

Supporting Information for
**Estimating the Electric Fields Driving Lightning Dart Leader
 Development with BIMAP-3D Observations**

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Contents of this file

1. Figures S1 to S15

Introduction

This document contains additional figures to support and expand our conclusions from the main text.

Figures S1 and S2 show results when applying an internal potential gradient to the channel rather than assuming a perfect equipotential. Figure S1 is in the format of Figure 3 in the main text, while Figure S2 is in the format of Figure 5 from the main text.

Figures S3 and S4 show results when assuming dart leader K-5 extends bidirectionally. The modeled leader channel starts between $s=-100$ m and $s=0$ m at time $t=0$, then it extends at equal speeds in both the $-\hat{s}$ and $+\hat{s}$ directions. The path in the $-\hat{s}$ direction (positive tip) follows the branch as observed by VHF earlier in the flash, in the direction away from the flash origin. The positive tip stops propagating once it has reached the previously observed end of the branch. The negative tip propagation matches the observed VHF dart leader development. Figure S3 is in the format of

Figure 3 in the main text, while Figure S4 is in the format of Figure 5 from the main text. Figure S3 includes the tip field and potential drop for both the negative and positive tips of the dart leader.

The remaining Figures (S5 through S15) show the results of applying our equipotential methodology to additional dart leaders from the same flash. For these dart leaders we are explicitly looking for an ambient field solution which matches both the measured field change and the measured leader speed. These figures are included to give a broader view of our results, both with cases that generally support our conclusions from the main text, and a few examples where we were not able to fit the observed field changes. Figures S5 through S15 are all in the format of Figure 3 from the main text.

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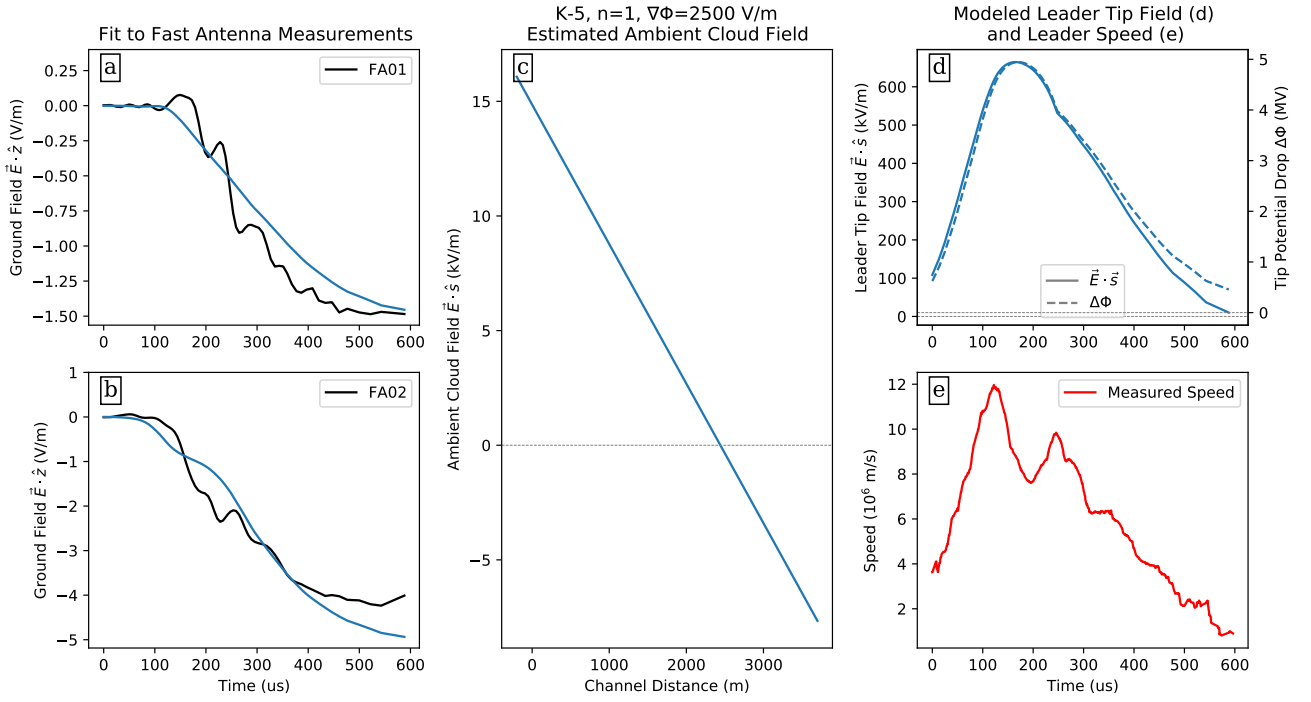


Figure S1. Field results for dart leader K-5 in the $n=1$ case when including an internal potential gradient of 2.5 kV/m. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e).

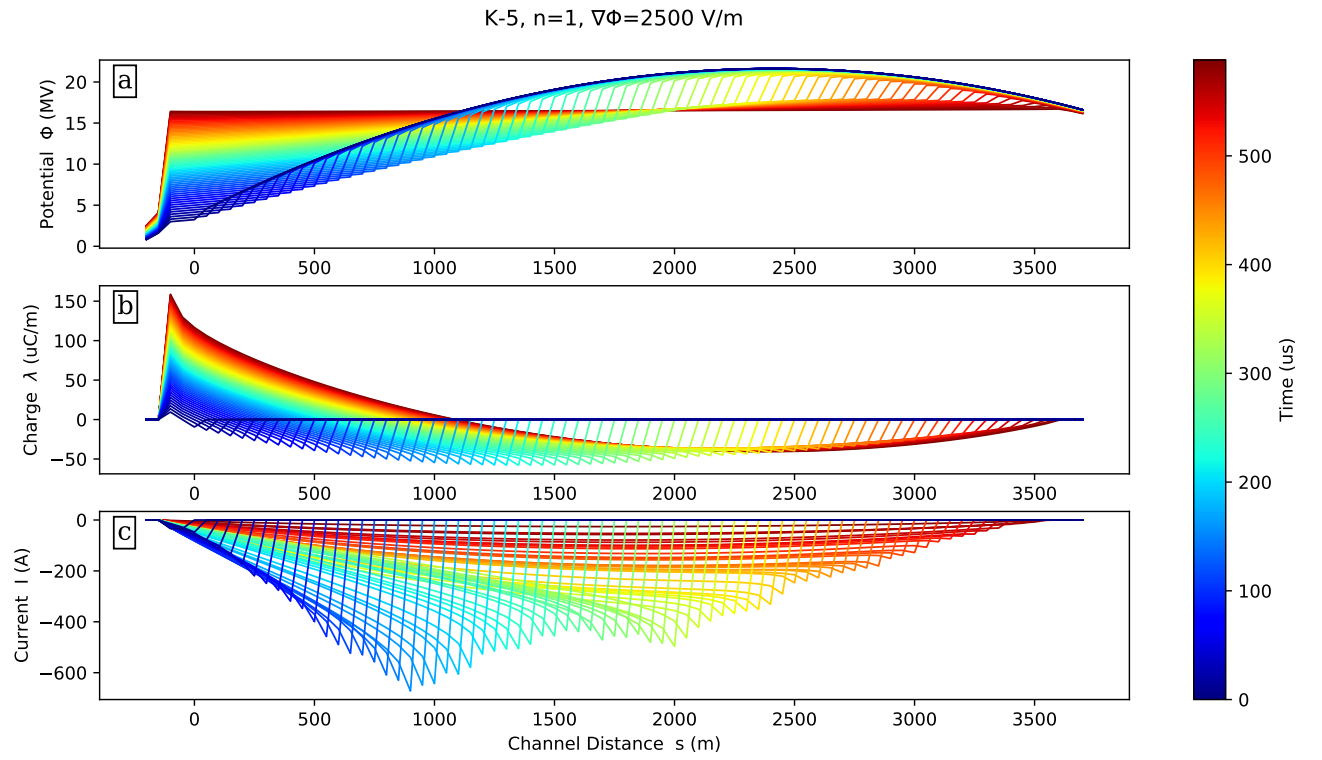


Figure S2. Modeling results for dart leader K-5 in the $n=1$ case when including an internal potential gradient of 2.5 kV/m. Plots show the potential distribution (a), charge distribution (b), and current distribution (c), all plotted vs channel distance and colored by time.

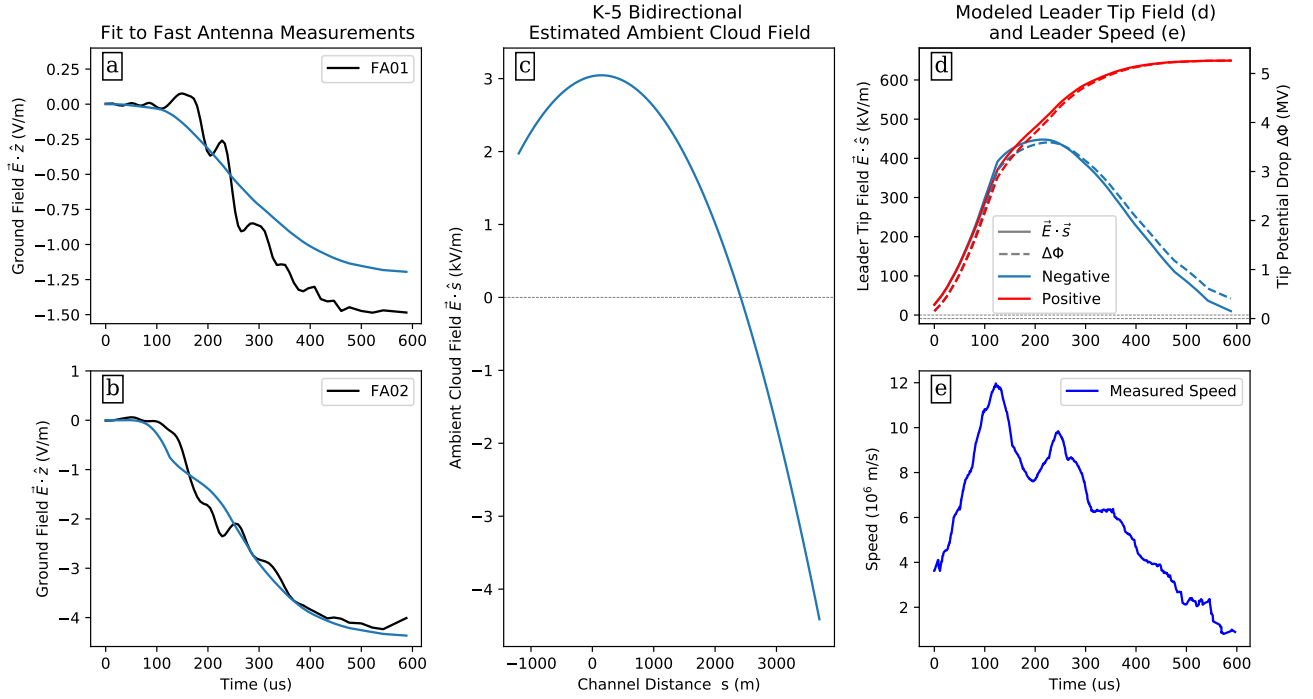


Figure S3. Field results for dart leader K-5 in the $n=2$ case when including bidirectional development. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time for the negative (blue) and positive (red) tips of the dart leader (d), and the measured leader speed vs time (e). Negative channel distance indicates extension of the positive tip in the direction away from the flash origin. The positive tip starts at $s=-100$ m and the negative tip starts at $s=0$ m.

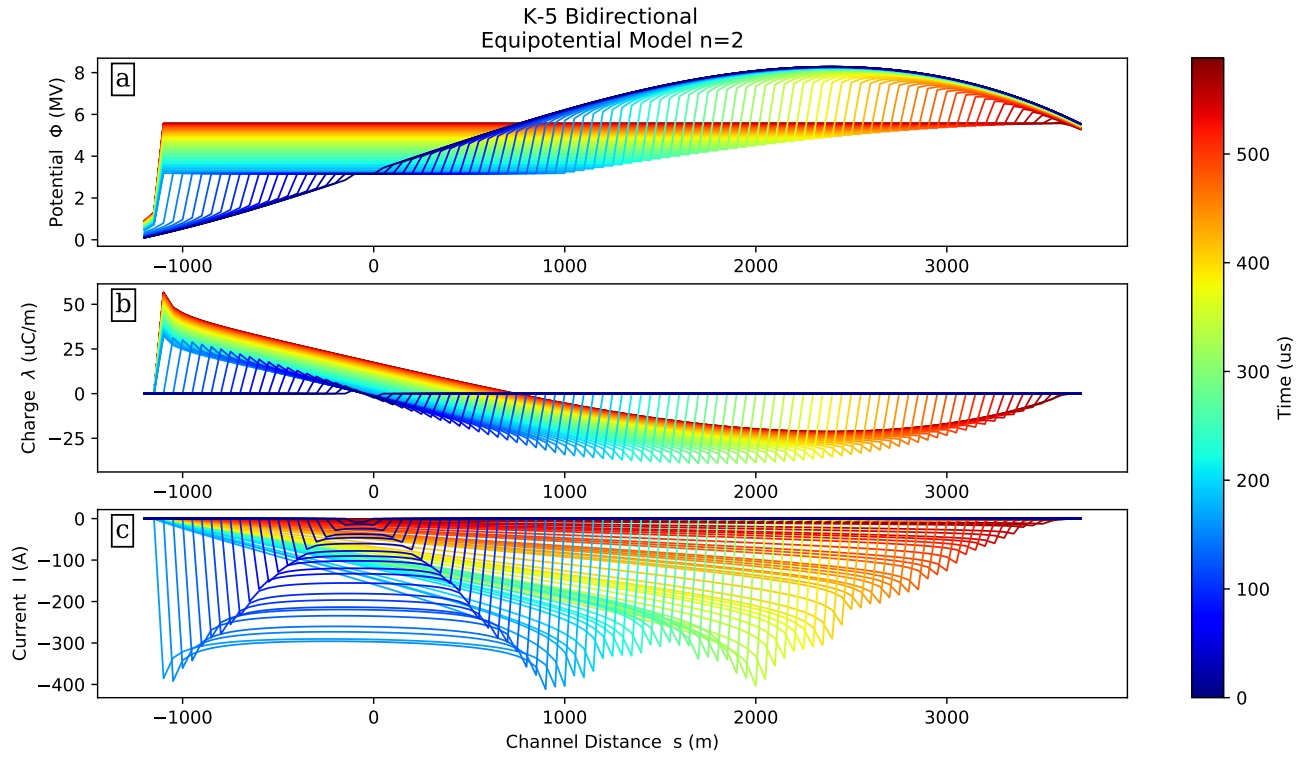


Figure S4. Modeling results for dart leader K-5 in the $n=2$ case when including bidirectional development. Plots show the potential distribution (a), charge distribution (b), and current distribution (c), all plotted vs channel distance and colored by time. Negative channel distance indicates extension of the positive tip in the direction away from the flash origin. The positive tip starts at $s=-100$ m and the negative tip starts at $s=0$ m.

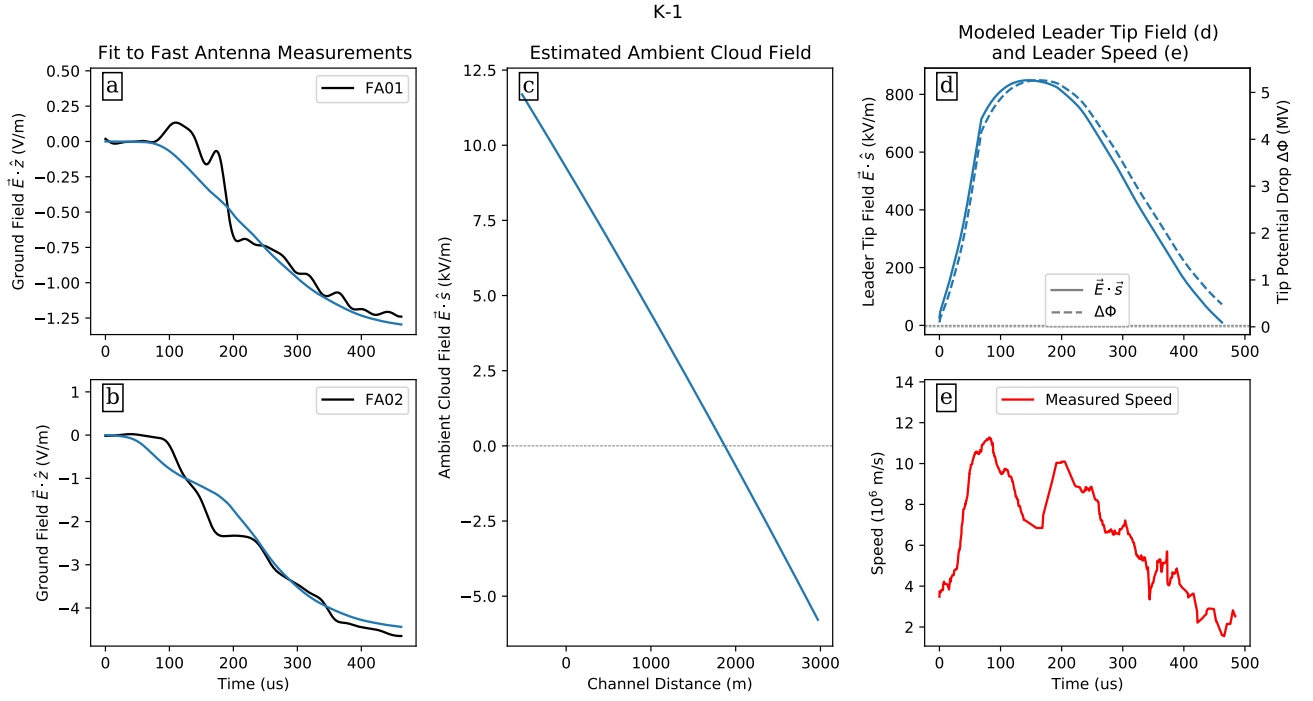


Figure S5. Results for dart leader K-1 with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader includes a 500 m extension of the positive tip to improve the field change and speed fit. The model matches the field change and speed fairly well.

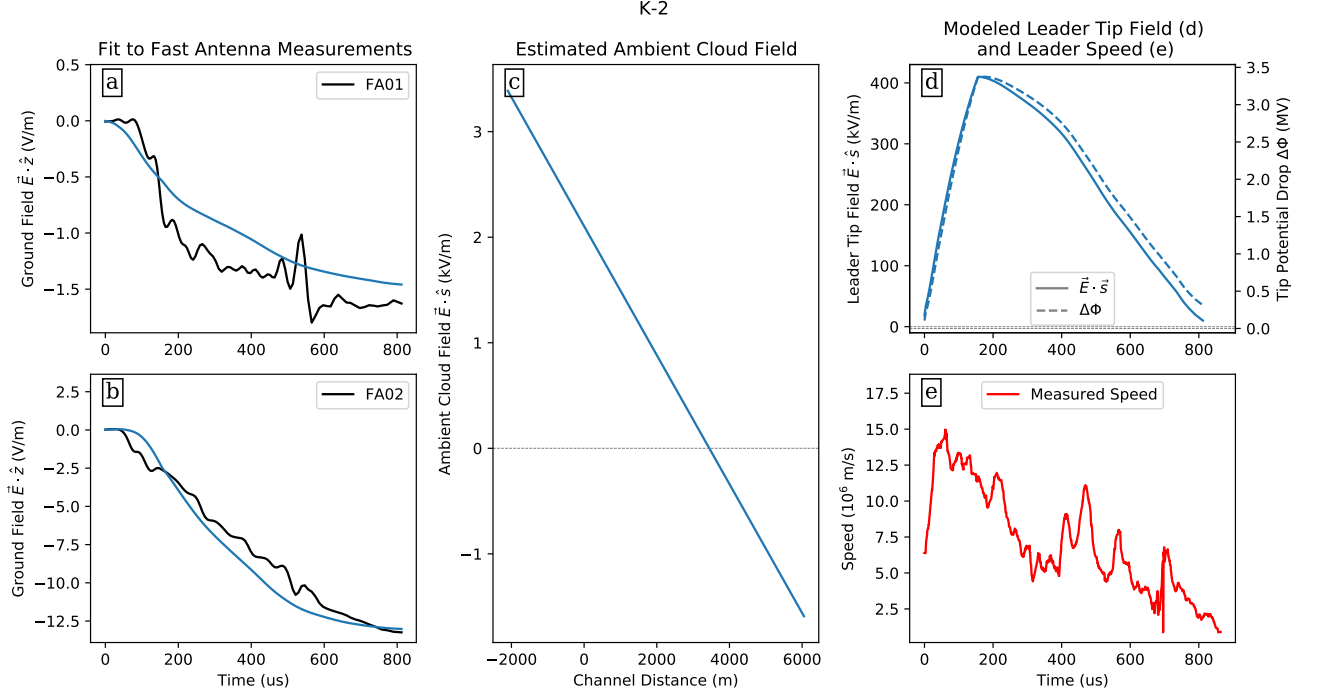


Figure S6. Results for dart leader K-2 with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader includes a 2000 m extension of the positive tip, without this the field changes at FA01 and FA02 cannot be fit simultaneously. The model fits the field changes fairly well, and the tip field is at least somewhat correlated with the leader speed.

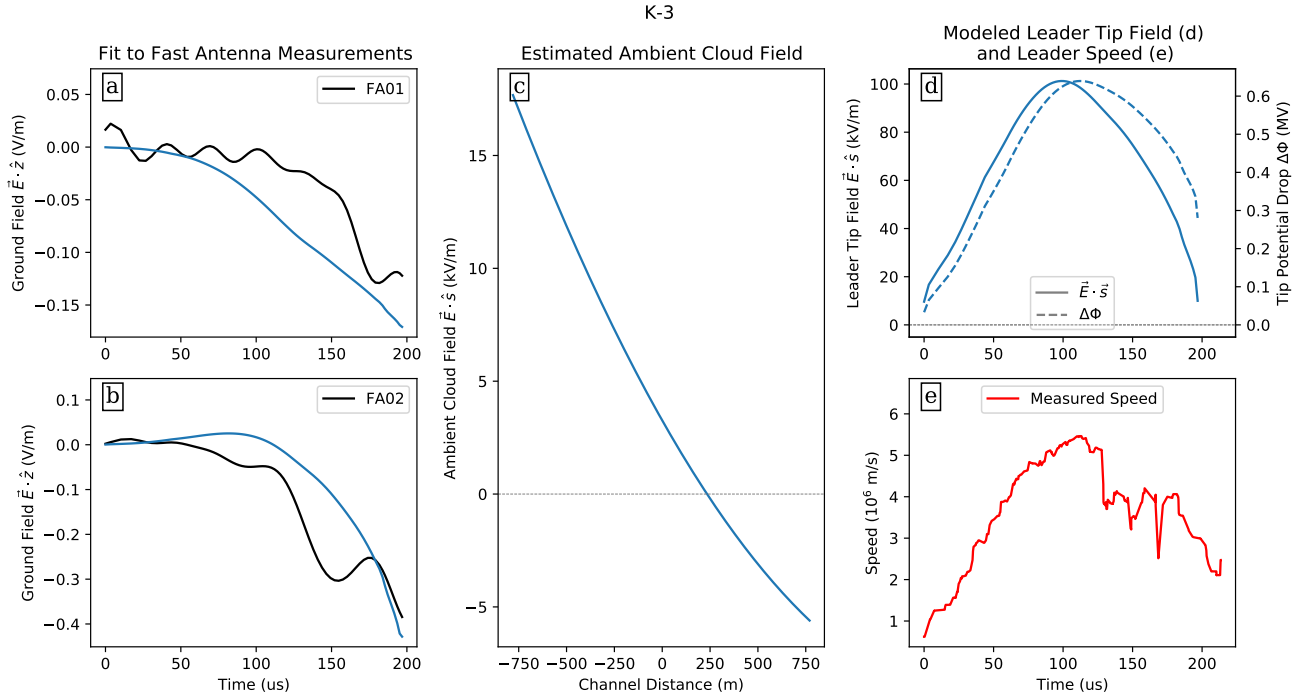


Figure S7. Results for dart leader K-3 with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does not include significant extension of the positive tip. Considering that the field change is small relative to the noise level the model fits fairly well, and there is a general correlation between the tip field and speed.

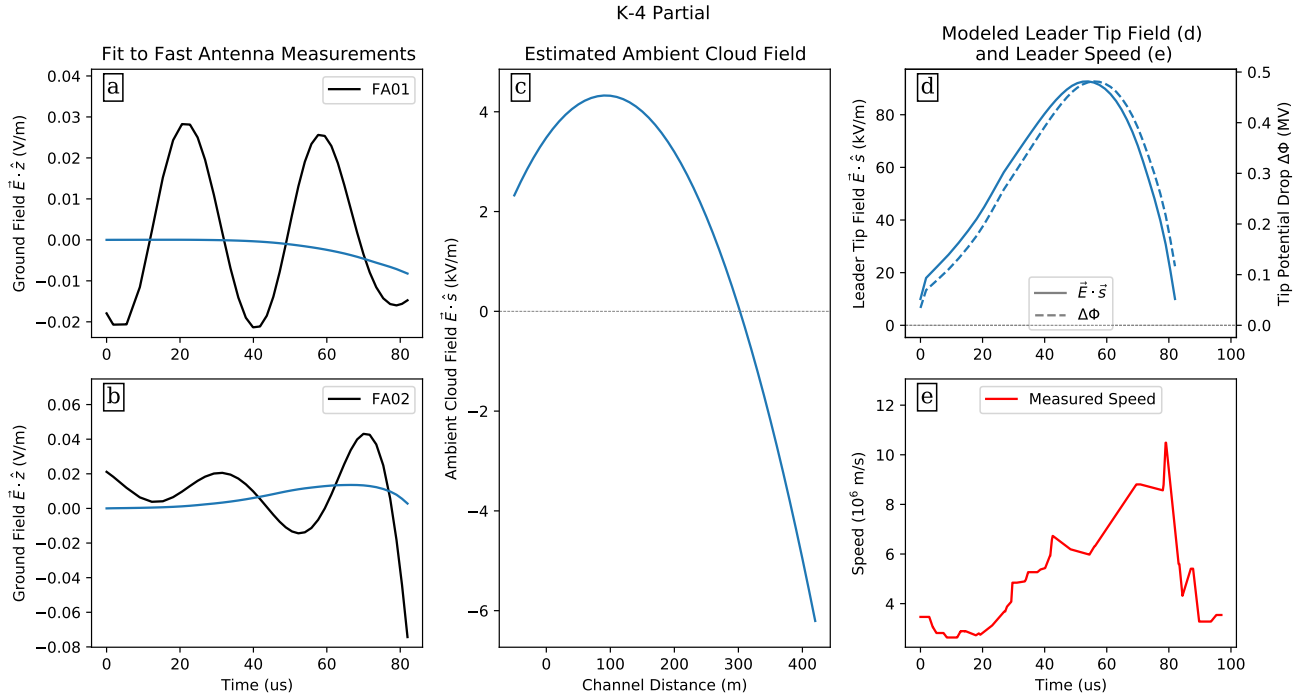


Figure S8. Results for the initial portion of dart leader K-4 before it splits into multiple branches, with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does not include significant extension of the positive tip. The net electrostatic field change is negligible compared to the noise level so the accuracy of the estimated field is likely very low, but the modeled tip field is well correlated with the speed.

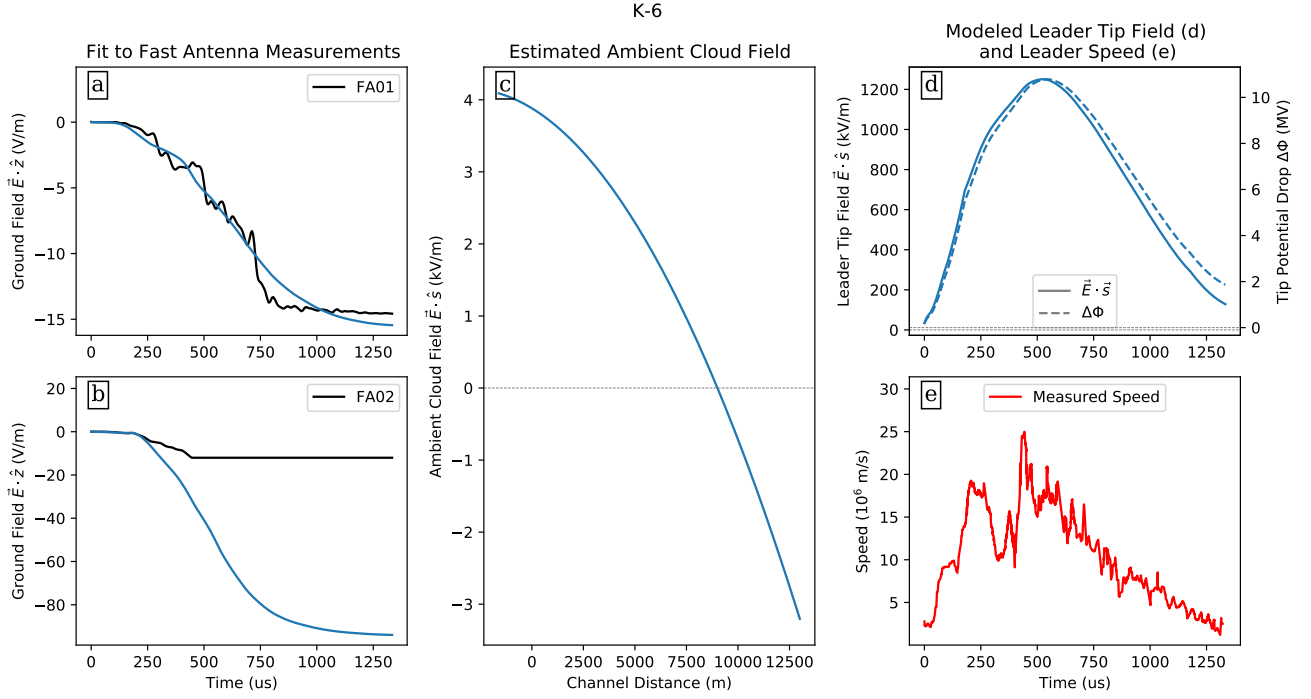


Figure S9. Results for dart leader K-6 with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does include a 1500 m extension of the positive tip in order to improve the tip field correlation with the speed. The field change at FA02 saturated the sensor, so the saturated portions are not included in the χ^2 fit. Even in the unsaturated portions between 250 μs and 500 μs the measured field change is much smaller than the modeled change, but it is difficult to properly constrain the results without a longer record to compare to. The tip field that results from matching FA01 does generally correlate with the speed, other than the branch junction variations.

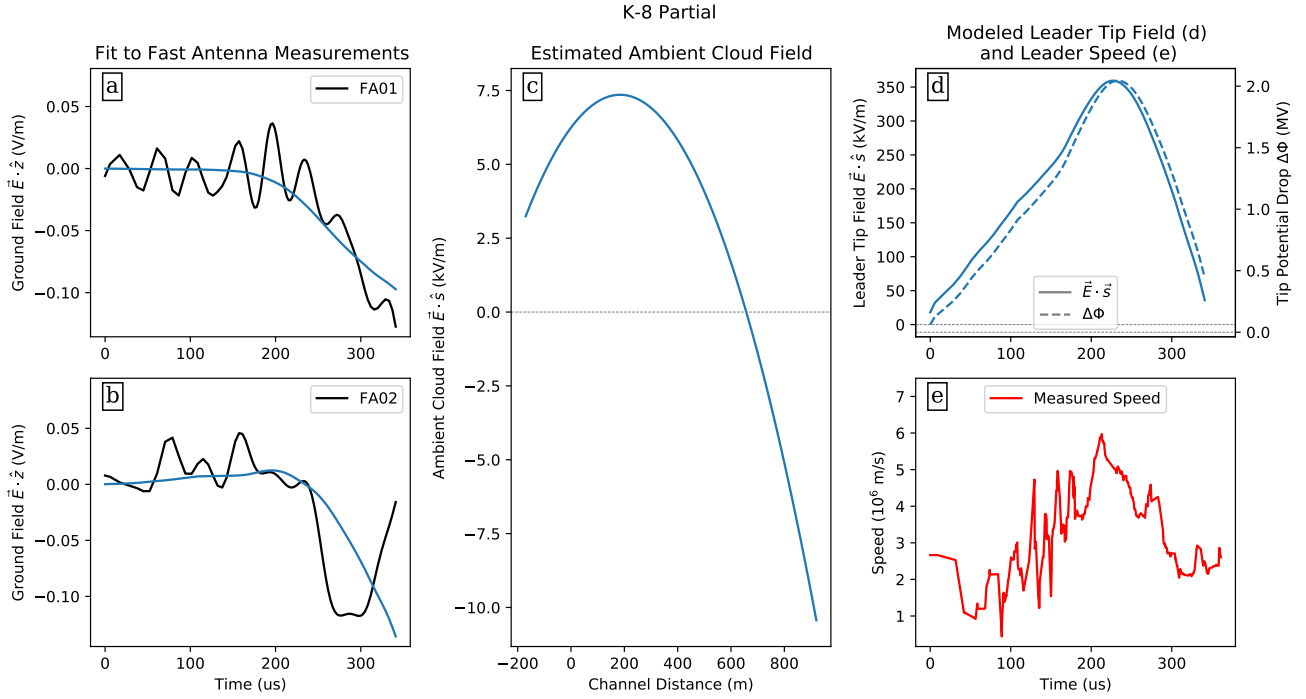


Figure S10. Results for the initial part of dart leader K-8, up to the first junction, with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does not include any significant extension of the positive tip. The net electrostatic field change is fairly small compared to the noise, but the fit to the field change and leader speed is fairly good for this portion of the leader.

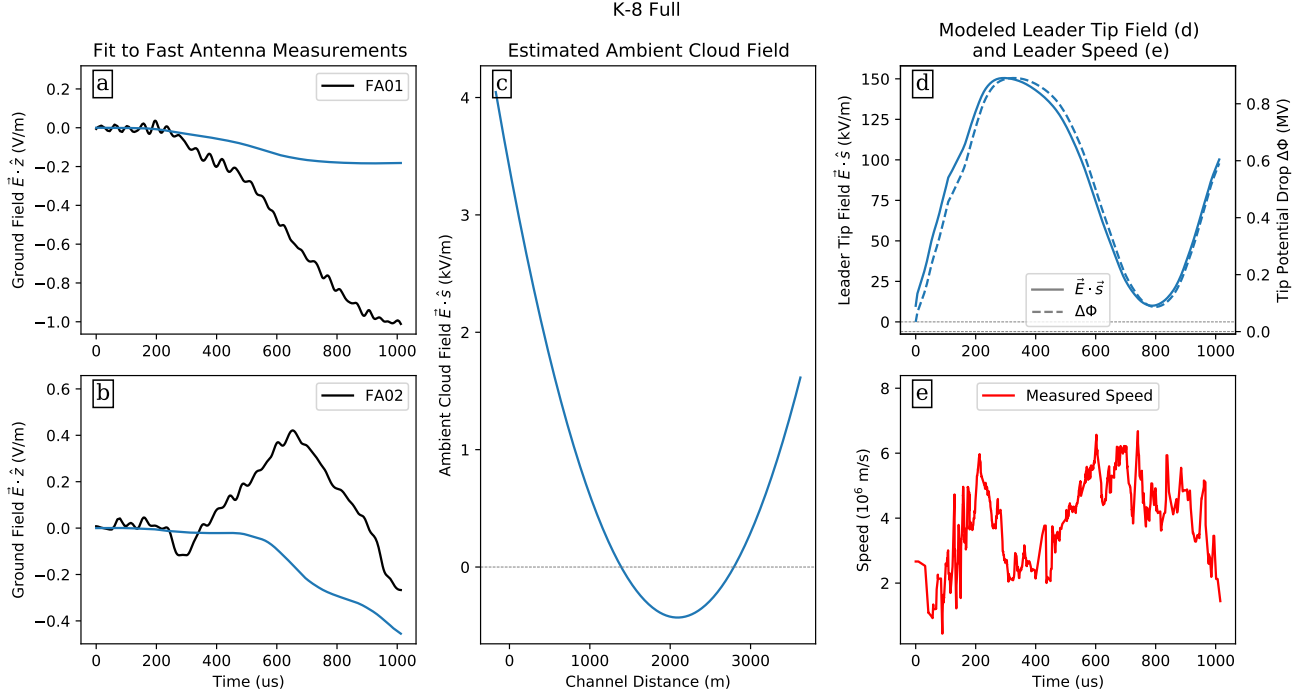


Figure S11. Results for the full development of dart leader K-8 with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does not include any significant extension of the positive tip. After the first junction around $300 \mu\text{s}$ FA02 measured a positive field change while FA01 continues to change in the negative direction, we were not able to match both of these field changes simultaneously. The tip field also does not match the measured speed well, but with such a poor fit to the field changes it is clear the model is not valid for the full leader development in this case, possibly due to conductivity extending down the other branch at the first junction. Otherwise it is not clear why the field changes are so different for this dart leader.

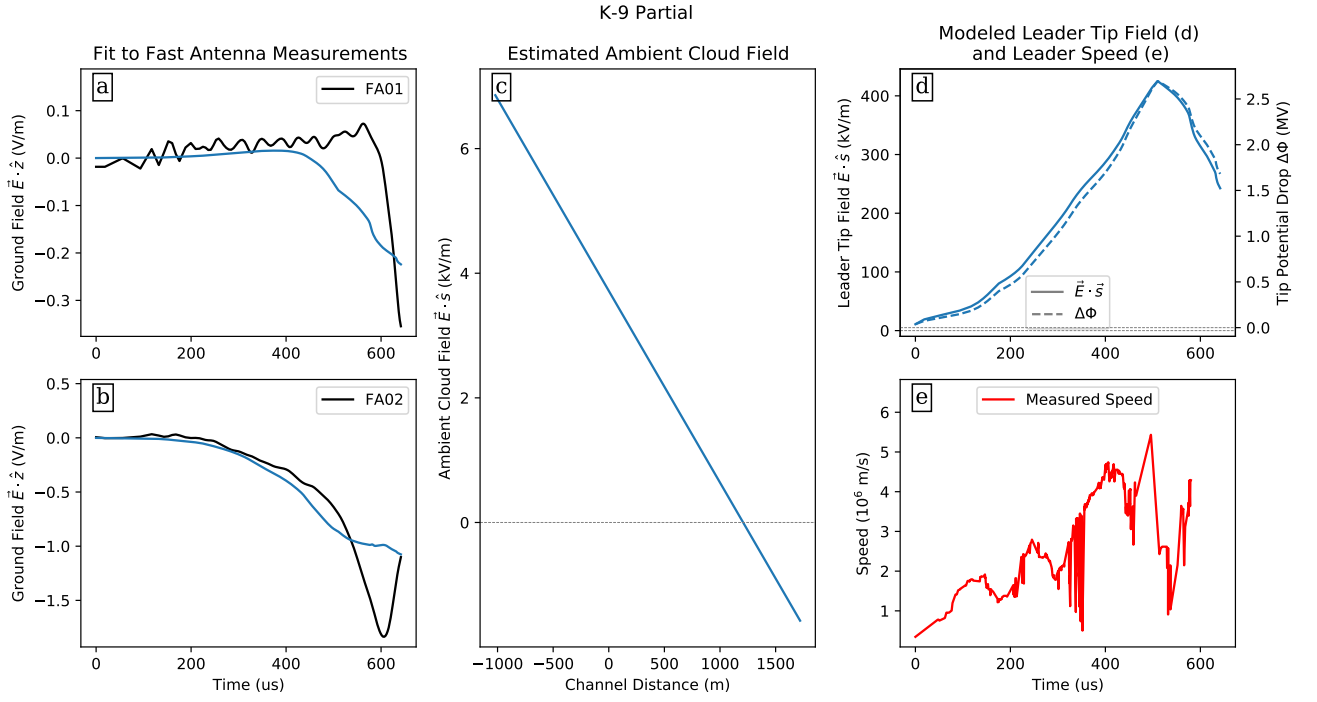


Figure S12. Results for the initial part of dart leader K-9, before it splits into multiple branches, with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does include a 1000 m extension of the positive tip in order to improve the fit to the FA02 field change. Both field changes show a sudden change around 600 μs, probably related to the other branches starting to develop. The tip field is well correlated with the general trend of the leader speed.

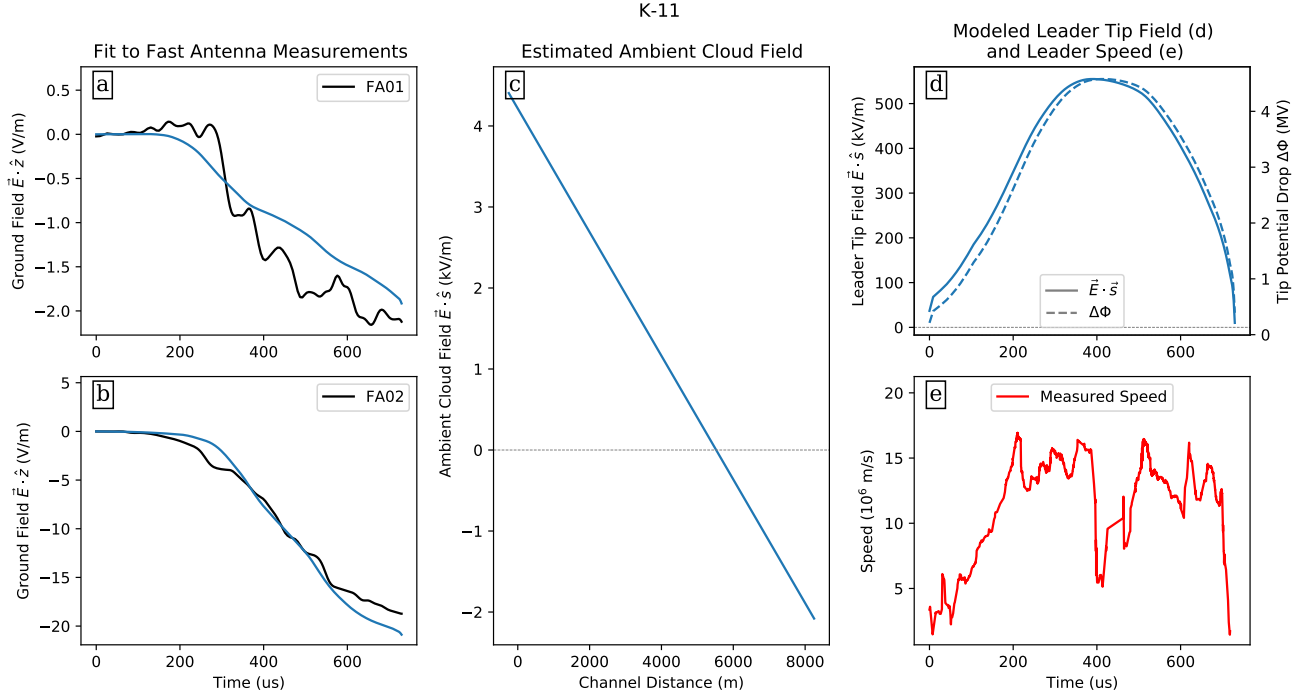


Figure S13. Results for dart leader K-11 with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does not have any significant extension of the positive tip. The fit to the field changes is fairly good, and the modeled tip field is somewhat correlated to the measured speed.

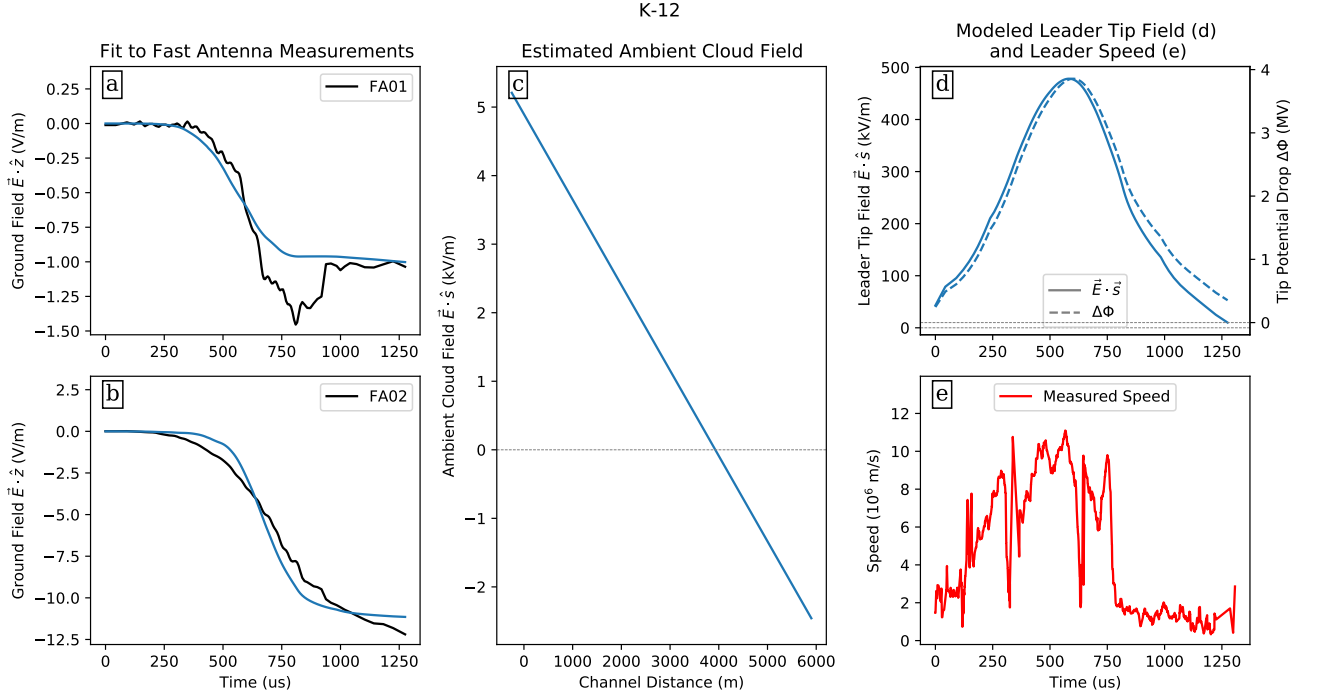


Figure S14. Results for dart leader K-12, with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does not have any significant extension of the positive tip. The fit to the field changes is fairly good, except the temporary deviation of FA01 around 750 μs , and the modeled tip field is somewhat correlated to the measured speed.

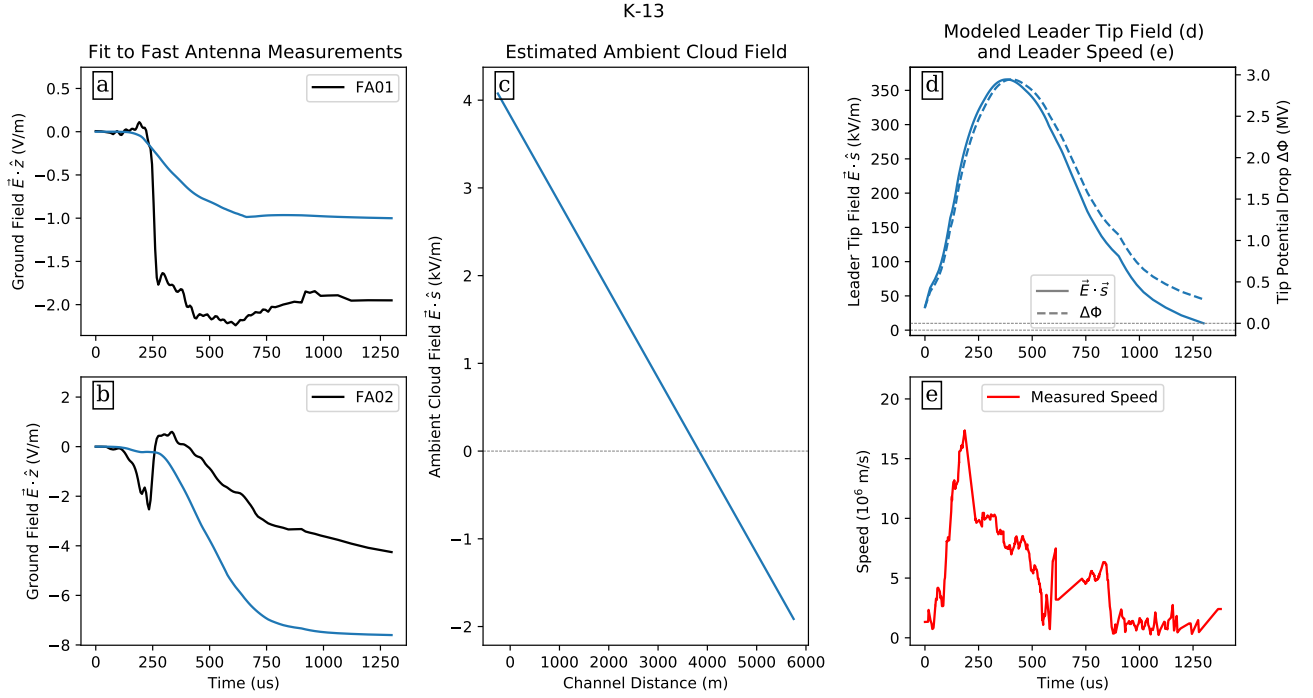


Figure S15. Results for dart leader K-13, with an equipotential channel. Plots show the measured and modeled field change vs time for FA01 (a) and FA02 (b), the estimated ambient field vs channel distance (c), the modeled tip field and tip potential drop vs time (d), and the measured leader speed vs time (e). The modeled leader does not have any significant extension of the positive tip. We were not able to simultaneously fit both the FA01 and FA02 field changes, possibly due to conductivity extending down the other branch at the first junction. Since the model does not fit the measured field changes the rest of the model results are clearly not valid.