

1 **Two-Phase Approach to Modeling the Grain-Fluid**
2 **Flows with Deposition and Entrainment over Rugged**
3 **Topography**

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8 **Key Points:**

- 9 • A two-phase mixture model for debris flows over erodible and rugged topography
10 is proposed.
- 11 • In the model, entrainment is induced by the basal shear stress and deposition takes
12 place mainly due to the sediment settling process.
- 13 • In addition to the significant impacts on the post-event morphology, levee forma-
14 tion and layered deposits can be reproduced.

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Abstract

We present a grain-fluid mixture for debris flows moving on a rugged (non-trivial) topography, where entrainment and deposition may take place. The model equations are derived with respect to a terrain-following coordinate system, which is constructed based on the topographic surface. The coordinates are fixed in space, and a "subtopography" is added on the coordinate surface to account for the variation in the local topography when entrainment or deposition takes place. Numerical implementation is made based on a GPU-accelerated simulation tool, into which the entrainment-deposition mechanism is integrated accordingly. Two numerical examples are assigned to investigate the key features of the proposed model. One is on a horizontal plane, on which a finite mass of grain-fluid mixture is released from the state of rest. In this example, debris flow deposits significantly impact the post-event morphology and the associated flow behaviors. The other concerns a moving mass down an inclined chute merging into a horizontal deposition plane, where the levee formation is reproduced. At the end, the model is validated against a debris flow experiment to evaluate its applicability.

Plain Language Summary

Debris flows are grain-fluid mixtures driven by gravity and moving in mountain areas. Generally, they deposit when entering the open area with gentle slopes, which is commonly either the agricultural regions or the residential areas. Because of their high density, debris flows threaten residents and destroy infrastructure, and the deposited material may bury farms and buildings. In the post-event surveys, a large difference between the initial volume and the deposit heap can often be identified, indicating entrainment on the flow paths during movement. In this study, we present a grain-fluid mixture for debris flows moving on rugged topography. The introduction of the entrainment-deposition process allows the dynamic evolution of the deposit heap to be described, and levee formation can be reproduced. Using two numerical examples, we confirm the impacts of the entrainment-deposition process on the flow behaviors and the deposit morphology. The applicability of the present model is validated by outdoor debris flow experiments.

1 Introduction

Debris flows are grain-fluid mixtures driven by gravity and moving in mountain areas (e.g., Hutter et al., 1996; Takahashi, 2007). Generally, they deposit in areas with gentle slopes, which usually are either agricultural regions or residential areas. Although the speed has decreased in the deposition area, the debris flows are still highly destructive and threaten residents and infrastructure due to the severe impacts of the high-density flows. The deposited material may bury farms, houses and associated infrastructure. Post-event measurements sometimes indicate a large difference between the initial volume and the final deposits, giving solid evidence of entrainment on the flow paths (e.g., Pierson et al., 1990; Hungr et al., 2005; Chen et al., 2006, 2012, 2014; Berger et al., 2010). In some circumstances, the entrained or deposited material may change the composition of the debris flow, increasing the complexity of determining the rheology. Hence, addressing the varying rheology during movement with a single-phase approach is highly challenging.

Numerical simulations for scenario investigation can be employed as a powerful tool to estimate the debris flow hazard area. Many numerical simulation tools have been developed based on depth-averaged models to improve the computational efficiency because the assessments generally cover large areas. However, the coordinate system used may play an important role in depth-averaged models because the depth-averaged velocities are parallel to the coordinate axes. High deviations may occur for highly rugged topography when the conventional Cartesian coordinate system is adopted. Hence, a terrain-following coordinate system is proposed (e.g., Bouchut & Westdickenberg, 2004; Tai &

65 Kuo, 2008; Tai et al., 2012; Luca et al., 2009), in which the axes coincide with the to-
66 pographical surface. A comprehensive guidance for modeling gravity-driven flows slid-
67 ing on rugged topography can be found in Luca et al. (2016).

68 Within the continuum-mechanical framework, the model equations for debris flows
69 over erodible beds are generally derived and simplified with the depth-averaged approach.
70 In addition, because the debris flows consist of grains and interstitial fluid, most mod-
71 els are based on the concept of mixture, and they can be approximately categorized into
72 two types: quasi-two-phase and two-phase (or multiphase) approaches. Although the flow
73 body should consist of solid and fluid phases in the quasi-two-phase approach, the re-
74 lative velocity between the constituents is assumed to be very small compared with the
75 barycentric velocity. Only the momentum balance for the entire mixture is considered.
76 Consequently, only the barycentric velocity needs to be computed and no individual ve-
77 locity for each constituent is available. The main drawback of the quasi-two-phase ap-
78 proach is the difficulty of describing the grain-fluid (phase) separation induced by the
79 different velocities of the phases. In addition, addressing the variation in rheology caused
80 by the change of the composition concentrations is also difficult. However, most entrainment-
81 deposition rates are based on this quasi-two-phase approach, such as Takahashi et al. (1992);
82 Pitman et al. (2003); Cao et al. (2004); Li and Duffy (2011); Tai and Kuo (2012) and Iverson
83 and George (2014). Recently, Nishiguchi and Uchida (2022) noticed the importance of
84 rheology variation and suggested the concept of "phase-shift" in modeling debris flows
85 with the quasi-two-phase approach, where the associated impacts of the fine sediments
86 on the flow dynamics are investigated with respect to a landslide-induced debris flow.

87 With the two-phase (or multiphase) approach, each constituent has its own veloc-
88 ity such that the grain-fluid separation can be well captured. Many of the two-phase mod-
89 els for debris flows are based on the pioneering work of Pitman and Le (2005), and many
90 extensions exist. For example, Pudasaini (2012) proposed a general two-phase model for
91 debris flows. In Meng and Wang (2016), the buoyancy force is given in a different form
92 for an appropriate expression in the static state. Tai et al. (2019) introduced the model
93 equations of Meng and Wang (2016) in the terrain-following coordinates, and the non-
94 hydrostatic pore-fluid pressure with nonlinear deformation of the granular skeleton is con-
95 sidered in Heß et al. (2019). In addition, with the two-phase approach, the dilatancy ef-
96 fects are considered in Bouchut et al. (2016, 2017). Multiphase (grain, fine sediments and
97 water) models can be found in Pudasaini and Mergili (2019) and Ma et al. (2022). Al-
98 though the above-mentioned models can address grain-fluid separation, they do not in-
99 clude the entrainment-deposition mechanism.

100 As elaborated in Pudasaini and Fischer (2020), entrainment-deposition rates specif-
101 ically proposed for two-phase solid-fluid models are rare. In addition to the complexity
102 of theoretical formulation, this problem is also fraught with the scarcity of reliable data
103 in experiments for parameter calibration. For instance, while the entrainment-deposition
104 rate can be approximated by analyzing the jump condition of the momentum balance
105 equation at the basal interface using the ratio of the difference of the shear stress to the
106 difference in velocities on either side of the interface (e.g., Fraccarollo & Capart, 2002;
107 Iverson, 2012; Issler, 2014), there is a scarcity of corresponding experimental data. Mea-
108 surements of the shear stresses for an erodible bed are rare, and the measurement of the
109 velocity for granular flows is mainly limited to the near-wall field (e.g., Shirsath et al.,
110 2015; Sarno et al., 2018). Nevertheless, Pudasaini and Fischer (2020) presented a me-
111 chanical two-phase erosion model in which the entrainment rates are considered for the
112 solid and fluid phases separately and addressed five aspects for the entrainment-deposition
113 rate in modeling geophysical mass flows. In addition to the continuum-mechanical ap-
114 proach, Suzuki and Hotta (2016) employed a particle method for simulating debris flows,
115 the Modified Particles Method (MPS), where the entrainment-deposition process is in-
116 cluded at the basal surface. Suzuki et al. (2019) extended this MPS method to 3D and
117 applied it to mimic the depositional process of alluvial fans in experiments. In Suzuki

118 et al. (2019), the entrainment-deposition rate follows the suggestion in Suzuki et al. (2009),
 119 where the concept of the equilibrium sediment concentration (cf. Egashira, 1997) is adopted.

120 In the present study, erosion (entrainment) and deposition are considered to be based
 121 on distinct mechanisms: erosion is basically induced by the basal shear stress, and de-
 122 position takes place mainly due to the process of sediment settling. This concept stems
 123 from Cao et al. (2004) and Li and Duffy (2011), which are based on quasi-two-phase mod-
 124 els. Here, we extend their concept for flows in a two-phase approach. In the new entrain-
 125 ment and deposition mechanisms, the Shields parameter (Shields, 1936) and the Hjul-
 126 ström-Sundborg diagram (Hjulström, 1935) are considered for the initiations of entrain-
 127 ment and deposition, respectively. All model equations are derived in terrain-following
 128 coordinates (cf. Tai et al., 2012; Luca et al., 2016). In addition, following Tai et al. (2012)
 129 and Tai and Kuo (2012), the momentum loss due to deposition is considered, while no
 130 momentum gain occurs due to the entrainment because the eroded material is at rest
 131 in the bed before being entrained. Numerical implementation is achieved with a high-
 132 resolution shock-capturing scheme, the anti-diffusive, nonoscillatory central scheme pro-
 133 posed by Kurganov and Tadmor (2000) and Kurganov and Petrova (2007). With the in-
 134 tegration of the present model into the GPU-accelerated simulation tool (MoSES_2PDF
 135 in Ko et al. (2021)), the computational efficiency is highly enhanced for scenario inves-
 136 tigation and/or parameter calibration. Two idealized topographies (horizontal plane and
 137 inclined curved chute) are assigned to investigate and highlight the key features caused
 138 by the introduction of entrainment/deposition, where its impacts on the morphology and
 139 flow behaviors are examined. On the horizontal plane, a finite mass of grain-fluid mix-
 140 ture is released from the state of rest, and the material subsequently deposits on the plane.
 141 In the example of inclined curved chute, the flow body slides down the inclined section
 142 merging into a horizontal deposition plane. In addition to idealized numerical examples,
 143 the present model is validated against debris flow experiments shown in Suzuki et al. (2019)
 144 and Tsunetaka et al. (2022).

145 The remainder of this paper is structured as follows. In section 2, we provide a brief
 146 introduction to the resultant equations of the proposed grain-fluid model with entrain-
 147 ment and deposition, where the flow body is treated as a fully saturated mixture. Two
 148 numerical examples and one experimental validation are numerically investigated in sec-
 149 tion 3, in which the key features and applicability of the proposed model are thoroughly
 150 examined. Finally, concluding remarks and perspectives are given in section 4.

151 2 Model equations

152 2.1 Two-Phase Grain-Fluid Model Equations

153 The model used in the present study is based on Tai et al. (2019), where the model
 154 equations are derived using a depth-averaged approach and presented in dimensionless
 155 form. A terrain-following coordinate system (cf. Tai & Kuo, 2012; Tai et al., 2012; Luca
 156 et al., 2012, 2016) is employed in the derivation. As shown in Fig. 1, the flow depth is
 157 defined in the normal ζ -direction, and the ξ - and η -coordinates are tangential to the basal
 158 surface, where the ζ -direction coincides with the unit normal vector $\mathbf{n} = n_x \mathbf{e}_x + n_y \mathbf{e}_y +$
 159 $n_z \mathbf{e}_z$ of the basal surface. Letting \mathbf{r}_{xyz} and $\mathbf{r}_{\xi\eta\zeta}$ be the position vectors in O_{xyz} and $O_{\xi\eta\zeta}$,
 160 respectively, they are related by the Jacobian (transformation) matrix $\mathbf{\Omega}$, i.e., $\mathbf{r}_{xyz} =$
 161 $\mathbf{\Omega} \mathbf{r}_{\xi\eta\zeta}$. The flow body is assumed to be shallow, i.e., a characteristic length \mathcal{L} of the flow
 162 body along the topographic surface and a respective characteristic flow thickness \mathcal{H} , where
 163 the aspect ratio $\epsilon = \mathcal{H}/\mathcal{L} \ll 1$ is small. As elaborated in Tai et al. (2019), we use $\mathbf{\Omega} =$
 164 $\mathbf{\Omega}_b + \mathcal{O}(\epsilon^\chi)$ with $\chi \in (0, 1)$ for a topographic surface of shallow curvature, where $\mathbf{\Omega}_b$
 165 is the Jacobian matrix (transformation of coordinates) for the basal surface. The two-
 166 phase approach gives two equations for the mass balance and six equations for momen-
 167 tum conservation in three-dimensional configuration. The depth-integration process re-
 168 duces the number of momentum equations from six to four. With the aid of scaling anal-

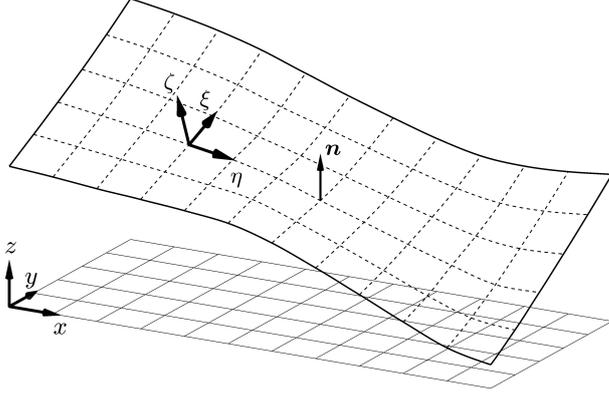


Figure 1. Coordinate system O_{xyz} and $O_{\xi\eta\zeta}$, in which the ζ -direction coincides with the unit normal vector $\mathbf{n} = (n_x, n_y, n_z)$ in O_{xyz} .

169 ysis, all physically insignificant terms are isolated in the resultant model equations (see
170 e.g. Tai et al., 2019).

171 The model equations are similar to the ones listed in (Tai et al., 2019). For ease
172 of distinguishing the differences, the additional terms and equation are marked in blue.
173 With $J_b = \det \mathbf{\Omega}_b$, the resultant mass balance equations for the solid and fluid phases
174 are

$$175 \quad \frac{\partial}{\partial t}(J_b h^s) + \frac{\partial}{\partial \xi}(J_b h^s v_\xi^s) + \frac{\partial}{\partial \eta}(J_b h^s v_\eta^s) = J_b \mathcal{E}^s \quad (1)$$

176 and

$$177 \quad \frac{\partial}{\partial t}(J_b h^f) + \frac{\partial}{\partial \xi}(J_b h^f v_\xi^f) + \frac{\partial}{\partial \eta}(J_b h^f v_\eta^f) = J_b \mathcal{E}^f, \quad (2)$$

178 respectively. In (1) and (2), $h^s = h\phi^s$ and $h^f = h\phi^f$, with h representing the mixture to-
179 tal depth, ϕ^s and ϕ^f denoting the depth-averaged volume concentrations of the solid and
180 fluid phases, respectively, $v_{\xi,\eta}$ is the tangential component of the depth-averaged veloc-
181 ity in the terrain-following coordinate system, and $\mathcal{E}^{s,f}$ represent the entrainment rates
182 of the solid and fluid phases. Here, the deposit is assumed to be fully saturated during
183 the depositional process, yielding $\mathcal{E}^f = \mathcal{E}^s \phi_b^f / (1 - \phi_b^f)$, where ϕ_b^f is the porosity of the
184 erodible bed. Hence, the evolution of the deposit heap is given by

$$185 \quad \frac{\partial \zeta_b}{\partial t} = \frac{\mathcal{E}^s}{\phi_b^s} = \frac{\mathcal{E}^f}{\phi_b^f} \quad \text{with} \quad \phi_b^s + \phi_b^f = 1. \quad (3)$$

186 The depth-averaged, leading-order momentum equations of the solid phase are given
187 by

$$188 \quad \begin{aligned} & \frac{\partial}{\partial t}(J_b h^s v_x^s) + \frac{\partial}{\partial \xi}(J_b h^s v_x^s v_\xi^s + \epsilon J_b h A_{11} \bar{N}^s) + \frac{\partial}{\partial \eta}(J_b h^s v_x^s v_\eta^s + \epsilon J_b h A_{21} \bar{N}^s) \\ &= \underbrace{J_b p_b^s n_x - \epsilon \alpha_\rho F_{1121}^B + J_b \alpha_\rho F_x^{\text{Rel}}}_{(i)} - \underbrace{\epsilon J_b \{N_b^s + \alpha_\rho \phi^s N_b^f\}}_{(ii)} \Phi_{1121} \\ & \quad - \underbrace{J_b p_b^s \tan \delta_b \frac{v_x^s}{\|\mathbf{v}^s\|}}_{(iii)} + \underbrace{J_b v_{x,b}^s \mathcal{E}^s}_{(iv)} \end{aligned} \quad (4)$$

189 and

$$\begin{aligned}
 & \frac{\partial}{\partial t}(J_b h^s v_y^s) + \frac{\partial}{\partial \xi}(J_b h^s v_y^s v_\xi^s + \epsilon J_b h A_{12} \bar{N}^s) + \frac{\partial}{\partial \eta}(J_b h^s v_y^s v_\eta^s + \epsilon J_b h A_{22} \bar{N}^s) \\
 &= \underbrace{J_b p_b^s n_y - \epsilon \alpha_\rho F_{1222}^B}_{(i)} + J_b \alpha_\rho F_y^{\text{Rel}} - \underbrace{\epsilon J_b \left\{ N_b^s + \alpha_\rho \phi^s N_b^f \right\} \Phi_{1222}}_{(ii)} \\
 & \quad - \underbrace{J_b p_b^s \tan \delta_b \frac{v_y^s}{\|\mathbf{v}^s\|}}_{(iii)} + \underbrace{J_b v_{y,b}^s \mathcal{E}^s}_{(iv)},
 \end{aligned} \tag{5}$$

191 where, with $\partial_\xi(\cdot) = \partial(\cdot)/\partial\xi$ and $\partial_\eta(\cdot) = \partial(\cdot)/\partial\eta$,

$$\begin{aligned}
 F_{1121}^B &= \phi^s \left\{ A_{11} \partial_\xi(J_b h \bar{p}^f) + A_{21} \partial_\eta(J_b h \bar{p}^f) \right\}, \\
 F_{1222}^B &= \phi^s \left\{ A_{12} \partial_\xi(J_b h \bar{p}^f) + A_{22} \partial_\eta(J_b h \bar{p}^f) \right\}, \\
 F_x^{\text{Rel}} &= c_D \phi^s \phi^f h (v_x^f - v_x^s), \quad F_y^{\text{Rel}} = c_D \phi^s \phi^f h (v_y^f - v_y^s),
 \end{aligned} \tag{6}$$

193 and

$$\Phi_{1121} = A_{11} \partial_\xi \zeta_b + A_{21} \partial_\eta \zeta_b, \quad \Phi_{1222} = A_{12} \partial_\xi \zeta_b + A_{22} \partial_\eta \zeta_b \tag{7}$$

195 are introduced. In (4) – (7), $\mathbf{A} = (A_{ij}) = \mathbf{\Omega}_b^{-1}$, and $\alpha_\rho (= \rho^f/\rho^s)$ stands for the den-
 196 sity ratio of flowing body. $\bar{N}^s = n_z(1 - \alpha_\rho)h^s/2$ and $\bar{p}^f = n_z h/2$ respectively denote
 197 the depth-averaged pressure of the solid and fluid phase. Notation $p_b^s = h^s n_z [(1 - \alpha_\rho) -$
 198 $\epsilon^X \kappa^s]$ represents the solid pressure at the basal surface, where $\kappa^s = v_x^s (\partial_\xi n_x) v_\xi^s + v_\xi^s (\partial_\xi n_y) v_y^s +$
 199 $v_\xi^s (\partial_\xi n_z) v_z^s + v_\eta^s (\partial_\eta n_x) v_x^s + v_\eta^s (\partial_\eta n_y) v_y^s + v_\eta^s (\partial_\eta n_z) v_z^s$ represents the centripetal accelera-
 200 tion (cf. Tai et al., 2012, 2019). Notations $F_{1121/1222}^B$ denote the buoyancy forces, $F_{x/y}^{\text{Rel}}$
 201 stand for the drags due to the velocity difference between the two phases, and c_D is the
 202 drag (between the constituents) coefficient. On the right-hand side of (4) and (5), terms
 203 (i) are the components of the normal pressure at the bottom, which are caused by the
 204 reaction force of gravity; terms (ii) represent the effects caused by the deposit heap; terms
 205 (iii) indicate the basal drags with δ_b being the angle of basal friction of the solid phase;
 206 terms (iv) stand for the momentum loss due to deposition.

207 The momentum equations for the fluid phase read as

$$\begin{aligned}
 & \frac{\partial}{\partial t}(J_b h^f v_x^f) + \frac{\partial}{\partial \xi}(J_b h^f v_x^f v_\xi^f + \epsilon J_b h A_{11} \bar{p}^f) + \frac{\partial}{\partial \eta}(J_b h^f v_x^f v_\eta^f + \epsilon J_b h A_{21} \bar{p}^f) \\
 &= \underbrace{J_b p_b^f n_x + \epsilon F_{1121}^B}_{(v)} - J_b F_x^{\text{Rel}} + \epsilon F_x^{\text{Vis}} - \underbrace{J_b h^f \frac{\vartheta_b^f v_x^f}{\epsilon N_R} - J_b \Pi M \frac{n^2 h^f v_x^f \|\mathbf{v}^f\|}{h^{f4/3}}}_{(vi)} \\
 & \quad - \underbrace{\epsilon J_b \phi^f N_b^f \Phi_{1121}}_{(vii)} + \underbrace{J_b v_{x,b}^f \mathcal{E}^f}_{(viii)}
 \end{aligned} \tag{8}$$

209 in the x -direction, and

$$\begin{aligned}
 & \frac{\partial}{\partial t}(J_b h^f v_y^f) + \frac{\partial}{\partial \xi}(J_b h^f v_y^f v_\xi^f + \epsilon J_b h A_{12} \bar{p}^f) + \frac{\partial}{\partial \eta}(J_b h^f v_y^f v_\eta^f + \epsilon J_b h A_{22} \bar{p}^f) \\
 &= \underbrace{J_b p_b^f n_y + \epsilon F_{1222}^B}_{(v)} - J_b F_y^{\text{Rel}} + \epsilon F_y^{\text{Vis}} - \underbrace{J_b h^f \frac{\vartheta_b^f v_y^f}{\epsilon N_R} - J_b \Pi M \frac{n^2 h^f v_y^f \|\mathbf{v}^f\|}{h^{f4/3}}}_{(vi)} \\
 & \quad - \underbrace{\epsilon J_b \phi^f N_b^f \Phi_{1222}}_{(vii)} + \underbrace{J_b v_{y,b}^f \mathcal{E}^f}_{(viii)},
 \end{aligned} \tag{9}$$

211 in the y -direction, where we introduce

$$\begin{aligned}
 F_x^{\text{Vis}} &= \frac{\phi^f}{N_R} \left\{ 2\partial_\xi \left[J_b h \left(A_{11} \partial_\xi v_\xi^f + A_{21} \partial_\eta v_\xi^f \right) \right] \right. \\
 &\quad \left. + \partial_\eta \left[J_b h \left(A_{12} \partial_\xi v_\xi^f + A_{22} \partial_\eta v_\xi^f + A_{11} \partial_\xi v_\eta^f + A_{21} \partial_\eta v_\eta^f \right) \right] \right\} \\
 F_y^{\text{Vis}} &= \frac{\phi^f}{N_R} \left\{ \partial_\xi \left[J_b h \left(A_{12} \partial_\xi v_\xi^f + A_{22} \partial_\eta v_\xi^f + A_{11} \partial_\xi v_\eta^f + A_{21} \partial_\eta v_\eta^f \right) \right] \right. \\
 &\quad \left. + 2\partial_\eta \left[J_b h \left(A_{12} \partial_\xi v_\eta^f + A_{22} \partial_\eta v_\eta^f \right) \right] \right\}
 \end{aligned} \tag{10}$$

213 for the viscous effects with $N_R = \rho^f \mathcal{H} \sqrt{g\mathcal{L}} / \mu^f$ and μ^f the fluid viscosity (cf. Tai et al.,
 214 2019). On the right-hand side of (8) and (9), terms (v) indicate the components of the
 215 normal pressure at the bottom; terms (vi) are the basal drags; terms (vii) represent the
 216 effects caused by the deposit heap; and terms (viii) represent the momentum loss due
 217 to deposition. Notably, the basal drags indicated in terms (vi) consist of two terms, the
 218 first of which refers to the Navier drag as employed in Tai et al. (2019), and the second
 219 term accounts for the Manning drag, as widely applied in open channel flows (see e.g.,
 220 Li & Duffy, 2011). In the Manning drag term, n is the Manning coefficient, and $\Pi^M =$
 221 $\mathcal{H}^{4/3} / (g\mathcal{L})$ is a factor for the consistency of dimension.

222 In addition, the maximum value of the solid concentration must be limited in the
 223 range of $[0.6, 0.8]$ in the computation because the grains have their own shapes. How-
 224 ever, when using the above model equations, the solid concentration may approach unity.
 225 Such a high value is unrealistic. Nevertheless, we also found that a large value of the drag
 226 coefficient c_D in (6) can alleviate the phase separation. In the present study, instead of
 227 a constant value, we suggest that

$$c_D = c_{d0} \left\{ 1.0 + e^{\left[n_c (\phi^s - 0.5) / \phi_{\text{max}}^s \right]} \right\} \tag{11}$$

229 to reduce the unrealistically high solid concentration. In (11), c_{d0} is a coefficient whose
 230 value is related to the resistance (inverse of the permeability) for viscous flows through
 231 a porous medium (e.g., Darcy's Law), and n_c is an empirical coefficient. The formula-
 232 tion (11) will synchronize the movements of the two constituents at high solid concen-
 233 trations, impeding the phase separation and therefore lowering the maximum solid con-
 234 centration in the computation. In our numerical tests, $n_c = 6.0$ can deliver satisfac-
 235 tory results, and this value is used for all computations in the following study.

236 2.2 Entrainment and Deposition

237 Here, based on the concept of Cao et al. (2004) and Li and Duffy (2011), we pro-
 238 pose a modified entrainment-deposition rate for the two-phase approach

$$\mathcal{E}^s = E^s - D^s \tag{12}$$

240 with

$$E^s = \begin{cases} \alpha_E \sqrt{gh} \left(\tilde{\Psi} - \tilde{\Psi}_{\text{crt}} \right) \phi_b^s, & \text{for } \tilde{\Psi} > \tilde{\Psi}_{\text{crt}} \\ 0.0, & \text{otherwise} \end{cases} \tag{13}$$

242 for the entrainment and

$$D^s = \begin{cases} \alpha_D \omega \left(\tilde{\Sigma}_b - \tilde{\Sigma}_{\text{crt}} \right), & \text{for } \tilde{\Sigma}_b > \tilde{\Sigma}_{\text{crt}} \\ 0.0, & \text{otherwise} \end{cases} \tag{14}$$

244 for deposition. In (13), α_E denotes the entrainment coefficient, and

$$\tilde{\Psi} = \frac{\tau_b}{(\rho^s - \rho^f)gd} \tag{15}$$

246 is related to the Shields parameter (Shields, 1936), where d indicates the sediment di-
 247 ameter and τ_b stands for the resultant basal friction (i.e., sum of the terms (iii) and (vi)
 248 of the model equations in Sect. 2.1). $\tilde{\Psi}_{\text{crt}}$ is the critical value whose magnitude depends
 249 on the the particle diameter of the sediments (cf. Berenbrock & Tranmer, 2008). In the
 250 present study, we set $\tilde{\Psi}_{\text{crt}} = 0.04$ for all numerical investigations since the mean grain
 251 size used in the experiment in Sect 3.3 is approximately 2.6 mm.

252 In (14), α_D is the deposition coefficient, $\tilde{\Sigma}_{\text{crt}}$ is the critical value, and we suggest

$$253 \quad \tilde{\Sigma}_b = \left(\tilde{v} - \frac{\|\mathbf{v}^s\|}{\sqrt{g d}} \right) \phi^s, \quad (16)$$

254 which introduces a sediment-diameter-dependent critical speed for the occurrence of de-
 255 position. That is, for large flow speeds and small sediment concentrations, $\tilde{\Sigma}_b$ decreases,
 256 and no deposition takes place. \tilde{v} is the dimensionless critical speed, whose value depends
 257 on the mean diameter of the sediment and can be determined with the help of the Hjul-
 258 ström-Sundborg diagram (cf. Hjulström, 1935; Earle, 2015). Following the Hjulström-
 259 Sundborg diagram for a 2.6-mm grain of sand, the transportation zone approximately
 260 lies in the velocity range [0.1, 0.6] m/s, yielding the range [0.627, 3.7] for \tilde{v} . In the present
 261 study, we set $\tilde{v} = 1.6$ for grains with a mean size of 2.6 mm. In addition, the impacts
 262 of the fluid viscosity are considered through the sediment settling velocity ω , which is
 263 given by an empirical formula (Zhang, 1989), namely,

$$264 \quad \omega = \sqrt{\left(\frac{13.95 \mu^f}{\rho^f d} \right)^2 + 1.09 g d \left(\frac{\rho^s}{\rho^f} - 1 \right)} - \frac{13.95 \mu^f}{\rho^f d}, \quad (17)$$

265 as employed in models for sediment transport, such as Li and Duffy (2011) or Cao et al.
 266 (2004).

267 With identical scalings for the model equations, (1) – (5) and (8) – (9), we have
 268 the dimensionless entrainment-deposition rate

$$269 \quad \mathcal{E}^{s*} = E^{s*} - D^{s*}. \quad (18)$$

270 Here, we use the superscript $(*)$ to denote the quantities after the scaling process. In
 271 (18), the dimensionless entrainment rate reads

$$272 \quad E^{s*} = \begin{cases} \alpha_E^* \sqrt{h^*} (\tilde{\Psi}^* - \tilde{\Psi}_{\text{crt}}^*) \phi_b^s, & \text{for } \tilde{\Psi}^* > \tilde{\Psi}_{\text{crt}}^* \\ 0.0, & \text{otherwise} \end{cases} \quad (19)$$

273 where $\alpha_E^* = \epsilon^{-1/2} \alpha_E$, $\tilde{\Psi}_{\text{crt}}^* = \tilde{\Psi}_{\text{crt}}$, $d = \mathcal{L} d^*$,

$$274 \quad \tilde{\Psi}^* = \frac{(\text{iii})^* + \alpha_\rho (\text{vi})^*}{(1 - \alpha_\rho) d^*}, \quad (\text{iii})^* = p_b^{s*} \tan \delta_b \quad \text{and} \quad (\text{vi})^* = h^{f*} \frac{\vartheta_b^f \|\mathbf{v}^{f*}\|}{\epsilon N_R} + \Pi^M \frac{n^2 h^{f*} \|\mathbf{v}^{f*}\|^2}{h^{f*4/3}}.$$

275 Furthermore, the dimensionless deposition rate is given by

$$276 \quad D^{s*} = \begin{cases} \alpha_D^* \omega^* (\tilde{\Sigma}_b^* - \tilde{\Sigma}_{\text{crt}}^*), & \text{for } \tilde{\Sigma}_b^* > \tilde{\Sigma}_{\text{crt}}^* \\ 0.0, & \text{otherwise} \end{cases} \quad (20)$$

277 with $\alpha_D^* = \epsilon^{-3/2} \alpha_D$,

$$278 \quad \tilde{\Sigma}_b^* = \left(\tilde{v} - \frac{\|\mathbf{v}^{s*}\|}{\sqrt{d^*}} \right) \phi^s \quad \text{and} \quad \omega^* = \sqrt{\left(\frac{13.95}{N_R d^*} \right)^2 + 1.09 \epsilon d^* \left(\frac{1 - \alpha_\rho}{\alpha_\rho} \right)} - \frac{13.95}{N_R d^*}.$$

279 Here, we note that $(\tilde{\Sigma}_b, \tilde{\Sigma}_{\text{crt}}) = \sqrt{\mathcal{L}/\mathcal{H}} (\tilde{\Sigma}_b^*, \tilde{\Sigma}_{\text{crt}}^*)$ and $\omega = \sqrt{g \mathcal{L}} \omega^*$.

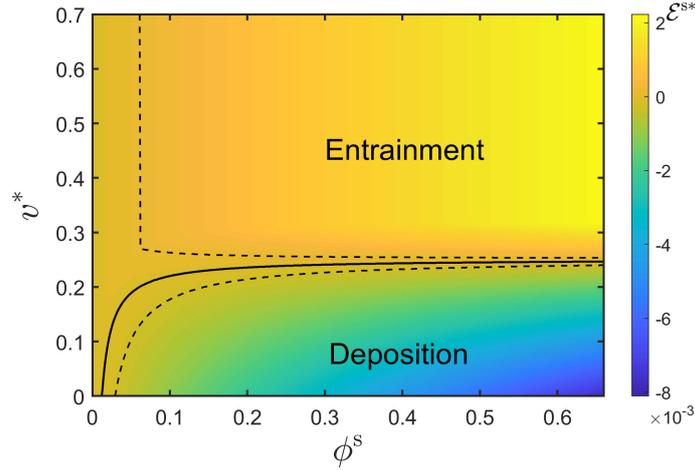


Figure 2. The resulting entrainment-deposition rate \mathcal{E}^{s*} , plotted against v^* and ϕ^s for a flow thickness $h^* = 1.0$. The solid line represents the boundary between entrainment and deposition, while the dashed lines indicate the locations of $\mathcal{E}^{s*} = \pm 2 \times 10^{-4}$, respectively.

280 The present formulation (18) (or (12)) provides a smooth transition between en-
 281 trainment and deposition. Figure 2 depicts the resulting entrainment-deposition rate \mathcal{E}^{s*} ,
 282 computed by (18) for a flow thickness $h^* = 1.0$ with a mean sediment grain size of 0.026.
 283 The parameter values used are identical to those used in the numerical investigation (Sect. 3)
 284 and are listed in Table 1. The solid line in the graph represents the boundary between
 285 entrainment and deposition. The entrainment-deposition phenomenon is not significant
 286 when it lies within the area between the two dashed lines, which indicate the locations
 287 of $\mathcal{E}^{s*} = \pm 2 \times 10^{-4}$ (about 0.02% of the flow thickness h^*). As shown in Fig. 2, en-
 288 trainment dominates the process at high speed, and its magnitude increases gently with
 289 the speed. Deposition becomes dominant as the flow speed decreases and approaches zero.
 290 For very low solid concentration (e.g. $\phi^s < 0.01$), neither entrainment nor deposition
 291 takes place.

292 3 Numerical Investigation

293 The equation system consists of (1) – (5), (8) – (9) together with (11) and (18),
 294 has been implemented and integrated into a CUDA-GPU-accelerated simulation tool called
 295 MoSES_2PDF, which is proposed by Ko et al. (2021). MoSES_2PDF is build on the foun-
 296 dation of the two-phase model by Tai et al. (2019) and developed using the anti-diffusive,
 297 nonoscillatory central scheme proposed by Kurganov and Tadmor (2000) and Kurganov
 298 and Petrova (2007). For high resolution in space, the Mimod TVD slope limiter is used
 299 for cell reconstruction. Kurganov and Tadmor (2000) proved that the Courant-Friedrichs-
 300 Lewy (CFL) number should be less than 0.125 to satisfy the *maximum principle*; con-
 301 sequently, CFL = 0.1 is set for all the following examples in the computation.

302 Two idealized numerical examples are available to explore the key features, and one
 303 experimental validation is available to examine the applicability. In the first idealized
 304 example (cf. Sect. 3.1), a heap with a parabolic shape in section view is released on a
 305 horizontal plane. The second example concerns a finite mass flowing down an inclined
 306 curved chute and merging into a horizontal deposition zone. In both examples, scenar-
 307 ios with and without the depositional mechanism are considered in the computation to
 308 investigate the impacts of the deposition mechanism on the flow behaviors. For the val-
 309 idation, we consider the debris flow experiment and the associated measurements and

Table 1. Parameters in computation for examples of horizontal flat plane and inclined curved chute

Parameters	Value	Description
$\epsilon = \mathcal{H}/\mathcal{L}$	1	Aspect Ratio
$\alpha_\rho = \rho^f/\rho^s$	0.4	Density Ratio (1.06/2.65)
ϕ_0^s	0.5	Initial solid concentration
δ_b	35°	Angle of basal friction (solid phase)
ϑ_b^f	1.0	Navier fluid friction coefficient
c_{d0}	4.0	Drag coefficient
μ^f	3.5 mPa·s	Viscosity of interstitial fluid
N_R	30,000	Viscous number
d^*	0.026	Sediment median diameter
$\tilde{\Psi}_{\text{crt}}^*$	0.04	Critical Shields parameter
\tilde{v}	2.0	Dimensionless critical speed for deposition
$\tilde{\Sigma}_{\text{crt}}^*$	0.02	Critical value for deposition
ϕ_b^f	0.38	Porosity of bottom
α_E^*	0.0002	entrainment coefficient
α_D^*	0.04	Deposition coefficient
n	0.03	Manning coefficient
Π^M	0.04736	Factor for Manning coefficient
$\Delta x = \Delta y$	0.1, 0.2	Mesh size (dimensionless)
CFL	0.1	CFL number

310 computed results presented in Suzuki et al. (2019) and Tsunetaka et al. (2022). Because
311 the model equations are given in dimensionless form, the first two examples are given
312 and discussed in a nondimensional manner. In the application to the outdoor debris flow
313 experiment, all variables and the employed topographic configurations are converted to
314 dimensional variables to meet the experimental setup.

315 3.1 Horizontal Flat Plane

316 In this numerical example, a finite mass of parabolic shape in sectional view is lo-
317 cated at a horizontal flat plane. The mass, initially at rest with a solid concentration of
318 0.5 (i.e., $\phi_0^s = 0.5$), is released and expands radially, forming a deposit heap at the bot-
319 tom. The initial height of the mass is 1.0, and its base covers a circle with a radius of
320 3.0. The center of the initial mass sits at the center $(x, y) = (0, 0)$ of the computational
321 domain $[-10, 10] \times [-10, 10]$, and mesh size is $\Delta x = \Delta y = 0.1$. The parameters used
322 in the computation are collected and listed in Table 1.

323 Figure 3 shows the contours of the evolutions at different time levels, where the re-
324 sults of flow/deposit thickness less than 0.02 are filtered. Those for flow thickness with-
325 out deposition are given in the panels in row (a), the panels in row (b) are for the flow
326 thicknesses with the depositional process, and the associated deposit heaps are given in
327 the panels in row (c). The corresponding sectional views along $y = 0$ are illustrated in
328 Fig. 4, in which the blue dashed line depicts the heap of deposit and the red line indi-
329 cates the flow surface. At $t = 2.0$, the deposit heap begins to develop from the center
330 and rapidly grows in the radial direction. At $t = 5.0$, the heap covers nearly the en-
331 tire bottom area of the flow body. From $t = 10.0$ to 35.0, the heap grows mainly in the
332 vertical direction. At $t = 35.0$, most of the sediments have been deposited, and a very
333 thin flowing layer remains on the top of the heap (cf. Fig. 4b). If no depositional pro-
334 cess is considered (panels of Figs. 3a and 4a), the entire flow body comes to a state of

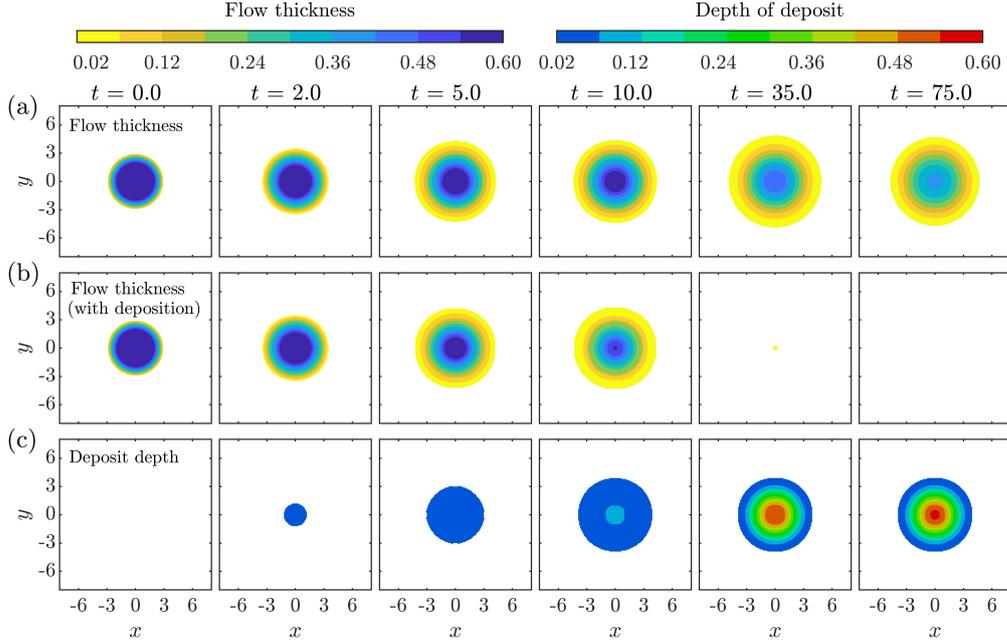


Figure 3. (a) Evolution of the flow body without considering the depositional mechanism; (b) Evolution of the flowing layer (with the depositional mechanism); (c) Evolution of the deposit heap under the flowing layer with respect to panels in (b).

335 rest at approximately $t = 75.0$ (the maximum speed is less than 0.01 in the computa-
 336 tional domain), while the deposit heap has completely developed shortly after $t = 35.0$.

337 The left panel of Fig. 5 shows a 3D view of the final flow body (at $t = 75.0$), while
 338 the right panel displays the final deposit at $t = 35.0$ or 75.0 , as shown in Fig. 4b. In
 339 addition to the longer duration of mobility, the geometry of the final flow body is much
 340 flatter, while the heap of final deposit is more concentrated. These findings reveal that
 341 the depositional process significantly impacts both the movement duration and the mor-
 342 phology, for which the deposit heap exhibits a concave shape in the sectional view (cf.
 343 Fig. 4b), as elaborated in Chen and Capart (2022).

344 3.2 Inclined Curved Chute

345 Besides studying the depositional pattern on the horizontal flat plane, we also ex-
 346 amine the depositional process of a flowing finite mass released from an inclined flat plane
 347 and merging into a horizontal plane, where a heap of sediment is formed. The chute cons-
 348 sists of three sections: the inclined part, the transition section, and the horizontal plane.
 349 The computational domain covers $x \in [0, 80]$ in the downslope direction, and $y \in [-18, 18]$
 350 in the transverse direction, where the mesh size is $\Delta x = \Delta y = 0.2$. The inclination
 351 angle is 40° , and the transition section lies in the range $x \in [24, 36]$. The finite mass
 352 is released from a parabolic cap with a circular base of radius 3.2 and height 2.0. The
 353 center of the cap is located at $(x, y) = (6.0, 0.0)$. The initial solid concentration is 0.5,
 354 and the mass is released with a given velocity in the downslope direction. Namely, the
 355 x - and y -components of the velocity read

$$\begin{aligned}
 356 \quad v_x(x, y, t = 0) &= \begin{cases} 1.2 + (x - 6.0)/3.2 & \text{for } h > 0, \\ 0.0 & \text{else,} \end{cases} \\
 v_y(x, y, t = 0) &= 0.0,
 \end{aligned} \tag{21}$$

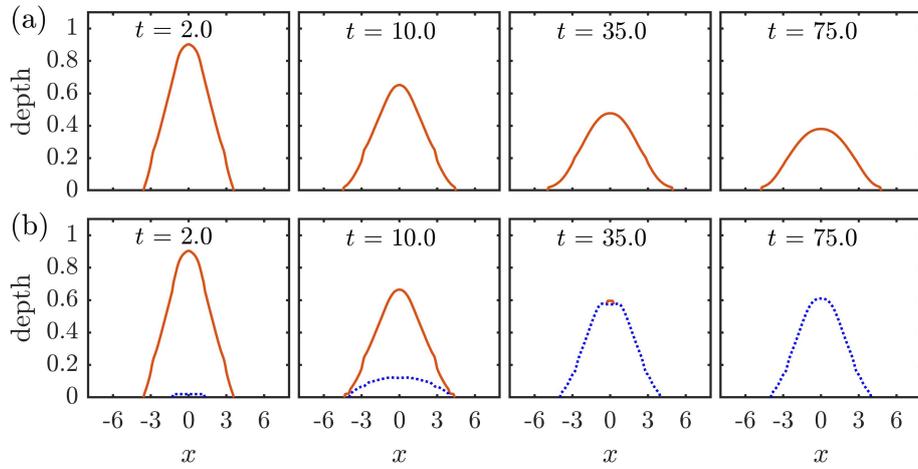


Figure 4. Sectional view (along $y = 0.0$) of the evolutions of the flow surface (red solid line) and deposit heap (blue dotted line). (a) Results computed without the depositional mechanism; (b) Results computed with the depositional mechanism.

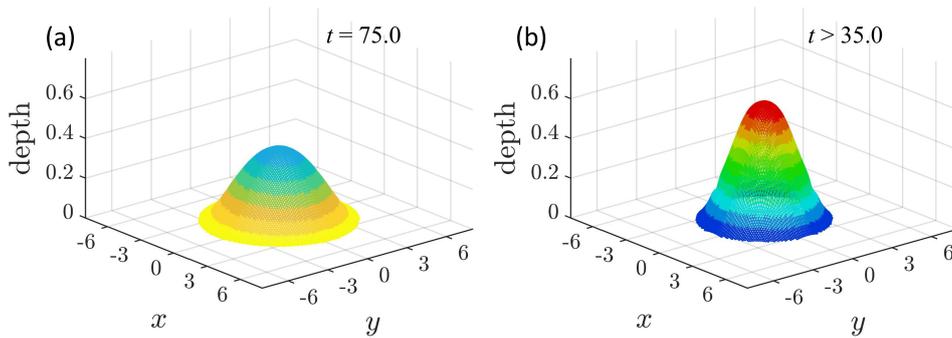


Figure 5. (a) Distribution of the final flow thickness at $t = 75.0$, where no depositional mechanism is considered in the computation; (b) Geometry of the final deposit (cf. the panels of (c) at $t = 35.0$ or 75.0 in Fig. 4).

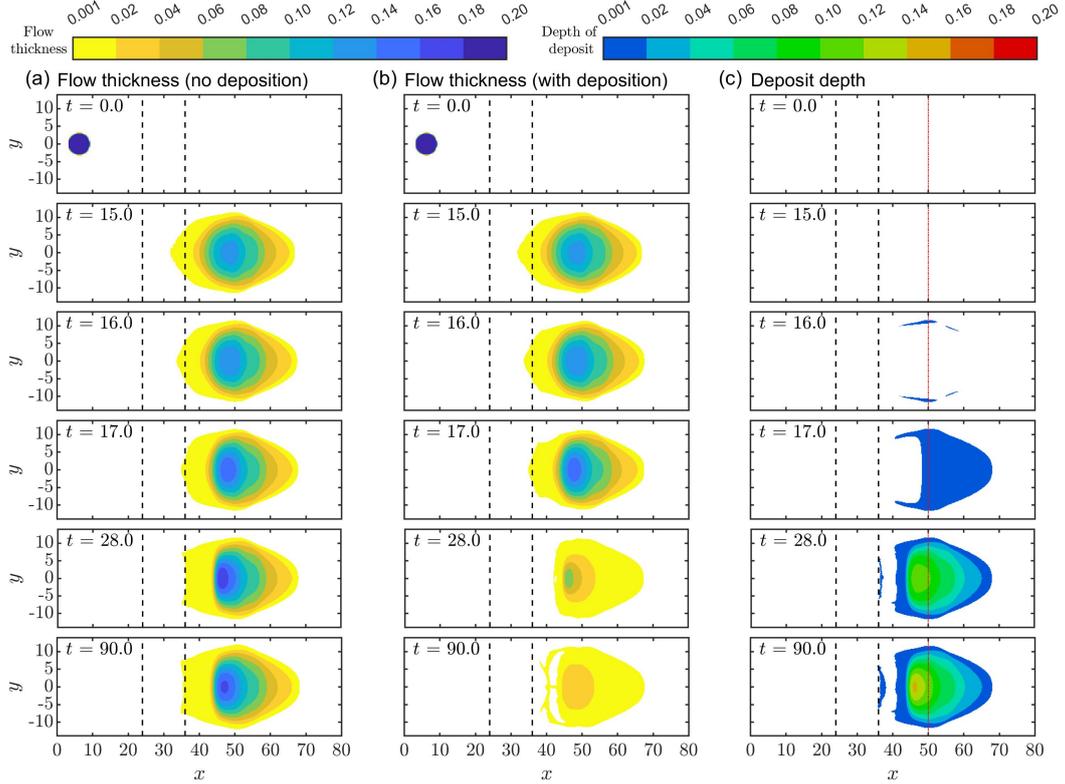


Figure 6. (a) Evolution of the flow body without considering the deposition; (b) Evolution of the flowing layer (with the deposition mechanism); (c) Evolution of the deposit heap under the flowing layer with respect to panels in Column (b). In all panels, the black dashed lines marked the transition section, and the red dash-dotted line indicates the location for the cross-sectional view in Fig. 7.

357 respectively. The parameters used in the computation are identical to those for the pre-
 358 vious example.

359 Figure 6 illustrates the evolution of the flow body and the associated heap of the
 360 deposit by contour plots from $t = 0.0$ to 90.0 at different time levels. The panels in the
 361 left column (indexed by a) are the flow thicknesses without considering the process of
 362 deposition/entrainment in the computation, the panels in the middle column (indexed
 363 by b) present the thickness distributions of the flowing layer where the deposition/entrainment
 364 is considered, and the associated heaps of deposits are depicted in the panels in the right
 365 column (indexed by c). The black dashed lines mark the transition section, and the red
 366 dash-dotted line in Panel c marks the location for the sectional view in Fig. 7b. The cor-
 367 responding longitudinal sectional views along the chute centerline ($y = 0.0$) are shown
 368 in Fig. 7a, while the cross-sectional views at $x = 50.0$ are depicted in Fig. 7b.

369 Although most of the flow body has passed the transition zone and slowed due to
 370 the basal drag at $t = 15.0$, no material deposited because the velocity remained too high.
 371 At $t = 16.0$, deposits begin to take place, where heaps of deposits are found on both
 372 side flank margins of the tail. The heaps of the deposit develop and extend rapidly, and
 373 nearly the entire bottom of the flowing layer of the front half part is covered by a thin
 374 heap of deposit (cf. the panels from $t = 16.0$ to 17.0). The levee formation, as presented
 375 in Pudasaini et al. (2005); Tai and Kuo (2012) and de Haas et al. (2015), can be observed
 376 on both flank margins in the panel at $t = 16.0$ (cf. Fig. 6c and Fig. 7b). Although the

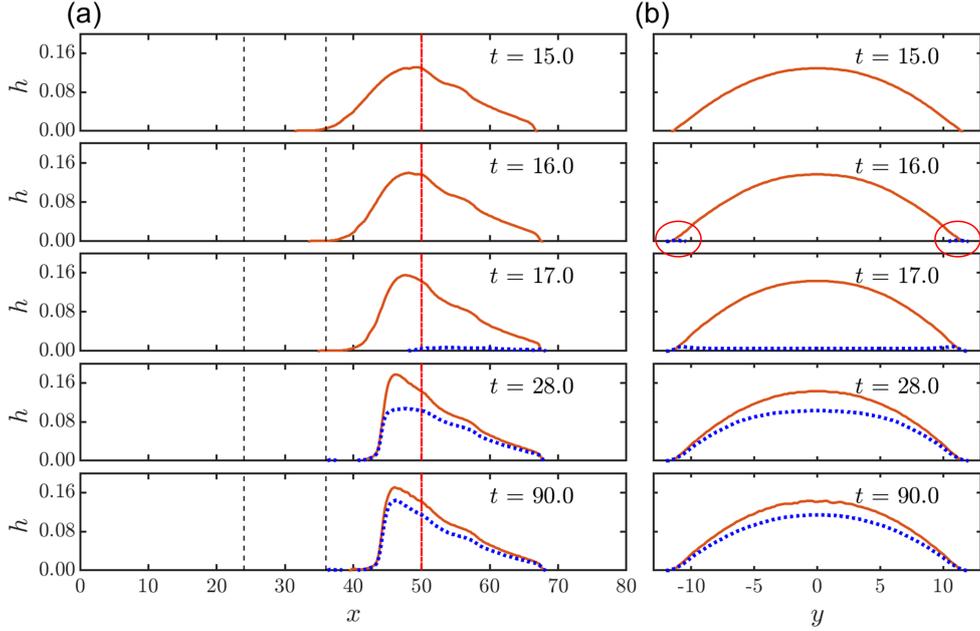


Figure 7. Sectional views of the evolutions of the flow surface (red solid line) and deposit heap (blue dotted line). (a) Sectional view at $y = 0.0$; (b) Sectional view at $x = 50.0$ as indicated by the red dash-dotted line in Panels (a) and in Fig. 6c, where the red circles mark the locations of levees.

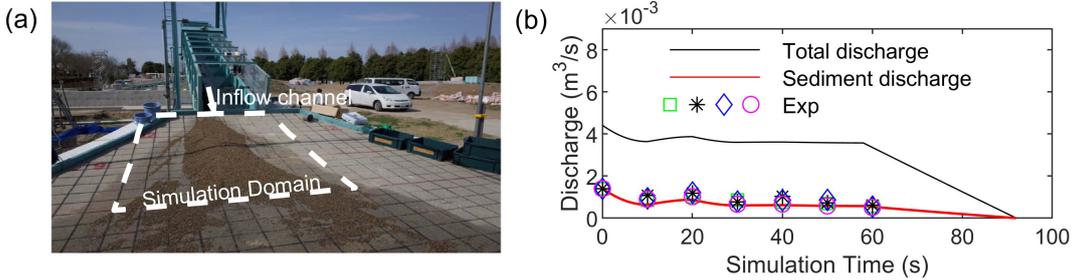


Figure 8. (a) Debris Flow Experiment (modified from Tsunetaka et al. (2019)); (b) The evolution of the discharge into the deposition plane.

377 levee deposits are not as significant (thick) as shown in Tai and Kuo (2012), the formulae used for the entrainment-deposition rates are totally distinct. From $t = 17.0$ to 28.0, the heap mainly grows in thickness without extending the coverage. At $t = 28.0$, an additional thin heap of deposit develops at the rear, and the main heap of deposit is covered by a thin flowing layer (see Fig. 7). From $t = 28.0$ to 90.0, the rear part of the thin layer slips backward down from the main heap of the deposit due to the high slope. It is interesting to note that the thin layer does not completely deposit because most of the sediments have already been deposited and the remaining concentration is too low for further deposition. At approximately $t = 90.0$, the maximum speed of the flowing layer is less than 0.01 in both cases (with or without considering the process of deposition/entrainment), and it is assumed to be at the state of rest. Moreover, the results here are filtered for flow/deposit thickness values less than 0.001 due to the relatively thin flowing body and deposit.

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Table 2. Parameters in computation for simulating the debris flow in experimental in Tsunetaka et al. (2022)

Parameters	Value	Description
$\epsilon = \mathcal{H}/\mathcal{L}$	0.1 m/0.1 m	Aspect Ratio
$\alpha_\rho = \rho^f/\rho^s$	0.37879	Density Ratio (1.00/2.64)
x	$\in[0, 6]$ m	Domain in x -direction
y	$\in[-2.2, 2.2]$ m	Domain in y -direction
$\Delta x = \Delta y$	0.02 m	Mesh size
δ_b	34.2°	Angle of basal friction (solid)
ϑ_b^f	0.1	Navier fluid friction coefficient
c_{d0}	1.5	Drag coefficient
μ^f	1.0526 mPa·s	Viscosity of interstitial fluid
N_R	94,048	Viscous number
d	0.0026 m	Sediment median diameter
$\tilde{\Psi}_{\text{crt}}^*$	0.04	Critical Shields parameter
\tilde{v}	1.6	Dimensionless critical speed for deposition
$\tilde{\Sigma}_{\text{crt}}^*$	0.016	Critical value for deposition
ϕ_b^f	0.38	Porosity of bottom
α_E^*	0.00008	Entrainment coefficient
α_D^*	0.04	Deposition coefficient
n	0.016	Manning coefficient
Π^M	0.04736	Factor for Manning coefficient
CFL	0.1	CFL number

3.3 Application to Debris Flow Experiment

390
 391 The proposed model is applied to the debris flow experiment in Suzuki et al. (2019)
 392 and Tsunetaka et al. (2022) for validation. In the experiment, a water-sediment mixture
 393 is released from a straight flume channel, which is 8.0 m long, 0.1 m wide and inclined
 394 with an angle of 15°. As shown in Fig. 8a, the channel is connected to a flat plane, which
 395 serves as the deposition zone. The inclination angle of this plane decreases at a rate of
 396 3° per meter in the longitudinal direction from 12° to 3°. The investigation focuses on
 397 the flat deposition zone, such that a computational domain $[0.0, 6.0] \times [-2.2, 2.2]$ (in m)
 398 is assigned, cf. Fig. 8a. The debris flow enters the computational domain from the flume
 399 channel at $x = 0.0$. The inflow condition is set based on the experimental data (cf. Suzuki
 400 et al., 2019; Tsunetaka et al., 2022), such that a uniform depth of 20 cm sediment of a
 401 median diameter $d = 2.6$ mm on the flume bed was flushed and allowed to flow into
 402 the computational domain by a water supply of 3×10^{-3} m³/s at the top of the flume
 403 for a duration of 60 s. Figure 8b illustrates the inflow condition in the computation, in
 404 which the total (water and sediment) discharge is depicted by the blue solid line, and
 405 the red line indicates sediment discharge. The sediment discharge is set by the mean value
 406 of four separate runs, whereas Suzuki et al. (2019) reported sediment concentrations of
 407 approximately 32% when the debris front reached the end of the flume, and ca. 16% at
 408 the end of the water supply (i.e., at $t = 60$ s). In the computation, the inflow vanishes
 409 at $t = 92$ s, and the simulation ends at $t = 100$ s. We refer the readers to Appendix
 410 A for details on the inflow condition used in the computation.

411 In the experiments, sediment particles (ca. 1 mm in diameter) were glued on the
 412 surface of the deposition zone to represent the roughness. Consequently, the Manning
 413 coefficient is set to 0.016 (cf. Chow, 1959) in the computation. The grain-water mixture
 414 consisted of water and quasi-mono-dispersed sediment particles, whose grain size ranges

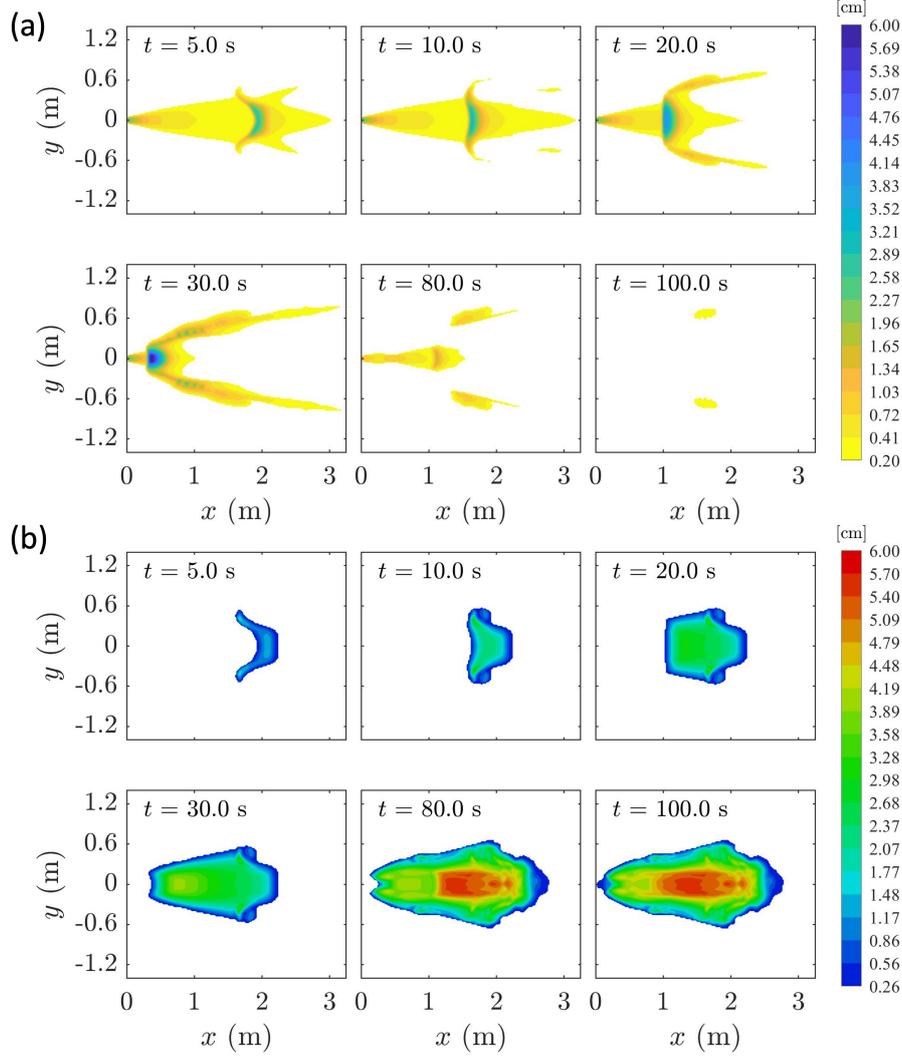


Figure 9. (a) Thickness distribution of the simulated flowing layer. (b) Evolution of the deposit heap in the computation.

415 from 2.02 to 3.24 mm, with a mean size $D_{50} = 2.6$ mm. Hence, we choose $\tilde{\Psi}_{\text{crt}} = 0.04$
 416 as the critical Shields value for entrainment in the computation. For the deposit, we set
 417 the critical value to $\tilde{\Sigma}_{\text{crt}} = 0.016$ in (14) and the dimensionless critical speed to $\tilde{v} =$
 418 1.6, which is equivalent to 0.2554 m/s for a sediment grain size of 2.6 mm. In the Hjul-
 419 ström-Sundborg diagram (Hjulström, 1935), this value lies in the transition band between
 420 erosion and deposition. According to Tsunetaka et al. (2022), the sediment particles have
 421 a density is 2,640 kg/m³, and an angle of internal friction of approximately 34°, which
 422 was used as 34.2° in the simulation. For a concise overview, all simulation parameters
 423 used in the computation are collected and listed in Table 2.

424 Figure 9a demonstrates the distributions of flow thickness at different time levels,
 425 where the associated deposits are given in Fig. 9b. At $t = 5.0$ s, a sharp change of flow
 426 thickness (similar to the hydraulic jump) is found at approximately $x = 1.9$ m, which
 427 moves backwards to ca. $x = 1.0$ m at $t = 20.0$ s and $x = 0.5$ m at $t = 30.0$ s. This
 428 migration of the hydraulic jump develops together with the migration of the heap of de-
 429 posit (cf. Fig. 9b). This finding indicates that the flow behavior is highly related to the

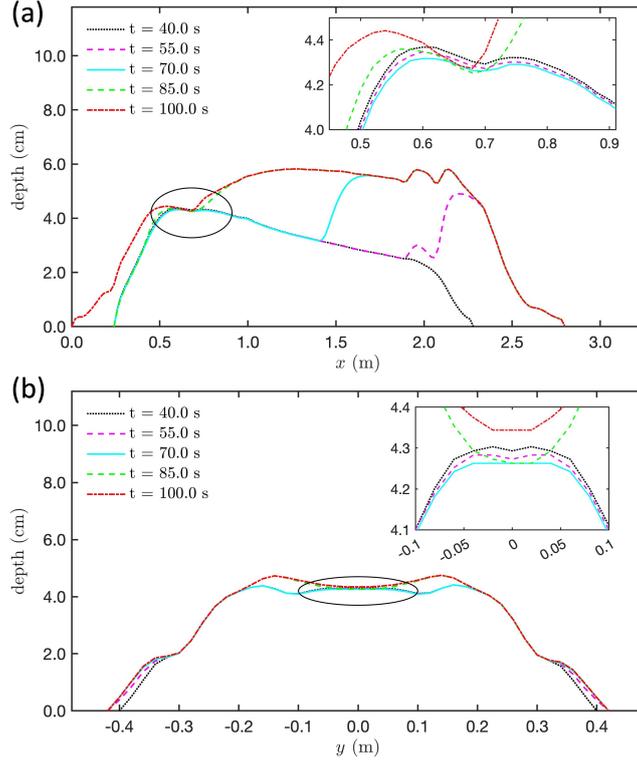


Figure 10. Evolution of the deposit pile, where the inlet plots show the local view of area marked with a black line. (a) Longitudinal sectional view at $y = 0.0$; and (b) transverse sectional view at $x = 0.7$ m (cf. Fig. 9b).

430 development of the heap deposit. When the deposit heap first develops, it is in a con-
 431 cave shape with the opening facing the stream ($t = 5.0$ s). The notch is filled as time
 432 increases ($t = 5.0$ s to 10.0 s). From $t = 10.0$ to 30.0 s, the heap mainly grows along
 433 the stream direction. At approximately $t = 80.0$ s, the area of the heap has reached the
 434 final shape, and the thickness increases significantly from $t = 80.0$ to 100.0 s. Either
 435 the moving mass has flowed out from the computational domain or the thickness of most
 436 flowing layer is less than 0.2 cm at $t = 100.0$ s, when the deposit heap has reached the
 437 final geometry. Exploring the flow behaviors and the development of the deposit heap
 438 shows that the jump patterns of the flow correspond to the fronts of the deposit heap
 439 developing at the bottom.

440 Another interesting finding is that the propagating front of the deposit heap is cor-
 441 related with the layered deposition. Figure 10 shows the longitudinal (at $y = 0.0$ m)
 442 and transverse (at $x = 0.7$ m) sectional views of the evolving deposit heap at various
 443 time levels. At around $t = 40.0$ s, the bottom layer of the deposit had fully developed,
 444 while the flow flux remained high and continued from above. As a result, the flowing ma-
 445 terial overrode the existing deposit pile and deposited both behind and on top of it ($t =$
 446 55.0 s in Fig. 10a). Between $t = 55.0$ s and 85.0 s, material continued to deposit on the
 447 top of the existing heap, so that a new pile grew as a traveling wave propagating upstreams
 448 on top of the previously deposited heap. This process resulted in a layered formation of
 449 the deposit (cf. Fig. 10a). While the deposit dominates the overall process, some entrain-
 450 ment (erosion) can still be observed in certain locations. As shown in the inlet plots in
 451 Fig. 10ab, which provide sectional views at $y = 0.0$ m and $x = 0.7$ m, respectively,
 452 erosion can be identified during the time interval $t \in [40.0, 70.0]$ s, followed by depo-

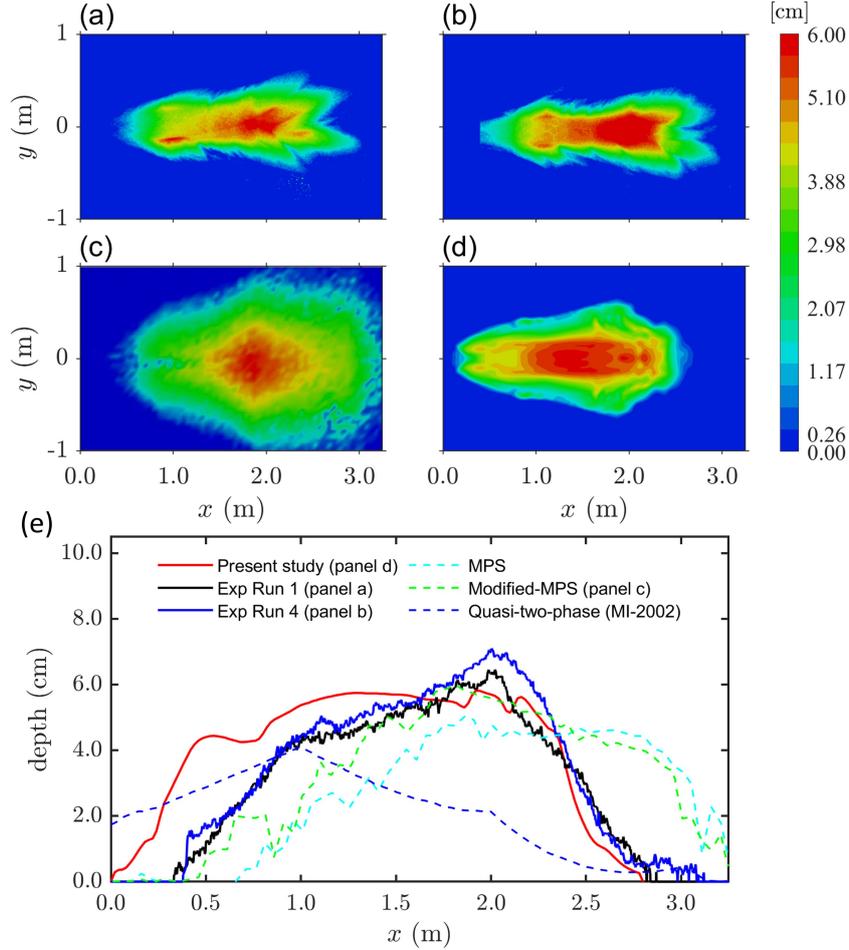


Figure 11. Top view of the final deposits, where (a) measurement in experiment (run1), (b) measurement in experiment (run4) (c) result computed by modified MPS in Suzuki et al. (2019), and (d) results computed by the present proposed model. (e) Longitudinal sectional views (along $y = 0.0$ m) of the various computed results and experimental measurements. (Panels (a) and (b) are extracted from Tsunetaka et al. (2022), Panel (c) is from Suzuki et al. (2019), and Panel (e) is modified based on Suzuki et al. (2019).)

453 sition from $t = 70.0$ s to 100.0 s. It is remarkable that the transverse sectional view of
 454 the deposit heap exhibits a translation from a flat plane to a concave shape between $t =$
 455 70.0 s and 100.0 s. This concave shape can also be regarded as a prototype for the for-
 456 mation of levees.

457 In addition to the development process of the deposit, we also examine the final
 458 geometry of the deposit heap computed by different models and the experimental de-
 459 posit heaps. The comparison includes the results illustrated in Suzuki et al. (2019), in
 460 which a Modified-MPS approach and a quasi-two-phase model (Miyamoto & Itoh, 2002)
 461 were employed. Figure 11abcd shows the top views of the final deposit. Panels (a) and
 462 (b) illustrate the measurements in experiments with mono-granular particles, where the
 463 data are extracted from Runs 1 and 4, as listed in Tsunetaka et al. (2022). Panel (c) presents
 464 the results computed by a Modified-MPS approach (see Suzuki et al., 2019), and Panel
 465 (d) shows the results given in the panel of $t = 100.0$ s in Fig. 9b which is regarded as
 466 the final geometry of the deposit heap computed by the proposed model. Because the

467 deposit computed by a quasi-two-phase model (Miyamoto & Itoh, 2002) is significantly
 468 worse than those computed by other models (cf. Fig. 11e), it is not considered in the present
 469 discussion. The Modified-MPS approach delivered a deposit (Panel c) that is in oval shape
 470 and features a single peak lying approximately at the center. In contrast, the deposit (Panel
 471 d) computed by the proposed model exhibits a long strip with multiple deposit peaks.
 472 Although the computed deposit covers a slightly larger area, the location of the high peaks
 473 mimics the locations measured in experiments.

474 The longitudinal sectional views of the final deposits are depicted in Fig 11e, in which
 475 the red solid line indicates the deposit computed by the present model, and the black
 476 and blue solid lines present the measurements in the mono-granular tests, the green and
 477 cyan dotted lines respectively are the results computed by the MPS and Modified MPS
 478 methods given in Suzuki et al. (2019), and the blue dashed line represents the results by
 479 the quasi-two-phase model (Miyamoto & Itoh, 2002, denoted by MI-2002). Although
 480 the MPS and Modified MPS methods can capture the peak at around $x = 1.9$ m, the
 481 present study (red solid line) also delivers sound agreement. Together with the feature
 482 of multiple peaks for the deposit heap, the present two-phase model can more appropri-
 483 ately describe the debris flows with deposition.

484 **4 Concluding remarks**

485 We presented a new two-phase fluid-grain model with entrainment and deposition
 486 for debris flows on rugged topographic surfaces, where the basal shear stress determines
 487 the entrainment and the sediment settling process yields deposition. The model equa-
 488 tions are derived with respect to a terrain-following coordinate system as employed in
 489 Tai et al. (2012); Tai and Kuo (2012); Luca et al. (2016) or Tai et al. (2019). Unlike the
 490 deformation coordinates used in Tai et al. (2012) or Tai and Kuo (2012), the heaps of
 491 deposit or the potholes of erosion are described by adding a "subtopography" over the
 492 topographic surface, i.e., as a deviation from the initial topography before entrainment/deposition.
 493 In addition, the entrainment-deposition mechanism is implemented and integrated into
 494 a CUDA-GPU-accelerated simulation tool (Ko et al., 2021) for high-performance com-
 495 putation.

496 This proposed model was investigated with two idealized numerical examples and
 497 one validation against experimental measurements. The example on the horizontal plane
 498 showed that the duration of movement is much longer if no depositional process is con-
 499 sidered because the material cannot arrive at the state of rest. Not only is this phenomenon
 500 related to the duration of movement, but the final geometry of the deposit heap also dif-
 501 fers from the one computed based on the depositional mechanism. This finding empha-
 502 sizes one of the key impacts of the depositional process on the associated flow behaviors
 503 and on the local morphology after a debris flow event. Remarkably, levee formation is
 504 observed in the example of chute topography, where a finite mass is released from the
 505 top of the inclined section and deposits on the horizontal plane. The simulation of the
 506 debris flow experiment not only reproduces the formation of levees but also exposes a
 507 layered pattern of deposits. This layered deposition indicates that a stratified pattern
 508 can form within a single debris flow event and provides insight into one of the causes for
 509 the stratified scree deposits (e.g., Sass & Krautblatter, 2007; Van Steijn, 2011; de Haas
 510 et al., 2018). While there were some discrepancies in simulating the debris flow exper-
 511 iments, the computed deposit heap ended up in roughly the same position as the exper-
 512 imental deposit heap. Furthermore, the computed heap featured multiple peaks (such
 513 as the concave shape in transverse sectional view), just like those observed in the long
 514 strip deposit heap.

515 Although the results have shown the key features of the present two-phase erodible
 516 model and shed light on simulating the entrainment-deposition process, this model
 517 is still limited by the idealized conditions given in the three examples. For example, in

518 addition to the complex rheology of debris flows, the in situ entrainment-deposition pro-
 519 cesses are highly related to the material composition and the local geological conditions.
 520 The complex material composition and heterogeneous conditions lead to a tremendous
 521 challenge in modeling. Although reasonable or strong assumptions have been imposed
 522 to simplify the model equations, the degree of uncertainty in the associated parameter cal-
 523 ibration remains extremely high. The present model is a compromised approach, but its
 524 numerical implementation is uncomplicated. For the sake of engineering applications,
 525 we can make good use of the GPU-accelerated facility for highly efficient computation.
 526 That is, simulations can help to investigate and evaluate plausible scenarios using abun-
 527 dant parameter sets, and can typically be performed and completed within a few hours
 528 or days, depending on the number of scenarios and the complexity of the parameter sets.
 529 With these advantages, engineering applications of the present model for hazard assess-
 530 ment or evaluation of plausible disasters mitigation countermeasures can be expected.

531 **Appendix A Inflow condition in the computation for the debris flow** 532 **experiment**

533 The inflow condition is set in two stages. A constant inflow rate of $Q_{\text{in}}^I = 3 \times 10^{-3}$
 534 m^3/s is maintained for $t \in [0.0, 58.0]$ s, while Q_{in}^{II} linearly reduces to zero from $t = 58.0$
 535 s to $t = 92.0$ s. More precisely,

$$536 \quad Q_{\text{in}}(t) = \begin{cases} Q_{\text{in}}^I(t) = Q_{\text{supply}} & \text{for } t \in [0.0, 58.0] \text{ s,} \\ Q_{\text{in}}^{II}(t) = Q_{\text{supply}} (92 - t)/34 & \text{for } t \in [58.0, 92.0] \text{ s,} \end{cases} \quad (\text{A1})$$

537 where $Q_{\text{supply}} = 3 \times 10^{-3} \text{ m}^3/\text{s}$. The sediment concentration is set according to the
 538 measurements given in Suzuki et al. (2019) using

$$539 \quad \phi^s(t) = \begin{cases} \phi_a^s - (\phi_a^s - \phi_b^s) \cos\left(\frac{t_b - t}{t_b - t_a}\right) & \text{for } t \in [0.0, 58.0] \text{ s,} \\ \phi_a^s - (\phi_a^s - \phi_b^s) \left[1.0 - \frac{(t_b - t)}{(t_b - t_a)}\right] & \text{for } t \in [58.0, 92.0] \text{ s} \end{cases} \quad (\text{A2})$$

540 with the values of t_a , t_b , ϕ_a^s and ϕ_b^s listed in Table A1.

541 Similar to the inflow rate, the inflow height is initially set to 0.022 m for $t \in [0.0, 58]$
 542 s, and then it linearly decreases to zero over the period of $[58.0, 92.0]$ s. That is,

$$543 \quad h_{\text{in}}^T(t) = \begin{cases} 0.022 & \text{for } t \in [0.0, 58.0] \text{ s,} \\ 0.022 \times (92 - t)/34 & \text{for } t \in [58.0, 92.0] \text{ s.} \end{cases} \quad (\text{A3})$$

544 As a result, the depths of the sediment and fluid phases in the inflow condition are given
 545 by

$$546 \quad h_{\text{in}}^s(t) = h_{\text{in}}^T(t) \phi^s(t) \quad \text{and} \quad h_{\text{in}}^f(t) = h_{\text{in}}^T(t) - h_{\text{in}}^s(t), \quad (\text{A4})$$

547 respectively. The inflow velocity is assumed to be uniformly distributed with zero rel-
 548 ative velocity between the phases. With (A1), (A4), and the flume width $B_{\text{flume}} = 0.1$
 549 m, the inflow velocity remains constant and is given by

$$550 \quad \begin{cases} v_{x,\text{in}}^s = v_{x,\text{in}}^f = \frac{Q_{\text{in}}(t)}{h_{\text{in}}^T(t) B_{\text{flume}}} = 1.364 \text{ m/s,} \\ v_{y,\text{in}}^s = v_{y,\text{in}}^f = 0.0 \text{ m/s.} \end{cases} \quad (\text{A5})$$

551 **Open Research Section**

552 The data and code for reproducing the results shown in the present manuscript are
 553 available at <https://doi.org/10.6084/m9.figshare.21893943.v4>.

Table A1. Parameters for computing the inflow condition

time period (s)	ϕ_a^s	ϕ_b^s	t_a	t_b
[0.0, 5.0]	0.32	0.175	0.0	5.0
[5.0, 20.0]	0.175	0.225	5.0	20.0
[20.0, 30.0]	0.225	0.167	20.0	30.0
[30.0, 40.0]	0.167	0.17	30.0	40.0
[40.0, 58.0]	0.17	0.16	40.0	58.0
[58.0, 92.0]	0.16	0.15	58.0	92.0

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