Estimating the Electric Fields Driving Lightning Dart Leader Development with BIMAP-3D Observations

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Key Points:

- We estimate cloud ambient electric fields by combining 3D mapping and electric field change observations with an equipotential leader model
- Our results are consistent with the hypothesis that dart leader speed is proportional to the leader tip electric field
- The modeled tip fields also explain why dart leaders typically follow previous channels and appear very narrow compared to step leaders

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Abstract

In this paper, a numerical dart leader model has been implemented to understand the leader's development and the corresponding electric field changes observed by the 3D Broadband Mapping And Polarization (BIMAP-3D) system. The model assumes the extending leader channel is equipotential and has a linear charge distribution induced by an ambient electric field. The charge distribution induced by the ambient field can be used to model the electric field change at the ground. We then find the ambient electric field which best fits the field change measurements at the two BIMAP stations. The estimated ambient electric field decreases in the direction of dart leader propagation. Our observations and modeling results are consistent with our earlier hypothesis that dart leader speed is proportional to the electric field at the leader tip. The model also supports our earlier analysis that leader speed variations near branch junctions were due to previous charge deposits near the junctions. The modeled tip electric field is generally lower than the breakdown field unless the pre-dart-leader channel has a significant temperature of ∼3000 K. This is consistent with the fact that dart leaders typically do not form new branches into the virgin air. Furthermore, the tip field is generally close to the negative streamer stability field at ambient temperatures, explaining the nature of the narrow and well-defined channel structure. In addition to the charge distribution and the ambient and tip electric field, the development of the channel potential and current distribution are also presented.

Plain Language Summary

A dart leader is a discharge process that occurs at the later stage of a lightning flash. It retraces the path established by earlier discharges and propagates at a high speeds of 1%-10% the speed of light. Recently we developed a system called BIMAP-3D that can map lightning radio sources in 3D with high time resolution. We also measured the electric field at the ground caused by lightning discharges. In this paper we modeled a 3D mapped dart leader as a perfectly conducting wire. A conducting wire placed in an external electric field disturbs the field around it. We used our wire dart leader model to find an electric field in the cloud so that the modeled disturbance matched the electric fields we measured at the ground. The estimated cloud field decreases in the direction of dart leader propagation. Our model suggests that the speed of a dart leader is closely related to electric field at the dart leader tip. The modeled electric field at the dart leader tip is also too low to form a new discharge path through the air, explaining why they follow previously established paths.

1 Introduction

In Jensen et al. (2023b) we analyzed 14 in-cloud dart leaders (also called K-leaders or recoil leaders) with 3D Broadband Interferometeric Mapping And Polarization (BIMAP-3D) observations. The dart leaders often exhibited an overall initial acceleration and a gradual deceleration, and some rapid speed variations as the leaders passed branch junctions in the flash structure. We proposed that the dart leader speed was proportional to the electric field strength at the leader tip when the leader is considered an equipotential channel growing through an ambient electric field. Based on the overall speed trend, it was inferred that the ambient electric field decreases in the direction of the leader propagation. We also used a simple two-point dipole charge model to estimate the development of the charge distribution along the leader channel based on the field change measured by two fast electric field change antennas. This paper expands on the work of Jensen et al. (2023b) by numerically modeling the development of equipotential dart leader channels constrained by our BIMAP-3D and fast antenna observations.

The leader model used in this study was first suggested by Kasemir (1960), who applied the basic concepts of electrostatics to approximate conductive lightning chan-

nels as equipotential and growing in an electric field, with qualitative descriptions of the induced charge distribution. Mazur and Ruhnke (1993, 1998) expanded upon Kasemir's initial work with numerical modeling of equipotential lightning channels. The model of Kasemir (1960) has historically been referred to as the bidirectional leader model, we will instead refer to it as the equipotential leader model, since the equipotential assumption is the most significant difference between this model and other non-physics-based leader models. A number of studies have used measured field changes to model simple charge configurations along leader channels without considering leader conductivity (Cai et al., 2022; Chen et al., 2013; Gao et al., 2020; Jensen et al., 2023b; Karunarathne et al., 2015; Lu et al., 2011). By contrast relatively few studies have attempted to compare the equipotential model to observed electric field changes associated with lightning (Mazur & Ruhnke, 1993; Pasko, 2014; da Silva & Pasko, 2015), and these studies have been somewhat limited by lack of knowledge of the extent or location of the leader channel.

The BIMAP-3D system has the capability to map out very high frequency (VHF) radio sources along lightning channels in 3D over time (Shao et al., 2023), and simultaneously measures the electric field changes with a fast antenna at each of the two BIMAP stations. If we assume the active and continuous dart leader channel sections with recent VHF activity are at equipotential, and choose some reasonable channel radius, the only remaining unknown in the model is the cloud electric field distribution along the channel. The cloud electric field can then be estimated through standard non-linear inverse problem techniques as described in Section 3.

This approach to indirectly measuring the ambient electric field is similar in concept to the work reported by Cummer (2020), but our use of time resolved leader lengths and field changes allows the estimated ambient field along the leader channel to be spatially resolved. This additional information leads to a number of insights on the physical conditions of the developing channel.

As a note on terminology, in our previous papers (Jensen et al., 2021, 2023b) we have discussed the use of the terms "K-leader", "dart leader", and "recoil leader" to refer to what is fundamentally the same phenomenon. Following some recent discussion and consensus within at least part of the lightning community (Hare et al., 2023b, 2024; da Silva et al., 2023) we now choose to use the term dart leader exclusively to refer to this phenomenon, as we previously suggested in Jensen et al. (2021). To avoid confusion about whether a dart leader is followed by a return stroke we suggest that dart leaders may be further classified as in-cloud (IC) dart leaders or cloud-to-ground (CG) dart leaders whenever the distinction is relevant, as we suggested in Jensen et al. (2021).

Following this terminology convention, in this paper we apply the equipotential model to IC dart leaders to understand the electric fields and other conditions that drive the leader's development. This work builds on recently reported observations of dart leaders (Jensen et al., 2023b), and serves as a more rigorous test of some of our hypotheses on the dart leader propagation physics. When applying the methodology discussed in Section 3 the model results are consistent with our hypothesis that dart leader speed is proportional to the leader tip electric field, as we first suggested in Jensen et al. (2021). We demonstrate that an ambient field which starts relatively high and decreases along the channel results in a tip field that matches the observed speed trends of initial acceleration and gradual deceleration. We also confirm that the branch junction speed variations can be explained by charge deposits near the primary channel. Our results also provide possible explanations to some other observed dart leader properties, such as their well-defined and narrow channel width in VHF and the fact that they typically do not exit the pre-conditioned channel structure to propagate through virgin air.

2 Instrumentation

In this paper we make use of lightning observations from the BIMAP-3D system (Shao et al., 2023) that has been deployed at Los Alamos National Laboratory (LANL) since 2021. It consists of two stations, each with four VHF antennas arranged in a Y configuration. The two stations are 11.5 km apart. Lightning data from each station is first processed separately to form a 2D lightning map, and then is combined and reprocessed to produce a 3D lighting map. Based on observations of dart leader channel widths the random uncertainty of BIMAP-3D can be better than 10 m in easting, northing, and altitude in ideal conditions (Shao et al., 2023). Systematic biases in the absolute location have not been evaluated, but the 3D results produced by the triangulation and DTOA location techniques typically differ by less than 30-50 m, and this gives an estimate of the absolute location error. For lightning channels several kilometers away a location bias of 50 m is negligible for this study.

BIMAP-3D also has a fast electric field change antenna, or fast antenna (FA) at each station. The fast antenna at station 1 (FA01) has a highpass time constant of 1 ms, and the fast antenna at station 2 (FA02) has a time constant of 0.2 ms. For a first order highpass filter the low frequency content can be reliably recovered by de-drooping (deconvolution) (Födisch et al., 2016; Sonnenfeld et al., 2006) as long as the low frequency signal is sufficiently above the noise level, the direct current (DC) offset or "zero-level" of the signal is reliably known, and the signal does not saturate. For both FA01 and FA02 the low frequency content can be recovered down to the 60 Hz noise from nearby power infrastructure. There is no explicit lowpass filter in the fast antennas, but above 20 MHz there is essentially no signal. In this study we are interested in the electrostatic field change associated with the dart leaders, so we digitally lowpass the signals at 25 kHz. The field change signals we are analyzing are thus more in the slow antenna regime. We also de-droop the field change for each dart leader separately, making use of the fact that there is essentially no field change in the interval between dart leaders for this flash.

In Jensen et al. (2023b) we found a relative calibration between FA01 and FA02 by comparing peak amplitudes for distant return strokes. This relative calibration was sufficient for qualitative modeling analysis. For quantitative modeling in this paper, we need an absolute calibration for each fast antenna. To achieve this we used 48 hours of National Lightning Detection Network (NLDN) peak current data for strikes within 100 km of our stations that were captured by either of our stations. We restricted the NLDN data set to strikes that were at least 15 km away from both stations, and with reported peak currents between 0 kA and -50 kA. With these restrictions we found 263 strikes for FA01 and 87 strikes for FA02. To compare NLDN peak currents to our field change measurements we used the empirical relation of Equation 4 in Nag et al. (2014) (first reported by Rakov et al. (1992) based on results from Willett et al. (1989)). Based on this equation we derived calibration factors for our fast antennas based on each NLDN strike, with units of $\left[\frac{V}{m}\right]$ per digital count. After finding the calibration factor for each match between the NLDN and our fast antennas, and removing any obvious outliers, we found the average calibration factor for each station. The calibration factors were 4±2 mV/(m·count) for FA01 and 2±1 mV/(m·count) for FA03. In each case the 1σ uncertainties are about 50% of the calibration value. This is a significant source of uncertainty for all the quantities calculated in this paper.

3 Methods

3.1 The Equipotential Leader Model

Following the approach of Mazur and Ruhnke (1998), we consider a lighting leader as a long cylinder with length L and effective capacitive radius r_C. If the cylindrical channel is placed in an ambient potential distribution $\Phi_{amb}(s)$ and a linear charge distribu-

tion $\lambda(s)$ is placed along the channel, then the new potential along the channel $\Phi(s)$ will be given by:

$$\Phi(s) = \Phi_{amb}(s) + \frac{1}{4\pi\varepsilon_0}\int_{s_a}^{s_b}\frac{\lambda(s')ds'}{\sqrt{(s-s')^2+r_C^2}} \tag{1}$$

where s is the coordinate along the length of the channel. s_a and s_b are the ends of the channel.

For an assumed equipotential channel we have $\Phi(s) = \Phi_{cha} = const.$ for $s_a \leq s \leq s_b$. We further assume that the leader has no net charge, since the transfer of charge between the leader and the cloud should be negligible at dart leader time scales. Under these assumptions the channel potential must be given by the average value (Mazur et al., 1995):

$$\Phi_{cha} = \frac{1}{s_b - s_a}\int_{s_a}^{s_b}\Phi_{amb}(s)ds \tag{2}$$

We then wish to find the charge distribution $\lambda(s)$ which satisfies this condition. To evaluate this numerically with the method of moments technique we discretize the leader channel into N segments of length Δs, where each segment has a uniform charge density. We then linearize Equation 1 as (da Silva & Pasko, 2015):

$$[\Phi_i] = [\Phi_{amb,i}] + [K_{i,j}]\,[\lambda_j] \tag{3}$$

where Φ_i is the net potential at s_i, $\Phi_{amb,i}$ is the ambient potential at s_i, and $[K_{i,j}]\,[\lambda_j]$ gives the approximate potential at s_i due to the linear charge density λ_j at every location s_j along the channel. Φ_i, $\Phi_{amb,i}$, and λ_j are the elements of column matrices of size $N \times 1$, and $K_{i,j}$ are the elements of an $N \times N$ matrix defined as

$$K_{i,j} = \frac{1}{4\pi\varepsilon_0}\int_{s_j-\Delta s/2}^{s_j+\Delta s/2}\frac{ds'}{\sqrt{(s_i-s')^2+r_C^2}} \tag{4}$$

where $K_{i,j}$ has units of meters per Farad.

The discretized charge distribution can then be obtained as:

$$[\lambda_j] = [K_{i,j}]^{-1}\,[\Phi_{amb,i} - \Phi_{cha}] \tag{5}$$

We can then apply this model repeatedly with leaders of increasing length $L(t)$, in order to approximate $\lambda(s,t)$ for a growing leader channel. The time step at each point in time will be $\Delta t_k = \Delta s / v_k$, where v_k is the speed of the leader growth at time t_k.

From the discrete charge distribution over time we can calculate the discretized current by solving the continuity equation for I_i as

$$I_i(t_k) = -\Delta s\sum_{j=0}^{i}\frac{\lambda_j(t_k) - \lambda_j(t_{k-1})}{t_k - t_{k-1}} \tag{6}$$

Calculation of the electric field at the leader tip is somewhat nuanced because of uncertainty about the effective channel radius r_C, so we will discuss this separately in Section 3.2.

The leader tip potential drop $\Delta\Phi_{tip}$ can also be calculated, it is simply defined as

$$\Delta\Phi_{tip} = \Phi_{cha} - \Phi_{amb}(s_{tip}) \tag{7}$$

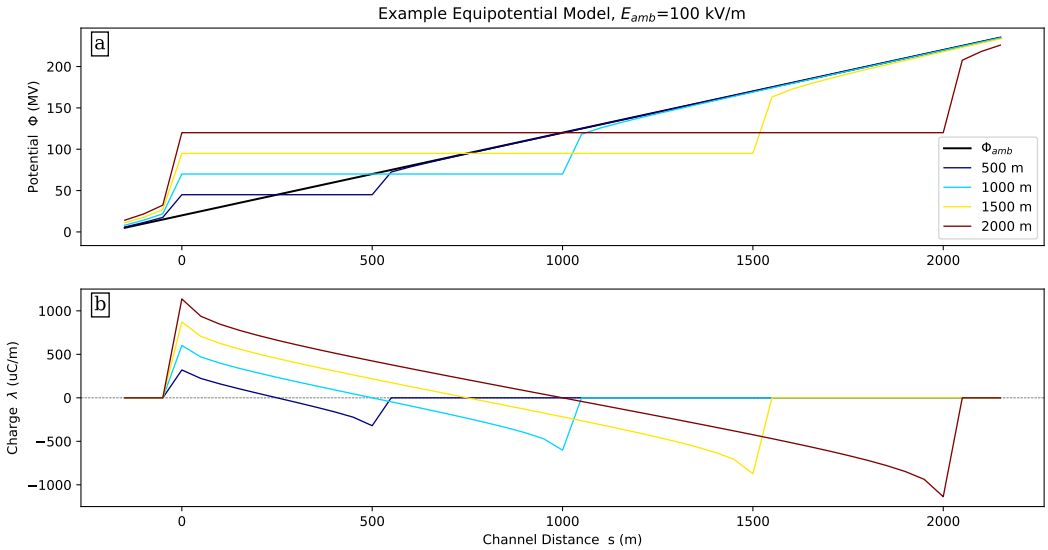

Figure 1. Examples calculations of the leader potential and charge distribution for various leader lengths in a uniform ambient field $E_{amb} = 100$ kV/m, following the atmospheric electricity sign convention $E = \nabla\Phi$. Plots show the potential along the leader channel (a), and the induced charge distribution (b).

This is the difference between the channel potential and the ambient potential that would exist at the tip if there was no leader present. Our $\Delta\Phi_{tip}$ is similar to the definition of ΔU_g^* in Celestin and Pasko (2011), or ΔU_t in Bazelyan and Raizer (2000) (Equation 4.3 and other uses throughout). The potential drop $\Delta\Phi_{tip}$ is generally proportional to the tip field E_{tip}, but has the advantage that it does not depend on the tip geometry. Nevertheless we will include E_{tip} estimates despite the much larger uncertainty because many other papers on leader and streamer propagation consider average electric fields over some distance rather than potential difference at a single point.

EXAMPLE: As an example of the model calculations, Figure 1 shows the leader potential and charge distribution for a negative leader with $r_C = 1$ m growing in a uniform ambient field $E_{amb} = 100$ kV/m, following the atmospheric electricity sign convention $E = \nabla\Phi$. The channel segment length is Δs=50 m. In this example the positive tip of the leader stays stationary, and the model values are plotted when the negative tip has reached distances of 500 m, 1000 m, 1500 m, and 2000 m. In Figure 1a the potential is uniform within the channel, but the value of Φ_{cha} increases as the negative leader tip extends through the uniform field. In a uniform field Φ_{cha} is equal to the potential at the middle of the channel. Beyond the ends of the leader the potential quickly returns to the ambient potential, as marked in black.

Figure 1b shows how the charge distribution changes as the leader propagates. In a uniform field the leader has equal amounts of positive and negative charge at the tips, tapering off to zero at the middle of the leader channel. As the leader propagates the symmetry of the charge distribution remains, but the magnitude of the charge at either end increases. Since only one tip is propagating in our example the zero charge point also moves. In a uniform field the zero charge point is always halfway along the leader. For an equipotential channel the induced charge $\lambda(s)$ at each point is proportional to the difference between the channel potential Φ_{cha} and the ambient potential at that point $\Phi_{amb}(s)$, $\lambda(s) \propto \Phi_{cha} - \Phi_{amb}(s)$ as pointed out originally by Kasemir (1960). So the point on the leader with zero charge is the point where $\Phi_{cha} - \Phi_{amb}(s) = 0$, at the middle of

the channel in this case. There is a slight enhancement to the charge density $\lambda(s)$ at the tips of the leader, beyond the value expected by a $\lambda(s) \propto \Phi_{cha} - \Phi_{amb}(s)$ relation. This charge enhancement at the tips is related to the change in capacitance per unit length at the tips of a cylinder (Jackson, 2000).

3.2 Leader Tip Electric Field

Having estimated the ambient field and associated charge density along the channel we also wish to estimate the electric field at the leader tip. Each segment of the leader is a uniformly charged cylinder of length Δs, radius r_C, and linear charge density λ_i. From first-principles electrostatics it can be shown that the electric field along the s-axis from each cylindrical segment is given by

$$E_i(s) = \frac{\lambda_i}{2\pi r_C^2 \varepsilon_0} \left[\sqrt{r_C^2 + s^2} + \Delta s - \sqrt{r_C^2 + (\Delta s + s)^2} \right] \tag{8}$$

where in this case $s = 0$ is defined as the forward end of the cylinder, and the equation is only valid ahead of the cylinder in the direction of propagation ($s \geq 0$).

Equation 8 demonstrates that the field for each segment drops off very quickly over distances on the order of r_C. Thus, the field at the tip of the leader can be approximated as due to just the final segment. This field is highest right at the edge of the cylinder, so for $s = 0$

$$E_{tip}(0) = \frac{\lambda_{tip}}{2\pi r_C^2 \varepsilon_0} \left[r_C + \Delta s - \sqrt{r_C^2 + (\Delta s)^2} \right] \tag{9}$$

If we approximate Equation 9 using $r_C^2 + (\Delta s)^2 \approx (\Delta s)^2$, we get

$$E_{tip}(0) \approx \frac{\lambda_{tip}}{2\pi\varepsilon_0} \frac{1}{r_C} \tag{10}$$

Without further knowledge the effective capacitive radius r_C for a dart leader channel could plausibly be anywhere from 1 mm to 100 m at the tip, so this $1/r_C$ dependence appears to pose a significant challenge in extracting any useful information about the magnitude of the tip field.

However, the electric field at a single point in a highly non-uniform field is not really useful in any case. More relevant to a discussion about streamer or breakdown activity at the tip would be the average field at the tip over some distance d

$$\overline{E_{tip}}(d) = \frac{1}{d} \int_0^d \frac{\lambda_{tip}}{2\pi r_C^2 \varepsilon_0} \left[\sqrt{r_C^2 + s^2} + \Delta s - \sqrt{r_C^2 + (\Delta s + s)^2} \right] ds \tag{11}$$

which evaluates to

$$\begin{aligned}
\overline{E_{tip}}(d) = &\frac{\lambda_{tip}}{4\pi\varepsilon_0 d}\left[\ln\left(\frac{\sqrt{r_C^2+d^2}+d}{r_C}\right)\right. && (A)\\
&+\ln\left(\sqrt{r_C^2+\Delta s^2}+\Delta s\right) && (B)\\
&-\ln\left(\sqrt{r_C^2+(\Delta s+d)^2}+\Delta s+d\right) && (C)\\
&-\frac{\Delta s+d}{r_C^2}\sqrt{r_C^2+(\Delta s+d)^2} && (D)\\
&+\frac{d}{r_C^2}\sqrt{r_C^2+d^2} && (E)\\
&+\frac{\Delta s}{r_C^2}\sqrt{r_C^2+\Delta s^2} && (F)\\
&\left.+\frac{2d\Delta s}{r_C^2}\right] && (G)
\end{aligned} \tag{12}$$

where we have labeled each term (A, B, C, etc.).

Assuming $\Delta s >> d > r_C$ then $(B)+(C) \approx 0$. Applying the binomial approximation for $\frac{r_C^2}{(\Delta s+d)^2} < 1$, $\frac{r_C^2}{d^2} < 1$, and $\frac{r_C^2}{(\Delta s)^2} < 1$ to terms (D), (E), and (F) respectively, yeilds $(D)+(E)+(F)+(G) \approx \frac{1}{4}$. So we are left with

$$\overline{E_{tip}}(d) \approx \frac{\lambda_{tip}}{4\pi\varepsilon_0 d}\left[\ln\left(\frac{\sqrt{r_C^2+d^2}+d}{r_C}\right)+\frac{1}{4}\right] \tag{13}$$

If we choose $d = 1$ m then the difference in tip fields between $r_C = 0.001$ m and $r_C =$ 1 m is about a factor of 7. This is still rather large, but at least gives an estimate of the tip field within about an order of magnitude, even across 3 orders of magnitude in radius. We do not use this approximation in the actual model calculations of E_{tip}, but it is useful for demonstrating the logarithmic dependency on r_C.

Additionally, the channel radius r_C is the effective radius of charge transported by streamers out into the corona sheath. The radius r_C should then approximately correspond to the radius at which the radial electric field is equal to the streamer stability field. Assuming the axial and radial fields are approximately equal, one should expect $\overline{E_{tip}}(r_C)$ to be close to the value of the stability field. If this condition is met then we can be more confident that we have chosen r_C correctly and our calculated E_{tip} values are essentially correct. A more sophisticated model would allow r_C to vary so that the radial field was always equal to the streamer stability field for each segment (e.g. Cooray et al. (2009)), but for now a simpler model with fixed r_C is used.

We note that E_{tip} as discussed in this section is essentially the "vacuum solution" field as defined by Celestin and Pasko (2011), we are not accounting for streamers forming ahead of the conductive leader tip. Streamers ahead of the tip would reduce the field at the tip by spreading the potential drop $\Delta\Phi_{tip}$ over a larger distance. Under typical approximations this would result in a constant E_{tip} equal to the streamer stability field E_{st}, which extends for a distance $L = \Delta\Phi_{tip}/(2E_{st})$ (Bazelyan and Raizer (2000), page 69).

For E_{tip} in this paper we will report the values calculated from Equation 12 over a distance of d=1 m.

3.3 Ambient Electric Field Estimation

For any ambient electric field distribution we can calculate the charge distribution along the channel with Equation 5. If we can map the channel path parameter s_i to 3D coordinates (x_i, y_i, z_i) in the sky we can then calculate the vertical electrostatic field at a point (X, Y, Z_{grnd}) on the ground (where Z_{grnd} is the altitude of the ground above sea level) due to the charge distribution λ_i as:

$$E_z(X, Y, Z_{grnd}) = \sum_{i=1}^{N} \frac{\lambda_i \Delta s}{2\pi\varepsilon_0} \frac{z_i - Z_{grnd}}{\left[(x_i - X)^2 + (y_i - Y)^2 + (z_i - Z)^2\right]^{3/2}} \tag{14}$$

where the ground is treated as an ideal infinite conducting plane. Since the lightning channel is far from our field measurement locations each segment of the channel can be approximated as a point charge. This calculation can be done for each time step t_k corresponding to a leader extension by Δs, to give the vertical field on the ground as a function of time, $E_z(X, Y, Z_{grnd}, t_k)$.

Typically for a lightning flash the ambient electric field $E_{amb}(s)$ is not known, but the vertical electric field at the ground $E_z(t)$ can be measured with an electric field change antenna. The extent of the conducting channel at any point in time can also be inferred from 3D lightning interferometer data. An initial guess can then be made for the ambient field $E_{amb}(s)$. For a conductive channel with endpoints at $s_a(t_k)$ and $s_b(t_k)$, the charge distribution $\lambda_i(t_k)$ induced by the guess $E_{amb}(s)$ can be calculated from Equation 5. The corresponding field change at the ground $E_z(t_k)$ can be calculated from Equation 14. The goodness-of-fit between the measured and modeled field changes can then be evaluated as

$$\chi^2 = \sum_{t_k} \frac{\left(E_{mod}(t_k) - E_{obs}(t_k)\right)^2}{\sigma_k^2} \tag{15}$$

where $E_{mod}(t_k)$ and $E_{obs}(t_k)$ are the modeled and observed fields, respectively, at time t_k, and σ_k is the estimated uncertainty in the observed field.

We can then find the ambient electric field $E_{amb}(s)$ which minimizes the χ^2 value iteratively using a non-linear optimization technique such as the Levenberg-Marquardt algorithm. The field on the ground does not depend strongly on each individual point $E_{amb}(s_i)$, so in order to limit the number of degrees of freedom we assume $E_{amb}(s)$ takes the form of a polynomial of order n, rather than individually fitting each value of $E_{amb}(s_i)$. We also include a penalty in the χ^2 value for solutions where E_{tip} changes signs, since a real leader should stop propagating if this condition ever occurred. This penalty is chosen to be large enough to suppress sign changes in E_{tip}, but small enough that it does not upset the convergence of the solution or lead to significantly increased errors in Equation 15.

3.4 Model Implementation

Dart leader K-5 from Jensen et al. (2023b) was chosen to implement the model since this is a relatively simple IC dart leader. Figure 2 shows the known information for K-5, including the full 3D extent of the leader channel over time relative to the two fast antennas (FA01 and FA02), along with the electric field change vs time at the ground at these locations. This is the known information from which we want to estimate the unknown ambient electric field along the channel.

The leader path is simplified by smoothing the measured VHF source locations with a rolling average of the location over ± 20 μs. Equally spaced and consecutive points $\Delta s =$ 50 m apart are selected to serve as the discrete leader segments in the model. The model

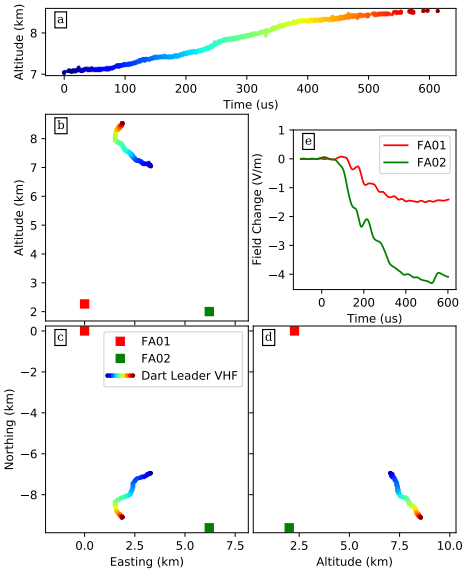

Figure 2. A plot of the path of the IC dart leader labeled K-5 over time relative to the two fast antennas. Panels are: altitude vs time (a), altitude vs easting (b), northing vs easting (c), and northing vs altitude (d). The points of K-5 are colored by time. The measured field change vs time at each station is also included (e), and the locations of the two fast antennas are marked in panels b, c, and d. The location of K-5 relative to the surrounding flash structure can be seen in Figure 3 of Jensen et al. (2023b).

includes a fixed channel radius of $r_C = 1$ m to account for charge being transported radially outward from the thin (∼1 mm, (Rakov, 1998)) conducting core into the corona sheath. To first order the capacitance of a long thin cylinder of length L and radius r_C is given by (Jackson, 2000):

$$C = \frac{2\pi\varepsilon_0 L}{\ln(L/r_C)} \tag{16}$$

The dependence of the estimated background field on the channel radius is thus weak, changing the modeled radius from 1 mm to 1 m only increases the capacitance by a factor of 2 for a L=1 km channel.

The channel of K-5 is obviously not perfectly straight. Since the axial field produced by each cylindrical segment (Equation 8) drops off quickly over distances on the order of $r_C << \Delta s$, only the nearest segments should contribute significantly to the potential at each point along the leader channel. Thus we assume that Equation 1 is still valid for the potential along a real tortuous leader channel as long the channel is approximately straight over distances of a few times Δs. Since the channel is ∼5 km above the ground we can ignore the contribution of image charges to the axial field along the channel.

We also assume that the dart leader channel starts with a conductive length of 100 m at t=0 in order to have some initial tip field and field change. Rather than skipping the first 100 m of dart leader development, we use the 3D map of the full flash to find a path along the same branch for 100 m in the direction opposite to the dart leader tip propagation. Negative values of channel distance s correspond to this "backward" direction along the branch. The negative dart leader tip starts at $s = 0$ at time $t = 0$.

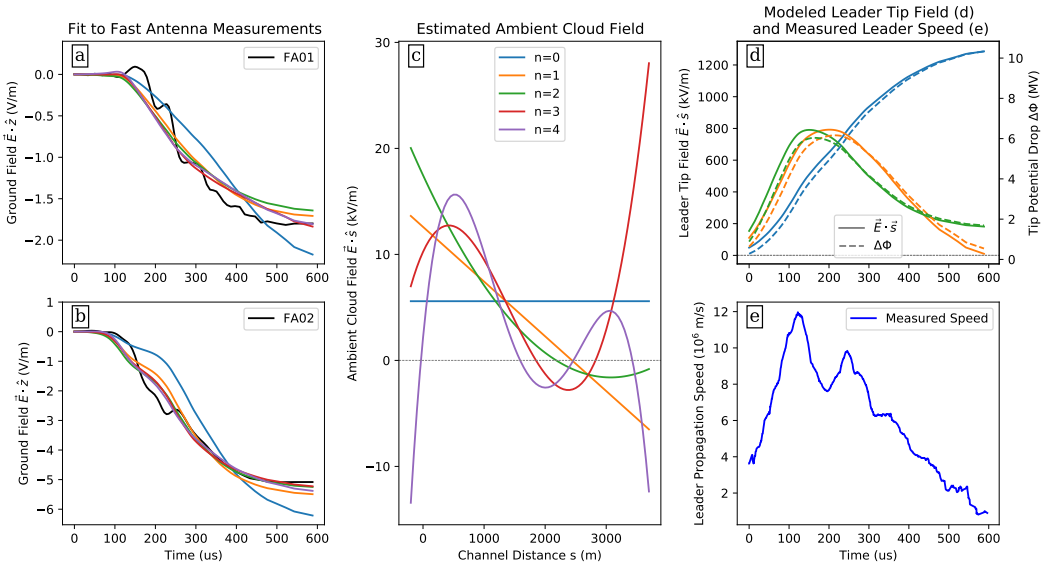

Figure 3. A plot of the modeled field change compared to the measured ground electric field change vs time (a and b), the estimated ambient cloud electric field vs channel distance (c), the estimated leader tip electric field E_{tip} vs time (d, left axis), and tip potential drop $\Delta\Phi_{tip}$ vs time (d, right axis), for various degrees n of polynomial ambient fields. We plot $|\Delta\Phi_{tip}|$ so the curves are not inverted compared to E_{tip}. The measured leader speed vs time (e) is included for comparison.

4 Results

4.1 Estimated Cloud and Tip Fields

All electric fields in plots use the atmospheric electricity sign convention, i.e. for the ambient cloud E-field and tip E-field a positive E-field will accelerate electrons in the direction of leader propagation, $\hat{s}$. Since the behavior of the negatively charged dart leader tip is the subject of interest this sign convention makes the plots easier to interpret.

Figure 3 shows the modeled field change (a and b) and estimated ambient field (c), where the ambient field is estimated using polynomials of various degrees n. The constant field ($n = 0$) is clearly a worse fit to the measured field changes than the higher polynomial degrees that result in similar field changes. All the modeled field changes seem to only fit the slow components of the measured field change, up to about 5 kHz. This is true even if we drastically increase the allowed degrees of freedom (e.g. $n = 50$). This suggests that the higher frequency components of the field change (above ~5 kHz) are not associated with the general extension of the leader, but some other process which is not captured by our model. Since the higher frequency components do not seem to match between the two stations they may also simply be local interference at each station.

The polynomial ambient fields themselves (Figure 3c) generally decrease in the direction of dart leader extension as predicted in Jensen et al. (2023b) based on dart leader speed trends, although the initial and final field values in Figure 3c diverge quickly for $n = 4$, with the final field values also diverging for $n = 3$. The field behaviors in the $n = 1$ and $n = 2$ cases are similar and are more physically reasonable. We will specifically consider the n=2 case for the rest of our analysis.

The ambient field values, which are mostly below 10 kV/m, are low compared to typical thunderstorm fields which are often measured to reach 50-100 kV/m (Marshall et al., 2001; Stolzenburg et al., 2007, 2015; Stolzenburg & Marshall, 2008). This is expected since the preceding leader, return stroke, and other discharge activities would have zeroed-out the field along the channel, and the ambient field estimated here therefore represents the recovery of the field along the channel as charge deposited in the corona sheath continues to expand radially outward between strokes.

The sign-change of the field in Figure 3c ($E_{amb} < 0$) is somewhat surprising, but the charge re-distribution during a lightning flash is complex, and it is possible dart leaders from other branches transported excess negative charge onto this branch while this branch was otherwise decayed and non-conducting.

Figure 3d shows the modeled leader tip field E_{tip} (solid, left axis) and tip potential drop $\Delta\Phi_{tip}$ (dashed, right axis) vs time. The $n = 3$ and $n = 4$ cases were omitted to make the plot less cluttered. The curves for E_{tip} and $\Delta\Phi_{tip}$ for a particular n value are nearly identical up to a scaling factor. In Figure 3d $|\Delta\Phi_{tip}|$ is plotted instead of $\Delta\Phi_{tip}$ for comparison with E_{tip} and easier interpretation. The $n = 1$ and $n = 2$ cases have an initial increase and gradual decreases of the tip field and potential drop, which is well correlated with the measured speed vs time of K-5 (Figure 3e), supporting our claim in Jensen et al. (2023b) that the leader speed is generally proportional to the leader tip field. With the current polynomial field approach, the model is incapable of catching small details such as the speed dip at 200 μs.

We include the $n = 0$ case in Figure 3d to demonstrate that in a uniform field the leader tip field and potential drop will increase indefinitely. The slope of the $n = 0$ E_{tip} vs time curve changes because leader length is converted to time using the observed leader speed. The equipotential model itself is time independent, and E_{tip} is directly proportional to leader length for a uniform field.

The tip field estimates are also lower than the traditional virgin air breakdown field. At the height of 7-8 km, the breakdown field is expected to be $E_k \cdot \delta \approx 1500$ kV/m, where $E_k = 3000$ kV/m for sea level air, and δ is the air number density compared to the sea-level/room temperature number density $\delta = n(h, T)/n(h = 0km, T = 300K)$ (da Silva et al., 2019). However, for a preconditioned pre-dart-leader channel the temperature may be significantly higher than 300 K, and the density will be correspondingly lower. Following

$$\delta(h, T) \approx \frac{300}{T} e^{-h/10.4} \tag{17}$$

we have $E_k \cdot \delta \approx 150$ kV/m for h=7 km and T=3000 K (Uman & Voshall, 1968), which is closer to the leader tip fields we estimate at the start of propagation (Figure 3d). The fact that the tip field never reaches the breakdown field for ambient air at 7 km may explain why dart leaders typically do not form new branches and instead follow the existing flash structure. The range of tip field values we found are generally in agreement with the range of tip fields measured by Miki et al. (2002) in triggered lightning strikes, although their measurements were made near sea level. The tip potential drop estimates in Figure 3d are also in reasonable agreement with the "typical value" of 15 MV given for dart leaders in Rakov and Uman (2003) Table 1.1.

The fact that the estimated leader tip field is initially lower than even the nominal streamer stability fields in virgin air (500-750 kV/m at 7 km, (Babaeva & Naidis, 1997; Briels et al., 2008; Qin & Pasko, 2014)) may also explain the observation that the width of dart leader channels resolved in VHF is much narrower than those of stepped leaders (Hare et al., 2023a; Jensen et al., 2021; Shao et al., 2023). The fact that negative leaders are detected much more readily than positive leaders in VHF suggests that the observed VHF predominantly comes from negative streamers. The axial field at the leader tip (E_{tip}) should also be the highest electric field at any point on the leader. So

if E_{tip} is below the negative streamer stability threshold in virgin air there should be no negative streamers anywhere on the leader, except within the "warm" (∼1000 K) pre-conditioned channel core with a radius on the order of centimeters (Uman & Voshall, 1968), where the air density is lower. Recalling that E_{tip} is the average field over 1 m, if E_{tip} is approximately equal to the virgin air stability field, and if the radial electric field near the tip is approximately equal to E_{tip}, then we may expect streamers out to a radius of about 1 m. This is in contrast to negative stepped leaders where E_{tip} is expected to be close to the breakdown field, and the streamer zone may have a radius of 10-100 m (Edens et al., 2014; Petersen & Beasley, 2013; Sonnenfeld et al., 2023).

In fact our streamer zone estimates agree very well with the high speed video observations of Petersen and Beasley (2013). While they observed a radial streamer/corona zone of 10-20 m on a descending negative stepped leader, a later dart leader in the same channel had no visible radial streamer zone, although there is some faint uniform luminosity which may be corona. Instead of a wide radial streamer zone the dart leader exhibited a long forward streamer zone confined to the pre-conditioned channel, extending ∼20-40 m ahead of the leader tip. This long forward streamer zone is expected. If the air density in the pre-conditioned channel is ∼1/10th the ambient density, then the forward streamer zone within the pre-conditioned channel should be about 10 times longer than the radial streamer zone.

As discussed in Section 3.2 there is a large uncertainty in the modeled tip field values due to the uncertainty in the effective channel radius r_C. For instance, the field values may be up to 7 times higher if the channel radius is 1 mm rather than 1 m. This would make E_{tip} much higher than the streamer stability field for r_C=1 mm. Recall however that r_C is the effective radius of the channel due to streamers transporting charge into the corona sheath. Our conclusions about the streamer zone radius based on E_{tip} should therefore be consistent with our initial assumption of r_C. Since we infer a streamer zone on the order of 1 m based on an assumption of r_C=1 m our results are self consistent.

4.2 Speed, Tip Field, and Tip Current

Figure 4 shows a direct comparison between the measured leader speed, the modeled tip field, and square root of the modeled tip current vs time for the $n = 2$ case. In the Figure we have normalized each variable to a maximum value of 1 to remove any constants of proportionality. It is clear that the speed and tip fields are closely correlated, except for the speed dip at 200 μs and a small variation at the end. We do note that the correlation is not as strong for other orders n of the polynomial ambient field, and the correlation is also somewhat sensitive to how the errors are weighted in Equation 15 and model parameters. In fact, due to the ill-posed nature of the inverse problem there are essentially infinitely many ambient fields which could reproduce the measured field changes. Among these infinite solutions there are many possible ambient fields which result in E_{tip} curves that do not match the leader speed, although many are physically unreasonable. While we cannot therefore definitively conclude that the leader speed is proportional to the tip field as guessed in Jensen et al. (2021, 2023b), we can at least claim that the observed field changes are consistent with a relationship of $v \propto E_{tip}$ for an equipotential leader.

If we frame the speed/field relationship as the leader mobility, $v = \mu E$, then we get a value of $\mu = 20\ m^2/Vs$ for this particular dart leader. As shown in Figure 3d the curves for E_{tip} and $\Delta\Phi_{tip}$ are essentially identical up to a scaling factor, so the dependence of the speed can be expressed in terms of either the tip field or potential drop. If we model the tip speed as $v = \eta\Delta\Phi_{tip}$ we get a value of about $\eta = 2$ m/Vs.

Since there is a fair amount of uncertainty in the correlation between tip field and speed we cannot completely rule out other power law relations such as the $v = a\Delta\Phi_{tip}^{1/2}$ relation suggested by Bazelyan and Raizer (2000) Equation 4.2. We note that the value

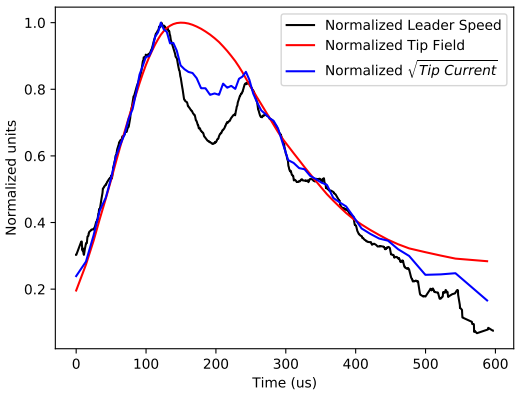

Figure 4. A plot of the normalized values of speed, tip field, and the square root of current vs time, for the $n = 2$ fit.

of $a = 15\ m/(sV^{1/2})$ suggested by Bazelyan and Raizer (2000) gives a speed 2 orders of magnitude too low even if we assume the relationship can be adjusted as $v = a\sqrt{\frac{\Phi_{tip}}{\delta}}$ for a 3000 K pre-dart-leader channel at 7 km altitude, where δ is defined in Equation 17. Regardless of the form of the relation, our results do suggest that the tip electric field or potential drop appears to be one of the main factors for the dart leader speed.

The square root of the tip current is also very well correlated with the speed in, Figure 4, but this should not be surprising. If we re-write Equation 6 we can see that

$$I_{tip}(t_k) = -\Delta s \frac{\lambda_{tip}(t_k) - \lambda_{tip}(t_{k-1})}{t_k - t_{k-1}} = -\frac{\Delta s}{\Delta t}\lambda_{tip}(t_k) \tag{18}$$

where $\Delta s/\Delta t$ is just the speed of the leader, and we are making use of the fact that $\lambda_j(t_{k-1}) = 0$ for the advancing leader tip. This would suggest that $I_{tip} \propto v$, but we must further consider that from Equation 12 $\lambda_{tip} \propto E_{tip}$, so we must have $I_{tip} \propto vE_{tip}$. We have also just established that $v \propto E_{tip}$, so therefore $I_{tip} \propto v^2 \propto E^2$ is exactly the form we should expect. This result also generally agrees with numerical modeling of the streamer to leader transition by da Silva and Pasko (2013), and empirical relations based on laboratory sparks suggested by Andreev et al. (2008) and Bazelyan and Raizer (1997)(page 213). Rather than implying that higher current somehow causes a higher speed, it is likely that the causal relationship is the other way around. A faster dart leader will generally be changing in potential more quickly, both at the tip and along the entire leader length, and this potential change will induce a larger current.

4.3 Other Model Results

Figure 5 shows the modeled potential, charge density, and current distributions along the leader colored by time, for the $n = 2$ model. Each curve in Figure 5 is a snapshot of conditions along the entire plotted distance at a particular time indicated by the color. Ahead of the negative leader tip at any point in time the charge and current are zero, and the potential is just the ambient potential. Thus in each plot a sharp increase along a curve of a particular color at a positive channel distance indicates the position of the negative leader tip at that time. Some distance past the positive tip of the leader (in the $-s$ direction) is also included to show the return to ambient conditions on that end.

Figure 5a shows that the potential along the active part of the channel is always a horizontal line due to the equipotential assumption, and the overall channel potential is increasing over time as the leader grows in length. If we consider the dart leader as

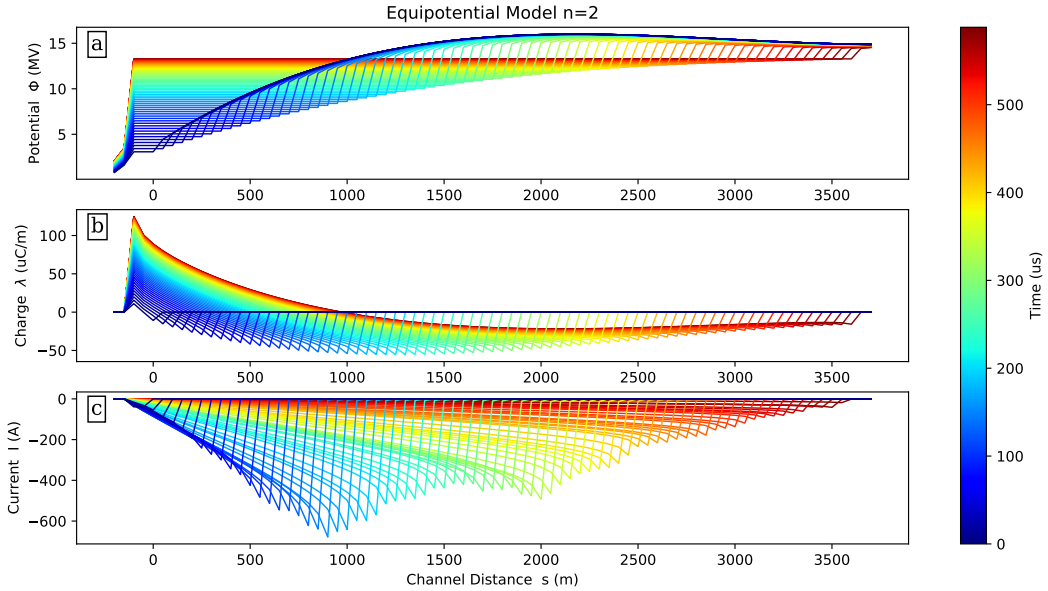

Figure 5. Plot of the potential distribution (a), charge density distribution (b), and current distribution (c) along the leader channel, colored by time, for the $n = 2$ polynomial ambient field. In all three plots the sharp increase at the leading edge indicates the leader tip position at that time.

transporting negative charge while leaving relatively stationary positive charge behind then this increase in channel potential over time corresponds to an overall decrease in potential energy, as expected. Ahead of the leader tip the potential essentially returns to the ambient potential within one step Δs. We can also see that the charge distribution (Figure 5) essentially satisfies $\lambda(s) \propto \Phi_{cha} - \Phi_{amb}(s)$.

The amount of charge at the negative leader tip in Figure 5b does increase initially as in our simple dipole model in Jensen et al. (2023b), but then the charge at the tip decreases somewhat as the leader progresses. However, there is a larger deposit of charge behind the tip which remains more constant, this may be the cause of the nearly constant tip charge in our previous dipole model as the leader slowed to a stop (Jensen et al., 2023b). The charge density at the positive tip on the other hand increases continually as the leader progresses, although the increase is fastest at the beginning. The scale of the charge density at tens of μC/m is much smaller than typical charge density estimates of about 1 mC/m, but these estimates are typically for stepped leaders in virgin air, where the ambient fields (and thus charge density) are much higher.

The current in Figure 5c is highest right at the negative tip. For a more realistic channel with some finite conductivity we would expect this peak to follow a little behind the leader tip as the charge density at the tip takes some finite time to build. The current then drops off towards the stationary positive tip of the leader since the potential is not changing as much at that end. The peak current magnitude is about 700 A, which is in reasonable agreement with the "typical" dart leader current of 1 kA given in Rakov and Uman (2003) Table 1.1.

5 Discussion

5.1 Validity of the Equipotential Assumption

Our modelling results assume a perfect equipotential channel, therefore the conclusions we draw about dart leader propagation are only valid if real dart leader channels are approximately equipotential. In this section we put forward three arguments that dart leader channels are approximately equipotential, and that our modeling results are therefore valid.

First, a channel with finite conductivity σ_R will approach an equipotential over time, so the key question is how the time scale at which the channel reaches equipotential compares to the timescale of the leader propagation.

For a channel of length L and conductive/resistive radius r_R the total channel resistance is given by

$$R = \frac{L}{\sigma_R \pi r_R^2} \tag{19}$$

We can then estimate the timescale at which the leader approaches equipotential as $\tau \approx RC/10$ (see Appendix A for a derivation). Writing the equation out fully combining Equations 16 and 19 we have

$$\tau \approx \frac{1}{10} \frac{2\pi\varepsilon_0 L}{\ln(L/r_C)} \frac{L}{\sigma_R \pi r_R^2} \tag{20}$$

This time constant is derived assuming a channel of fixed length L suddenly becomes conductive in a uniform field, which does not really match the conditions of an extending channel in a non-uniform field, but it is still useful to consider the results of this simple approximation.

High speed spectroscopy observations of dart leaders suggest that they reach temperatures of ∼20 kK (Chang et al., 2017; Orville, 1975), which corresponds to equilibrium conductivity σ_R of about 10 kS/m (Chang et al., 2017; Yos, 1963). The conductive radius r_R of a dart leader channel is estimated to be about 1-4 mm (Rakov, 1998), and for a well developed dart leader it is likely to be closer to 4 mm. The channel for K-5 is about 3500 m long by the end of its propagation, so with r_R=4 mm and σ_R=10× 10^3 $(\Omega m)^{-1}$ we get a time constant of τ=10 μs. Since 10 μs is short compared to the propagation timescale of dart leaders (100-1000 μs) the dart leader channel should be close to equipotential, at least by the time it stops propagating.

As a second argument, τ is also an estimate of how long it should take the field change to stop after propagation ceases. Any significant current in the channel will lead to a changing field at the ground. Ohm's Law $j = \sigma E$ suggests that the current will only drop if either the field or conductivity drops by several orders of magnitude. The channel conductivity is kept high by current heating the channel, so we should not expect the conductivity to drop while there is still significant current on the channel. Thus for a hot plasma channel the current will only stop when the field along the channel drops to essentially zero. Therefore the channel must be close to an equipotential when the field on the ground has stopped changing, and τ must also be low enough to allow the channel to reach equipotential by this time. Since the measured fields in Figures 3a and b have essentially stopped changing even before the leader has stopped propagating, we can assume that the leader is in fact close to an equipotential.

Finally, we consider the non-linear resistance of a plasma channel. For a plasma channel to remain hot and conductive ohmic heating must balance heat losses. The associated resistance will cause the current in the channel to be somewhat less than the ideal equipotential current, and the channel will take longer to reach the equipotential charge distribution. Therefore there will be some remaining potential gradient along the

channel $\nabla\Phi_{cha}$. We emphasize that in lightning $\nabla\Phi_{cha}$ is ultimately caused by the ambient potential gradient $\nabla\Phi_{amb}$.

Laboratory experiments of free burning arcs suggest that for currents of about 100 A to 1000 A the steady state potential gradient is between 1 kV/m (King, 1962; Mazur & Ruhnke, 2014) and 2.5 kV/m (Montano et al., 2006). These potential gradients are small enough that they should not significantly change our results. To verify this we model dart leader K-5 while including such a potential gradient. We set the channel potential gradient $\nabla\Phi_{cha}$ to be equal to the constant gradient $\nabla\Phi_{const}$ as long as the resulting $\Phi_{cha}(s)$ is between $\Phi_{amb}(s)$ and the ideal equipotential value at each point. Otherwise the channel remains at $\Phi_{cha}(s) = \Phi_{amb}(s)$. In keeping with our second argument for the equipotential model, we allow $\nabla\Phi_{cha}$ to approach zero as the leader slows to a stop. Based on Figure 11 of Montano et al. (2006) we drop $\nabla\Phi_{cha}$ to zero linearly over about 350 μs. Figures in the format of Figures 3 and 5 are included in the supporting information for $\nabla\Phi_{const}$=2.5 kV/m. We show the $n = 1$ ambient field because the $n = 2$ case has convergence issues when including the potential gradient. The results shown in those plots are very similar to the ideal equipotential results. Therefore the results of our equipotential modeling are a good approximation of the true leader properties even if a real leader has some internal potential gradient.

5.2 Branch Junctions

In Jensen et al. (2023b) we hypothesized that the rapid speed variations as dart leaders passed branch junctions may be caused by charge deposits near those junctions. The negative dart leader tip would be repelled by a negative charge deposit near the junction, so that the dart leader might decelerate while approaching the junction and accelerate after passing it. It is clear from the results in Section 4 that rapid variations in the ambient field are not resolved based purely on fitting to the measured field changes at the ground.

In order to test our branch junction hypothesis explicitly we need to modify our approach. Since the results in Section 4.2 do suggest a correlation between the tip field E_{tip} and the leader speed, we add this as another constraint using our assumed relationship of $v = \mu E_{tip}$, leading to a χ^2 value

$$\chi^2_{speed} = \sum_{t_k} \frac{\left(\mu E_{tip}(t_k) - v_{obs}(t_k)\right)^2}{\sigma^2_{speed}} \tag{21}$$

We also add a term to the ambient field which corresponds to the charge configuration in Figure 6. For a branch junction at location s_{junc} with a point charge q_{junc} at a distance h_{junc} along the perpendicular second branch the resulting electric field on the leader channel is

$$E_{junc}(s) = \frac{q_{junc}}{4\pi\varepsilon_0} \frac{s - s_{junc}}{\left[(s - s_{junc})^2 + h^2_{junc}\right]^{3/2}} \tag{22}$$

where we are treating the channel as perfectly straight for simplicity, and calculating the $\vec{E}_{junc} \cdot \hat{s}$ component of the field.

To avoid having the junction charge significantly modify the field fit away from the junction, we find the fitting parameters in two stages. First h_{junc} and q_{junc} are fit using the Levenberg-Marquardt algorithm, while using an assumed value of s_{junc}=1675 m since this is where the change in speed is observed along the channel. As a background field the same $n = 2$ ambient field previously found in section 4 is used. The value of $\mu = 20\ m^2/(Vs)$ from Section 4.2 is also used so that the proportionality between the speed and tip field remains the same. The time range of Equation 21 is limited to $t =$ 100 μs to $t = 260$ μs, since this is the range of the dip in observed speed. This way we

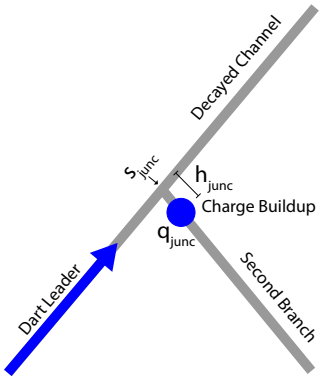

Figure 6. A diagram showing the configuration of the branch junction charge. The junction between branches is located at a point s_{junc}. A charge q_{junc} is located a perpendicular distance h_{junc} away from the junction point, along the second branch.

are fitting the dip specifically without trying to optimize the fit for other times. For the fast antenna field change fit we continue to use the full time range. The combined goodness of fit parameter is then $\chi^2_{tot} = \chi^2_{FA01} + \chi^2_{FA02} + \alpha\chi^2_{speed}$ where α is a weighting term which we adjusted manually to achieve a reasonable balance between fitting the field change and the speed.

After finding a reasonable fit for the junction point charge parameters, we then refit the background field with a 2nd order polynomial in order to find a better fit with both terms present. This process could be repeated iteratively, alternating between junction charge and background field fits, but we found one iteration was enough in this case. It may also be possible to fit the junction charge and background field both at the same time, but due to issue with convergence to the optimal result and the need for human judgement in weighting the χ^2 values the two stage approach was more tractable.

The results from this process are shown in Figure 7. Figures 7a and 7b show only a modest change in the modeled field change at the ground from the $n = 2$ results in Figures 3a and 3b. The ambient field in Figure 7 includes the 2nd order polynomial background field , the junction charge field from Equation 22, and the sum of the two field terms. The fit values are h_{junc}=332 m and q_{junc}=−115 mC. The tip field and tip potential drop vs time in Figure 7d show a pronounced dip around the location of the dip in speed in Figure 7e. We further include the modeled $v = \mu E$ in Figure 7e, this model slightly over-estimates the speed but in general there is now an excellent agreement with the measured leader speed for the whole leader duration.

We note that there is no branch visible in the BIMAP-3D sources at the s_{junc}=1675 m location of the simulated charge, even when we include all VHF sources from the full recorded flash. There are multiple small side branches within a few hundred meters of this location that appear in VHF either before or after K-5, and possibly the speed variation we observe is due to the combined influence of these multiple side branches. It is also possible that some previous leader activity deposited charge directly along the channel without the need for a branch junction. Either way, we have at least demonstrated that our hypothesis from Jensen et al. (2023b) is generally viable. We have reinforced our conclusion from Section 4.2 that using the equipotential model a leader speed relationship of $v = \mu E_{tip}$ is consistent with both our observations of field changes at the ground and the observed leader speed. Further, under the equipotential model it is possible for a charge deposit a relatively short distance from the primary channel to cause E_{tip} to exhibit a

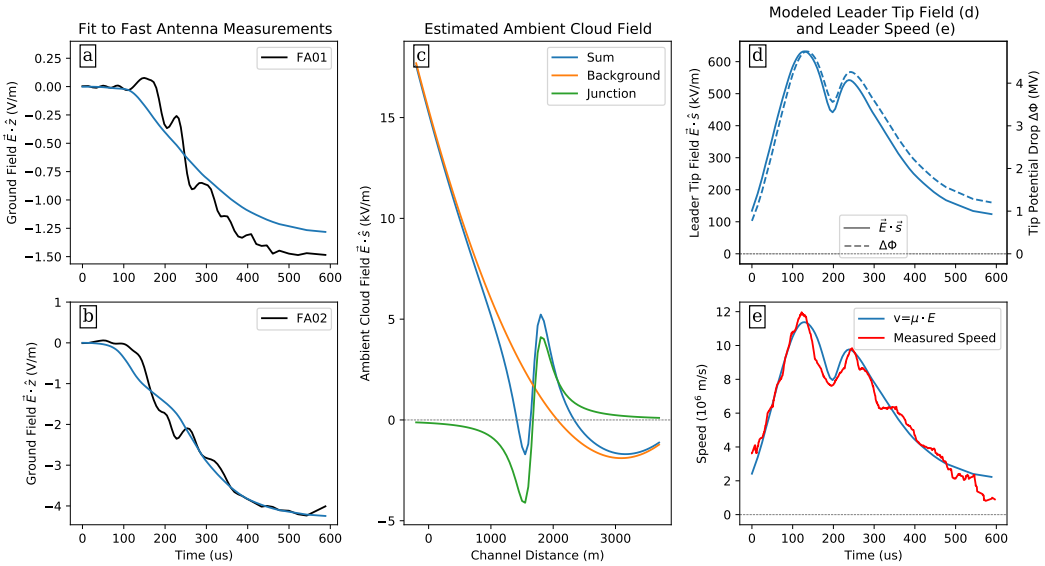

Figure 7. Equipotential model results when adding the junction charge term to the n=2 ambient field from Section 4. In the same format as Figure 3, the plot shows the measured and fit field change vs time for FA01 (a) and FA02 (b), the ambient field vs channel distance (c) including the background 2nd order polynomial field, the junction charge field, and the sum of these two components. The leader tip field vs time (d, left axis) and tip potential drop vs time (d, right axis) is shown, along with the measured and modeled leader speed vs time (e).

rapid drop and recovery, like the branch junction speed changes reported in Jensen et al. (2023b), without significantly changing the field change measured at the ground.

We note that the fit presented in Figure 7a seems to slightly under-estimate the magnitude of the field change for FA01, but this could be due to the large uncertainty in gain calibration for each fast antenna. A similar underestimation does also occur in Figure 3a but it is less obvious since multiple modeled field changes are shown. Additionally, in Figure 7a different values of μ might allow a better simultaneous fitting of the speed and field changes. Since this is a first-of-its-kind comparison of the equipotential model with observed 3D leader propagation, and since the uncertainties in fast antenna calibration are so large, we will not attempt to refine the fit further.

5.3 Bidirectional Development

High speed camera observations of dart leaders initiating outside of clouds indicate that the bright channel initially extends bidirectionally, but the extension in the positive tip direction quickly halts once it reaches the previously observed end of the preconditioned channel (Ding et al., 2024; Mazur, 2016). Unfortunately this extension in the positive direction is not observed in VHF by BIMAP-3D. We have performed some tests assuming the positive tip extends at the same speed as the negative tip until it reaches the end of the channel as observed in earlier VHF.

Plots showing the K-5 results when including this bidirectional development are included in the supporting information, in the style of Figures 3 and 5. The estimated ambient field is somewhat lower in magnitude when including bidirectional development, but still generally decreases in the direction of propagation. The modeled leader tip field is slightly lower, peaking at about 500 kV/m rather than 800 kV/m, but it is still gen-

erally correlated with the leader speed. Thus the inclusion of this bidirectional development does not significantly change any of our conclusions. The most significant change is that the current is high at both leader tips while they are propagating, with a more uniform current through the middle of the channel. After the positive tip stops propagating the current distribution is similar to the distribution shown in Figure 5.

Figure S3d in the supporting information also includes the tip field and potential drop for the positive tip of the dart leader. The positive tip field is below the the ambient air breakdown field. This explains why the fast bidirectional development stops once it reaches the end of the pre-conditioned channel. As reported by Jensen et al. (2023b) the positive tip appears to continue extending at 2×10^4 m/s throughout the dart leader phase of the flash. Our new modeling results suggest this 2×10^4 m/s positive tip extension may occur with a tip field above the positive streamer stability threshold but below the breakdown field in virgin air.

In a few cases for other dart leaders shown in the supporting information (most notably K-2) significant bidirectional development needs to be added to the model in order to match the field change at both stations. This suggests that there was more bidirectional development for those dart leaders, as compared to some of the other dart leaders where adding a bidirectional component made little difference to the field change or speed fits. The differences seem to be both in the geometry of the channel and in the overall shape of the resulting field change.

5.4 Other Dart Leaders

We also estimated ambient fields and tip fields for several other IC dart leaders from the same flash analyzed in Jensen et al. (2023b). Figures for these are included in the supporting information to avoid an excessive number of figures in the main text. We excluded a few cases where the dart leader development involved multiple simultaneous branches (K-10 and K-14), or there were large gaps in time with no located VHF sources (K-7) since our methodology depends on the leader following one single path with a well defined tip location at each point in time. For a few other dart leaders which split into multiple branches we were able to model the initial portions before they branched (K-4 and K-9). These are marked as "partial" fits. The path each dart leader follows can be seen in the figures of Jensen et al. (2023b), or the figures and animations in the supplementary material for that paper (Jensen et al., 2023a).

For the other dart leaders we explicitly look for an ambient field which fits both the measured field changes and the measured leader speed. We do this by adding the χ^2_{speed} term from Equation 21, although in some cases this additional constraint seems to cause convergence issues for the Levenberg-Marquardt algorithm, and we actually obtained better fits to the leader speed without the explicit χ^2_{speed} constraint. We allow the μ value to be determined as the median value of $v(t_k)/E_{tip}(t_k)$ for each iteration.

In most cases shown in the supporting information we were able to find ambient fields such that the modeled leader fit both the measured field changes, while also having a tip field which was generally correlated with the leader speed. In all these valid cases the estimated cloud field generally decreases along the channel length, similar to Figure 3c. The highest field values are also similarly low, less than about 10 kV/m, with the exception of K-3 which peaks at about 45 kV/m. If K-3 is modeled with equal development in the positive and negative directions then the estimated ambient field peaks at <20 kV/m, while the field change remains similar, and the tip field is actually slightly better correlated with the leader speed. The modeled E_{tip} values are much lower for K-3 if we assume the leader extension is symmetrical about the starting point.

Among the valid cases the tip field and leader speed correlations ranged from being quite close to only being vaguely correlated. These generally support our conclusion

that the observed leader speed trends can be explained as $v \propto E_{tip}$ for an equipotential leader, especially considering we are estimating the ambient field with only a few degrees of freedom, so we cannot expect to match complicated variations in speed. It may be possible to allow more degrees of freedom while fitting to the leader speed in addition to the measured field changes, but in our testing there seemed to be issues converging to an optimal solution. A more robust approach might be to start with a simple linear ambient field fit and then use the linear field as an initial guess while gradually adding degrees of freedom. Adding degrees of freedom as piece-wise linear fits rather than polynomials might also improve the convergence behavior, while avoiding the undesirable polynomial divergence near the endpoints.

For the empirical relation $v = \mu E_{tip}$ we find μ values ranging between 10-30 m^2/Vs. For the relation $v = \eta \Delta\Phi_{tip}$ we find η values ranging between 1-4 m/Vs. For K-4 we find extreme outlier values of μ and η, but the net field change in K-4 is a small fraction of the noise level, so the results are not reliable. Since our uncertainties are large and the quality of fit varies for each leader we cannot say whether the differences in these values among different dart leaders are caused by random uncertainty, or if they reflect something more fundamental like the temperature of the pre-dart-leader channel in each case.

In a few cases (the full K-8 and K-13) we were not able to match the measured field changes from both stations. These cases indicate that it is possible for a dart leader to have a more complicated field change even if the observed development in VHF seems fairly simple. Comparing the field change timing (supporting information for this paper) to the leader development (Jensen et al., 2023a), for both K-8 Full and K-13 the shift towards a positive field change at FA02 occurs close to the time that those dart leaders reach junction J1. This strongly indicates that the more complicated field changes are caused by VHF invisible development into the other branch at J1. For these cases where the measured field changes at the ground could not be reproduced the corresponding model results are not valid. These cases are included only to show that while our equipotential model constrained by the BIMAP-3D observations works in most cases, there are some exceptions.

6 Summary

Due to the integral nature of the field change at the ground in Equation 14, there are essentially an infinite number of ambient field solutions which will fit the observed field changes, even when constrained by the path and speed of leader development as observed by BIMAP-3D. Solving for this ambient field is thus an "ill-posed" inverse problem. The dart leader channel properties we model are therefore not definitive, but are at least consistent with our observations. The fact that our modeled results seem to explain more general observed properties of dart leaders, and the fact that we obtained most of these model results using only simple linear or quadratic ambient fields lends further credibility to our claims.

The following conclusions are consistent with our observations:

Section 4.1

1. A physically plausible ambient field E_{amb} which matches VHF observations of channel development and electric field changes at the ground can be found
2. The estimated ambient field along the dart leader channel is generally low, less than 15 kV/m
3. The ambient field generally decreases in the direction of dart leader propagation
4. The modeled E_{tip} and $\Delta\Phi_{tip}$ are essentially proportional to each other
5. E_{tip} is generally less than $E_k \cdot \delta$ unless the pre-dart-leader channel has a significantly elevated temperature ($\sim$1000-3000 K) compared to ambient air

6. Dart leaders are typically confined to pre-conditioned channels because their tip fields are too low to propagate into virgin air ($E_{tip} < E_k \cdot \delta$)
7. The modeled E_{tip} values are close to the negative streamer stability field in ambient air $E_{st-} \cdot \delta$, suggesting that negative streamers should only extend a few meters radially outward from the channel, in agreement with VHF observations of narrow dart leader channels (Hare et al., 2023a; Jensen et al., 2021; Shao et al., 2023)

Section 4.2

8. E_{tip} and $\Delta\Phi_{tip}$ are correlated with the observed leader propagation speed
9. The square root of the current at the leader tip is also correlated with leader speed, this is expected since the model equations yield $I_{tip} \propto v \cdot E_{tip}$ and we have observed $E_{tip} \propto v$

Section 4.3

10. The equipotential model allows calculation of the potential of the leader channel, as well as the charge and current distributions, all resolved in time and space

Section 5.1

11. The equipotential model is a good approximation of the true leader properties

Section 5.2

12. A charge deposit near the channel can produce tip field variations similar to the speed variations we observed associated with branch junctions in Jensen et al. (2023b)

Section 5.3

13. In most cases including some initial bidirectional extension in the dart leader development does not significantly change the model or fit results
14. At the end of dart leader propagation the field E_{tip} at the positive tip of the dart leader is between the positive streamer stability field E_{st+} and the breakdown field E_k in ambient air
15. The positive tip field $E_{st+} < E_{tip} < E_k$ seems to correspond to the 10^4 m/s positive leader extension between dart leaders (Jensen et al., 2023b)
16. In a few cases significant extension of the VHF invisible positive tip of the dart leader is needed to simultaneously match the measured field change at both fast antennas

Section 5.4

17. Similar results can be obtained for several other dart leaders from the same flash
18. For the empirical relation $v = \mu E_{tip}$ we find typical values of μ=10-30 m^2/Vs
19. For the empirical relation $v = \eta \Delta\Phi_{tip}$ we find typical values of η=1-4 m/Vs
20. In a few cases the model cannot fit the observed field changes at both stations simultaneously, these cases seem to correspond to VHF invisible development along other branches in the flash structure

Appendix A Time Constant Derivation

To first order, the self-capacitance C_{tot} of a long cylindrical leader channel is given by Equation 16. The total resistance R_{tot} of the channel is then given by Equation 19. If we split this leader channel into N discrete segments then each segment has capacitance C_{tot}/N and resistance R_{tot}/N. We note that the capacitance C_{tot}/N is not between the channel and some hypothetical co-axial shell, but rather the self-capacitance between each cylindrical segment of length L/N and every other segment of the channel.

If the leader is initially non-conductive in a uniform electric field, and then suddenly becomes conductive, this is analogous to being driven by equal and opposite voltages at the two ends, and then having the voltage supplies suddenly disconnected. We set the potential at the center of the leader to 0 for convenience since the channel will approach the central potential in a uniform field (Equation 2).

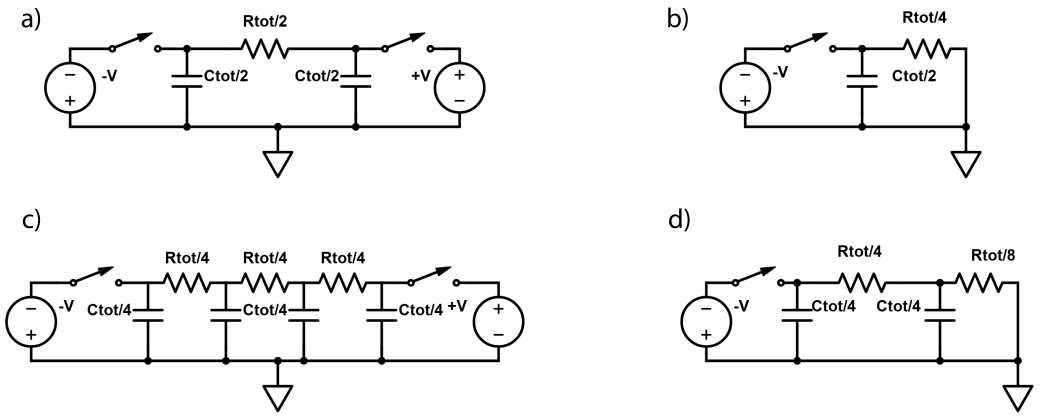

Figure A1. Circuit diagram showing the N=2 (a, b) and N=4 (c, d) simple model of a leader in a uniform field as N capacitors connected by N-1 resistors, with the capacitors driven by equal and opposite voltages across switches. On the right (b, d) we see the equivalent circuits reduced by symmetry.

First we consider the simple case of N=2 segments. We then have an electrical circuit with two capacitors of value $C = C_{tot}/2$, separated by a resistor of value $R = R_{tot}/2$, as shown in Figure A1a. If this circuit is driven by equal and opposite voltages $+V$ and $-V$ (analogous to a leader channel in a uniform field), then by symmetry the voltage must always be 0 in the middle of the resistor, and the circuit in Figure A1a is equivalent to the circuit in Figure A1b (up to the sign of the voltage). The circuit in Figure A1b is a regular RC circuit, so we can immediately see that the time constant is $\tau = (R_{tot}/4)(C_{tot}/2) = R_{tot}C_{tot}/8$. For N=2 our choice of $R = R_{tot}/2$ is somewhat contrived, but as N gets larger the difference between R_{tot} and $R_{tot}(N-1)/N$ becomes negligibly small.

We then consider the N=4 case, shown in Figure A1c. Again by symmetry we can see that the voltage at the center of the middle resistor must always be zero, and thus the discharging circuit is equivalent to Figure A1d. After applying Kirchoff's node law for this circuit and substituting the relevant terms in voltage and $\frac{dV}{dt}$ we get a system of ordinary differential equations

$$\frac{dV_1}{dt} = \frac{16}{R_{tot}C_{tot}}(-V_1 + V_2) \tag{A1}$$

$$\frac{dV_2}{dt} = \frac{16}{R_{tot}C_{tot}}(V_1 - 3V_2) \tag{A2}$$

where V_1 is the voltage of the capacitor closest to the voltage source and V_2 is the capacitor closest to the ground point.

This system of differential equations can be re-framed as an eigenvalue problem by writing the system as

$$\frac{d}{dt}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{16}{R_{tot}C_{tot}}\begin{bmatrix} -1 & 1 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{A3}$$

which has a solution of the form

$$\vec{V} = \vec{X}e^{\lambda t} \tag{A4}$$

where $\vec{X}$ is an eigenvector and λ is the corresponding eigenvalue of the matrix in Equation A3. In this case the eigenvalues and corresponding eigenvectors are

$$\lambda = \frac{16}{R_{tot}C_{tot}}(\pm\sqrt{2} - 2); \quad \vec{X} = \begin{bmatrix} 1 \pm \sqrt{2} \\ 1 \end{bmatrix} \tag{A5}$$

The full solution will be a linear combination of solutions of the form given in Equation A4 for the two eigenvalue/eigenvector pairs, but for our purposes we are interested only in the time constants. The slower time constant will dominate over longer times, this time constant is

$$\tau = \frac{R_{tot}C_{tot}}{16(2-\sqrt{2})} \approx \frac{R_{tot}C_{tot}}{9.37} \tag{A6}$$

Checking higher orders of N with numerical simulations we find that the decay time remains within the range

$$\frac{R_{tot}C_{tot}}{8} < \tau < \frac{R_{tot}C_{tot}}{10} \tag{A7}$$

We thus suggest $\tau = RC/10$ as a convenient rule of thumb for the timescale at which a lightning channel becomes an equipotential. Strictly speaking this approximation is only valid for a stationary channel which suddenly develops in a uniform field, but it may still be a useful reference for a more realistic model of leader development.

Appendix B Open Research

The 3D mapping and field change data used for this paper has previously been made available online (Jensen et al., 2023a). All data files are in text format with headers that describe each data column. A PDF is included which describes the included files, and gives examples of the headers and column format.

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