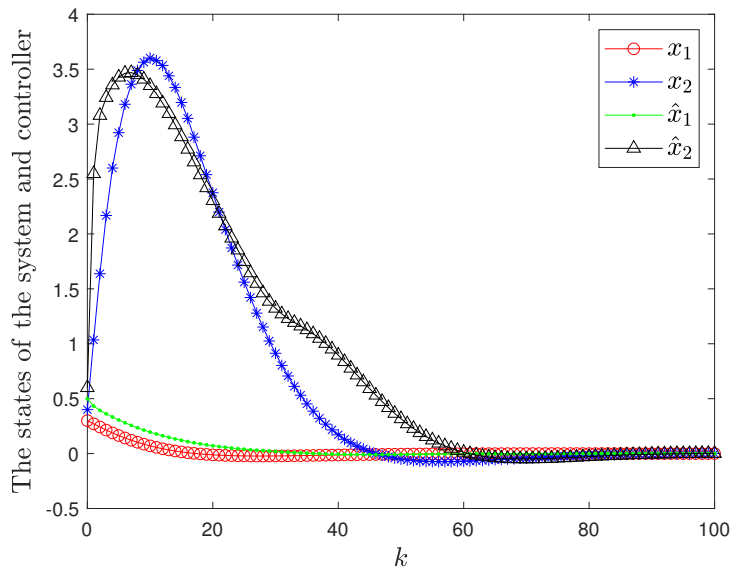


Figure 8 The system states and controller states.

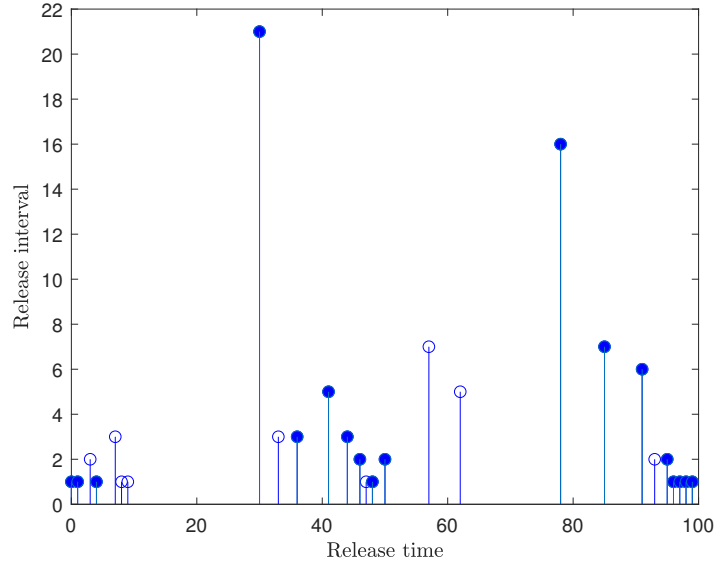


Figure 9 The trigger times with $\mathcal{J}_1 = 5$.

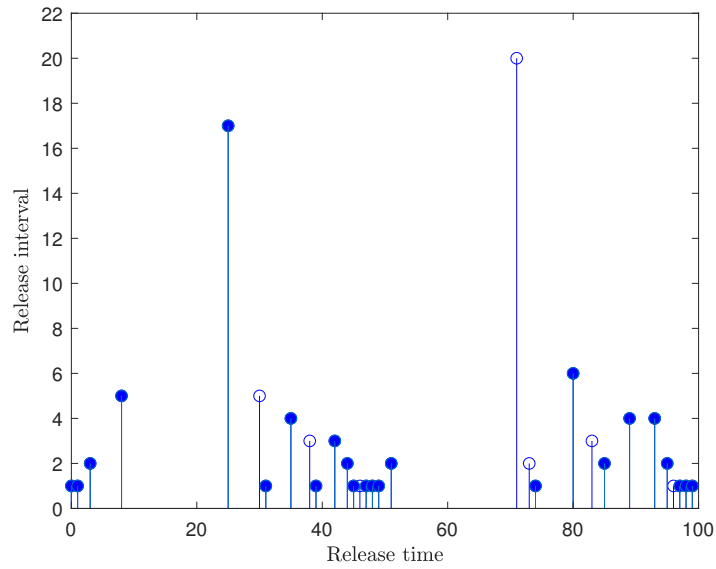


Figure 10 The trigger times with $\mathcal{J}_1 = 8$.

The bounded disturbance $\omega(k) = 0.1\sin(\pi k)$ and the deception attack signal $f(\hat{u}(k)) = \tanh(0.15\hat{u}(k))$, $F = 0.15$. Select the initial states $\hat{x}_0 = [0.5 \ 0.6]^T$ and $x_0 = [0.3 \ 0.4]^T$. The parameters to characterize the SAETM are $\mathcal{J}_1 = 8$, $\varepsilon_1(k) = 0.5$ and $\mathcal{J}_0 = 11$, $\varepsilon_0(k) = 0.1$. The other parameters and pre-conditions are the same to Example 1.

The state responses of the system and the controller are presented in Figure. 8, which reflects that the designed controller is able to address the stochastic stability of the real plant.

To illustrate the effectiveness of the provided technique, two groups of comparison experiments have been carried out.

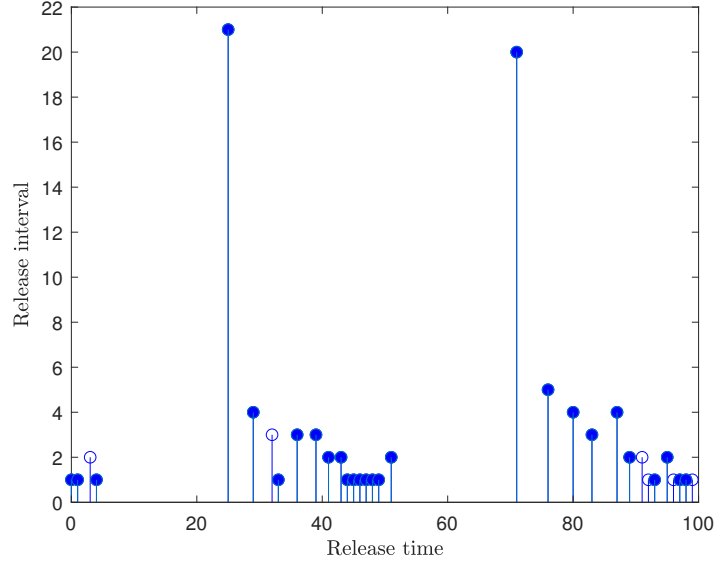


Figure 11 The trigger times with $\mathcal{J}_1 = 12$.

On one hand, the comparison experiments under different initial adaptive parameters, i.e., $\mathcal{J}_1 = 5$, $\mathcal{J}_1 = 8$ and $\mathcal{J}_1 = 12$, have been performed. The trigger times under different initial values of \mathcal{J}_1 are given in Figure. 9, Figure. 10, Figure. 11 and Table 2. We can easily conclude that the larger initial adaptive parameter \mathcal{J}_1 is, the smaller adaptive trigger threshold $\varepsilon_1(k)$ and the more trigger times are.

Table 2 Trigger times under different initial value \mathcal{J}_1

| \mathcal{J}_1 | $\mathcal{J}_1 = 5$ | $\mathcal{J}_1 = 8$ | $\mathcal{J}_1 = 12$ |
|--------------------|-------------------------------------|-------------------------------------|------------------------------------|
| $\varepsilon_1(k)$ | $\varepsilon_1(k) \rightarrow 0.32$ | $\varepsilon_1(k) \rightarrow 0.18$ | $\varepsilon_1(k) \rightarrow 0.1$ |
| Trigger times | 27/100 | 31/100 | 33/100 |

On the other hand, the comparison experiments between the SAETM provided in this paper and the traditional AETM in¹⁹ have been conducted. Select the trigger threshold $\varepsilon_0(k) = 0.1$ and initial adaptive parameter $\mathcal{J}_0 = 11$. Then, the trigger times under traditional AETM is shown in Figure. 12 and the trigger times is 38/100, which is more than the trigger times of the proposed SAETM in Table 2 and indicates that SAETM addressed in this paper can save resource of communication network effectively than that of traditional AETM.

6 | CONCLUSIONS

In this paper, the output feedback-based H_∞ control for IT2 T-S fuzzy system over network with hybrid attacks and Markovian packet losses via SAETM is investigated. The SAETM can not merely dynamically adjust the trigger threshold according to the change of system, but nimbly adapt to the trigger threshold based on whether Markovian packet loss occurs or not. In this way, the conservatism of the communication mechanism is greatly reduced. By applying iterative method and the packet loss dependent multiple Lyapunov function technique, the sufficient condition of the system stability as well as the controller design are obtained to address the closed-loop stability and the expected H_∞ performance.

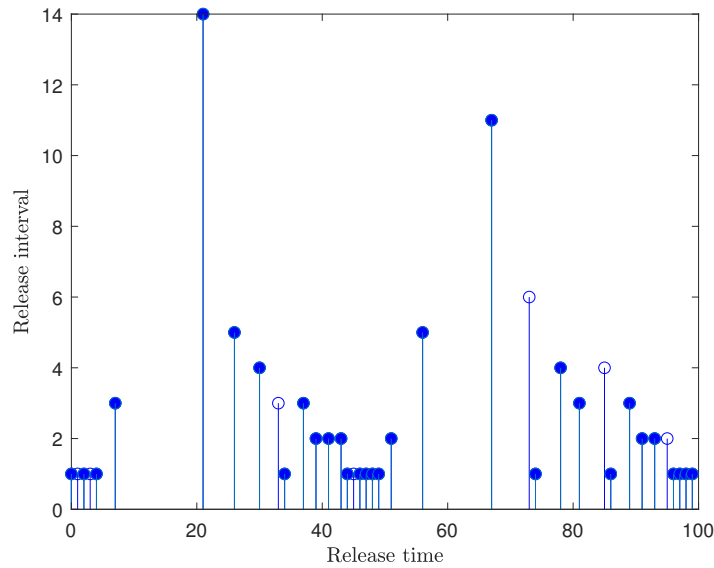


Figure 12 The trigger times under traditional AETM.

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