AGU PUBLICATIONS

Water Resources Research

Supporting Information for

Hydraulic model calibration using CryoSat-2 observations in the Zambezi catchment

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Introduction

The supporting information contains additional information concerning the in-situ data used for validation of the hydrological model, as well as a more detailed description of the mathematical equations used for the steady-state hydraulic solver.

Text S1.

The Saint-Venant equations express the mass balance and momentum balance equations for gradually varied one-dimensional flow in an open channel

$$\frac{\partial A}{dt} + \frac{dQ}{dx} = q \qquad (C.1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{\beta Q^2}{A}\right) + gA\frac{\partial h}{\partial x} - gA\left(S_0 - S_f\right) = 0 \qquad (C.2)$$

x is the chainage or distance along the channel [m], *t* the time [s], *h* the channel depth [m], *Q* the discharge [m^3/s], *A*, the flow cross-sectional area [m^2], *q* the lateral inflow [m^3/s], *g*, the acceleration due to gravity (set to 9.81 m^2/s) and β the momentum coefficient (set to unity). The bed slope, S_0 [m/m] is given by

$$S_0 = -\frac{dz}{dx} \qquad (C.3)$$

z is the channel datum or bed elevation above a given height. The friction slope, S_f [m/m], is given by

$$S_f = \frac{Q^2}{K^2} \qquad (C.4)$$

K, the conveyance [m^3/s], can be expressed as a function of channel cross-section geometry using Manning's equation

$$K = \frac{A^{\frac{5}{3}}}{n \times P^{\frac{2}{3}}} \qquad (C.5)$$

P is the wetted perimeter [m] and *n* is Manning's friction coefficient [s/$m^{1/3}$].

If we assume steady flow, i.e., constant Q, and no lateral inflow, the mass balance equation (Eq. C.1) becomes equal to the lateral inflow and Saint-Venant equations simplify to

$$\frac{dh}{dx} - \frac{Q^2}{gA^3}\frac{dA}{dx} - S_0 + \frac{Q^2}{K^2} = 0 \qquad (C.6)$$

By taking the partial derivative of the area relative to the chainage and width, and expanding the first term, Eq. C.2 becomes:

$$\left(1 - \frac{Q^2}{gA^3}\frac{\partial A}{\partial h}\right)\frac{dh}{dx} - \frac{Q^2}{gA^3}\frac{\partial A}{\partial x} - S_0 + \frac{Q^2}{K^2} = 0 \qquad (C.7)$$

Isolating the change in depth over the chainage gives the general form of the equation to solve

$$\frac{dh}{dx} = \frac{\left(\frac{Q^2}{gA^3}\frac{\partial A}{\partial x} + S_0 - \frac{Q^2}{K^2}\right)}{\left(1 - \frac{Q^2}{gA^3}\frac{\partial A}{\partial h}\right)}$$

$$\frac{dh}{dx} = RHS\big(x, h(x)\big) \qquad (C.8)$$

Where RHS (Right Hand Side) is the collection of terms not containing the derivative of the depth with respect to the chainage. We can replace $\frac{\partial A}{\partial h}$ and $\frac{\partial A}{\partial x}$ with channel properties. For a rectangular channel with variable width $w = w(x)$

$$\frac{dh}{dx} = \frac{\left(\frac{Q^2}{gA^3}\frac{dw}{dx} + S_0 - \frac{Q^2}{K^2}\right)}{\left(1 - \frac{Q^2}{gA^3}b\right)} \qquad (C.9)$$

If we apply this method to larger river networks, there will be lateral inflow, q, at certain points. Therefore, we must take into account

$$\frac{d}{dx}\frac{Q^2}{A} = \frac{A\frac{dQ^2}{dx} - Q^2\frac{dA}{dx}}{A^2} \qquad (C.10)$$

$$\frac{dQ^2}{dx} = \frac{d(Q^2)}{dQ}\frac{dQ}{dx} = 2Q \times q \qquad (C.11)$$

Eq. C11 becomes

$$\frac{dh}{dx} = \frac{\left(\frac{Q^2}{gA^3}\frac{\partial A}{\partial x} + S_0 - \frac{Q^2}{K^2} + \frac{2Q \times q}{gA^2}\right)}{\left(1 - \frac{Q^2}{gA^3}\frac{\partial A}{\partial h}\right)}$$

$$\frac{dh}{dx} = RHS\big(x, h(x)\big) \qquad (C.12)$$

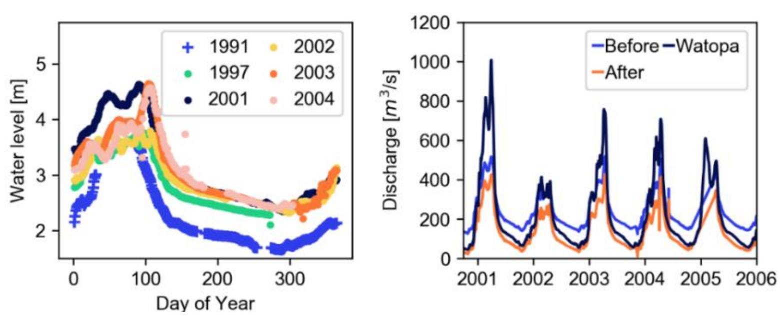

Figure S1. Water level at Kabompa Boma over the years of observation with at least 250 days of record and discharge record before and after bias correction of the level observations. The observed shift between the pre-1990 and post-2000s records is likely due to a shift in reference height, resulting in a bias of 65 cm. Comparison with the closest downstream station, Watopa, confirms this discrepancy. The station rating curve is applied to obtain the corrected post-2000 discharge records.

Main river and tributaries	Drainage area [km²]	Length [km]/ tributary to	Observations	Stations	Time of operation	Mean annual discharge [m³/s]	CryoSat-2 observations after outlier removal
Zambezi	238,667	468.9	Discharge, stage	Chavuma (1105)	1959-2019	656	140
				Zambezi Pump house (1150)	1990-2006	911	
				Lukulu (2030)	1950-2018	886	
Kabompo	72,068	491.0		Kabompo Boma* (1650)	2000-2008	165	83
				Watopa (1950)	1958-2019	273	
Lungwebungo	47,071	754.1					375
Kafue	102,714	739,1	Discharge, stage	Chilenga (4350)	1962-2007	153	180
				M'Swebi (4435)	1953-2005	162	
				Lubungu (4450)	1959-2007	147	
				Hook Pontoon (4670)	1973-2008	231	
Lunga	24,517	Kafue		Chifumpa Pontoon (4560)	1959-2007	87	
Luangwa	149,523	989.4	Discharge, stage	Great East Rd. Bridge (5940)	1948-2002	168	230

Table S1. Summary of In-situ stations used to calibrate the hydrologic (discharge records) and validate the hydrodynamic model (stage records) and of the number of CryoSat-2 observations available for calibration of the hydraulic model are indicated for the three study areas. The mean annual discharge is over the time of simulation, 2001-2018. *: the discharge record at Kabompo Boma has been manually bias-corrected based on historical records from 1990-1992.

	Calibration zones	Calibration and validation stations	RMSD of discharge climatology	Flow duration curve	Non-parametric KGE
Upper Zambezi	1. Low slope, dominant forest cover 2. Low slope, land cover mosaic 3. High slope, dominant forest cover	4. Watopa (C) 5. Chavuma (C) 6. Kabompo Boma (C)	1. 0.63 2. 1.05 3. 0.79	1. 0.12 2. 0.21 3. 0.15	1. 0.82 2. 0.74 3. 0.84
Kafue	7. Low slope, forest cover > 75% 8. Low slope, land cover mosaic 9. High slope, forest and shrub mosaic 10. High slope, forest cover > 80%	11. Lubungu (C) 12. Hook Pontoon (C) 13. Chilenga (C) 14. Chifumpa Pontoon (C)	1. 1.47 2. 0.83 3. 0.57 4. 1.64	1. 0.08 2. -0.36 3. -1.04 4. -0.72	1. 0.88 2. 0.87 3. 0.90 4. 0.74
Luangwa	15. High cover, forest mosaic	16. Great East Rd. Bridge (C)	1. 0.58	1. 0.002	1. 0.37

Table S2. Summary of calibration setup of the rainfall-runoff model and performance statistics at the calibration and validation stations