The role of surface potential vorticity in the vertical structure of mesoscale eddies

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ABSTRACT: The vertical structure of ocean eddies is generally surface-intensified, commonly attributed to the dominant baroclinic modes arising from boundary conditions (BCs). Conventional BC consideration mostly focuses on either flat- or rough-bottom conditions. The impact of surface buoyancy anomaly—often represented by surface potential vorticity (PV) anomaly—has not been fully explored. Here, we study the role of the surface PV in setting the vertical distribution of eddy kinetic energy (EKE) in an idealized adiabatic ocean model. The simulated EKE profile in the extratropical ocean tends to peak at the surface and have an e-folding depth typically smaller than half of the ocean depth. This vertical structure can be reasonably represented by a single surface quasi-geostrophic (SQG) mode at the energy-containing scale resulting from surface-trapped baroclinic instability and inverse energy cascades. The surface meridional PV gradient—key to oceanic baroclinic instability—induces surface-trapped unstable modes that decay faster with depth for smaller horizontal scales. Subsequently, through an inverse energy cascade from scales close to or smaller than the deformation radius to the energy-containing scale, the vertical structure grows deeper while remaining surface-trapped. These results indicate that the vertical EKE distribution depends on the horizontal scale since smaller eddies tend to be shallower. Guided by this interpretation, an SQG-based scale-aware parameterization of the EKE profile is proposed here. Preliminary offline diagnosis of a high-resolution simulation shows the proposed scheme successfully reproducing the dependence of the vertical structure of EKE on the horizontal grid resolution.

1. Introduction

Ocean mesoscale eddies including horizontal scales of tens to hundreds of kilometers account for a majority of oceanic kinetic energy (e.g., Ferrari and Wunsch 2009; Storer et al. 2022). These eddies are important to the transport and mixing of momentum, heat, salt, carbon, as well as other biogeochemical tracers, and impact the large-scale circulation and climate (Wolfe and Cessi 2010; Marshall and Speer 2012; Griffies et al. 2015; Gnanadesikan et al. 2015). These eddies are yet to be fully-resolved in climate models due to their relatively small size and as a result, their effects must be parameterized. Observational and modeling studies have shown that the strength of mesoscale eddies tend to peak at the surface and decay with depth (Wunsch 1997; de La Lama et al. 2016), with the exception for certain mode-water or topographically trapped eddies that intensify in the interior or near the bottom (Zhang et al. 2017; Radko 2023). Parameterizations that properly account for the surface-intensified vertical structure are crucial for simulations of large-scale ocean circulations and density structures (Danabasoglu and Marshall 2007; Eden et al. 2009). This effort requires a better understanding and representation of the eddy vertical structure, which is the topic of this paper.

The vertical structure of eddy motions is often represented using vertical normal modes (Wunsch 1997; Wortham and Wunsch 2014; de La Lama et al. 2016). Traditionally, these modes—including the barotropic and baroclinic modes—are solutions to an eigenvalue problem with flat-bottom and rigid-lid boundary conditions (Gill 1982; Vallis 2017). The surface-intensified structure of ocean eddies is then decomposed as a linear combination of these eigenmodes in the vertical.

Notably, the barotropic and first baroclinic modes together can capture the bulk of mid-latitude eddy structure and variability (Wunsch 1997; Zhang et al. 2013). That these two modes are dominant is consistent with geostrophic turbulence theory, which reveals that eddies emerging from baroclinic instability tend to transfer energy from higher baroclinic modes to the lower modes and then to the barotropic mode at scales comparable to the deformation radius (Salmon 1980; Smith and Vallis 2001). Due to surface-intensified stratification and bottom friction, ocean eddies are usually not fully barotropized and the first-baroclinic contribution remains considerable (Fu and Flierl 1980; Smith and Vallis 2001). Although the barotropic and first baroclinic modes are useful for diagnosing the vertical structure of eddies, they are inefficient for quantitative predictions since their relative contribution to eddy energy varies substantially in space and time (Wunsch 1997).

Vertical mode structures can change significantly in the presence of bottom topography (Hallberg 1997; Lacasce 2017), leading to recent studies promoting a different set of baroclinic modes subject to zero horizontal velocity at the bottom due to rough bathymetry (Lacasce 2017). The inclusion of the *rough bottom* tends to decouple the bottom pressure from the interior, yielding modified baroclinic structures that are more surface-intensified (Rhines 1970; Hallberg 1997; Samelson 1992; Tailleux and McWilliams 2001). The first baroclinic mode derived under such rough-bottom setup is more consistent with observations than the traditional mode (de La Lama et al. 2016; Ni et al. 2023), and has recently been used for diagnoses and parameterizations of the vertical structure of eddy velocity and mixing (Adcroft et al. 2019; Groeskamp et al. 2020; Stanley et al. 2020; Holmes et al. 2022).

Still, these rough-bottom modes assume a zero buoyancy anomaly at the ocean surface, whereas ocean eddies exhibit strong surface temperature and salinity anomalies (Lapeyre 2009; Hausmann and Czaja 2012; Frenger et al. 2015). Theoretical studies indicate that a complete vertical-mode analysis should include surface buoyancy anomalies that give rise to surface-trapped responses that decay quasi-exponentially from the surface (Smith and Vanneste 2013; Yassin and Griffies 2022a). The surface-trapped mode is governed by the surface quasi-geostrophic (SQG) dynamics in the presence of a surface horizontal buoyancy gradient (Blumen 1978; Held et al. 1995; Lapeyre and Klein 2006; Lapeyre 2017). Previous studies noted that the SQG mode can capture the surface intensification of the observed eddies better than the traditional baroclinic structures, pointing to surface buoyancy anomalies being key to establishing the vertical distribution of ocean eddies (Lapeyre and Klein 2006; Lapeyre 2009).

The SQG framework has been adapted to infer the eddy properties in the vertical from surface observations and hydrography in the midlatitude ocean in both modeling (Lapeyre and Klein 2006; Isern-Fontanet et al. 2008; Wang et al. 2013; Ponte and Klein 2013; Liu et al. 2014; Qiu et al. 2016; Fresnay et al. 2018; Qiu et al. 2020; Miracca-Lage et al. 2022) and observational studies (LaCasce and Mahadevan 2006; Liu et al. 2017). Such inference, however, depends on the spectra of the surface eddy energy or buoyancy—seemingly limiting the application of the SQG framework to the parameterization of eddies for ocean models in which mesoscale features are not fully resolved. Here, we argue that the energy-containing scale comparable to the deformation radius is sufficient to determine the dominant SQG mode and its vertical distribution. Assuming the energy-

containing scale is given and using this in lieu of the full surface spectra, an SQG-based, scale-aware parameterization for ocean eddy is proposed in this study. Focusing on the dependence of vertical structure on horizontal scales, offline analyses with output from an idealized adiabatic ocean model demonstrate that the proposed scheme reproduces the desired distributions of unresolved eddies by models with different horizontal resolutions. The only remaining step for a completely closed parameterization is the determination of the horizontal energy-containing scale.

Why does the SQG mode at the energy-containing scale play such an important role in the vertical eddy structure? The geostrophic turbulence theory provides a potential explanation. Surface buoyancy anomalies are often represented as surface-confined potential vorticity (PV) anomalies (Bretherton 1966). The horizontal PV gradient—important for baroclinic instability—tends to be substantial at the ocean surface, yielding surface-trapped unstable modes (Smith 2007; Fox-Kemper et al. 2008a; Tulloch et al. 2011; Callies et al. 2016; Feng et al. 2021, 2022). Surface-trapped instabilities can efficiently energize submesoscale processes, which in turn energize the mesoscale eddies via an inverse energy cascade (Roullet et al. 2012; Callies et al. 2016; Capet et al. 2016; Schubert et al. 2020; Khatri et al. 2021). That is, a surface PV (buoyancy) gradient can indirectly supply energy to mesoscale eddies via the surface-trapped submesoscale instabilities.

This hypothesized link between baroclinic instability and the vertical structure of mesoscale eddies are also investigated here in an idealized adiabatic ocean model. We examine the eddy kinetic energy (EKE) in circulation regimes analogous to those in the Atlantic Ocean. The simulated EKE exhibits i) a surface-intensified profile consistent with the SQG mode at the energy-containing scale; and ii) a dependence of vertical distribution on the horizontal eddy scale—smaller eddies tend to have shallower structures. Although this model does not have surface buoyancy forcing and a mixed layer, the isopycnal outcropping due to wind forcing gives rise to a strong surface PV gradient, which generates surface-trapped unstable modes. These unstable modes supply energy to the energy-containing eddy through the inverse cascade and in turn determines the EKE profile.

This manuscript is organized as follows. Section 2 covers equations of the baroclinic and surface-trapped modes for theoretical background. The numerical model configuration and WKB solutions for the SQG mode are given in Section 3. In Section 4, we reveal the key role of SQG modes in setting the vertical structure of EKE, which can be determined by the horizontal

energy-containing scale. In Section 5, we argue that the dominance of the SQG mode arises from baroclinic instability and the corresponding inverse energy cascade, which are surface-trapped due to the upper-ocean density structures. Based on these understandings, in Section 6 we propose a SQG-based parameterization for the subgrid EKE profile that depends on the model horizontal resolution. The results are summarized, and their implications for understanding the eddy vertical structure are discussed in Section 7.

2. Theoretical Background

In this section, we recap elements from quasi-geostrophic (QG) theory, focusing on (i) the vertical structure of baroclinic modes; and (ii) modifications to take into account the surface buoyancy anomaly.

a. Baroclinic Modes

The vertical structure of ocean eddies is commonly represented using the vertical normal modes of the linearized QG equations for a quiescent ocean state (e.g., chapter 6 of Vallis 2017),

$$\frac{\partial}{\partial t}\left[\nabla^2\psi + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial\psi}{\partial z}\right)\right] + \beta\frac{\partial\psi}{\partial x} = 0, \tag{1}$$

where $\psi(x,y,z,t)$ is the streamfunction perturbation, f the Coriolis parameter, β the meridional derivative of f, and $N^2 = \partial B/\partial z$ the squared buoyancy frequency defined via the large-scale buoyancy B. Note that ψ and $\partial\psi/\partial z$ are proportional to the pressure and buoyancy anomalies, respectively.

Substituting the wave-like ansatz

$$\psi = \hat{\psi}_0\Phi(z)\exp[i(k_x x + k_y y - \omega t] \tag{2}$$

into equation (1) yields an eigenvalue problem for the vertical structure function Φ

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{f^2}{N^2}\frac{\mathrm{d}\Phi}{\mathrm{d}z}\right) = -\lambda^2\Phi. \tag{3}$$

Here, $\hat{\psi}_0$ is proportional to the 2D Fourier transform of ψ at the surface, k_x and k_y the zonal and meridional wavenumber, respectively, ω the frequency, and λ^2 the reciprocal of squared deformation radius.

Solving equation (3) requires surface and bottom boundary conditions (BCs). It is common to assume the flat-bottom and rigid-lid conditions in which the buoyancy vanishes

$$\frac{\mathrm{d}\Phi}{\mathrm{d}z} = 0 \text{ at } z = 0,\ -H. \tag{4}$$

Equations (3) and (4) constitute a Sturm-Liouville problem for Φ, admitting a set of orthogonal normal modes $\{\Phi_m\}$ with the corresponding eigenvalues $\{\lambda_m^2\}$. Here the index $m \geq 0$ counts the number of zeros of $\Phi_m(z)$ in the interior. The solution with $m = 0$ denotes the barotropic mode, and the infinity of $m \geq 1$ solutions are the baroclinic modes. The first few modes are shown in Fig. 1a as computed with an exponential stratification profile whose e-folding scale is a third of water depth. General solutions of the eddy satisfying equation (1) and the same BCs (4) can be uniquely represented as a linear combination of these modes.

An alternative structural consideration includes the bottom topography (Rhines 1970; Hallberg 1997) with which the horizontal velocity vanishes at the bottom, yielding

$$\frac{\mathrm{d}\Phi}{\mathrm{d}z} = 0 \text{ at } z = 0 \text{ and } \Phi = 0 \text{ at } z = -H. \tag{5}$$

That is, the bottom BC is now imposed on the pressure anomaly. These alternative BCs result in a different set of normal modes shown in Fig. 1b, referred to as the *rough-bottom* modes or "surface modes" in Lacasce (2017) since the bottom horizontal flow is at rest. Notably, there is not a depth-independent barotropic mode among these—an important distinction from the flat-bottom modes in Fig.1a. The barotropic mode is replaced by bottom-trapped topographic waves (e.g., Rhines 1970; Lacasce 2017; Yassin and Griffies 2022a) not illustrated here. The lowest rough-bottom mode ($m = 1$; blue line in Fig.1b)—because it does not change sign in the interior—is often referred as the equivalent barotropic (EBT) mode (Killworth 1992; Hallberg 1997; Adcroft et al. 2019). The EBT mode has been adapted for the parameterization of the vertical distribution of mesoscale eddy diffusivity, e.g., in the 0.5° version of the GFDL ocean climate model OM4.0 (Adcroft et al. 2019).

A notable feature of the two Sturm-Liouville problems considered thus far is that the horizontal wavenumbers do not appear in them, except for the bottom topographic waves. Therefore, the vertical structure of the normal modes do not depend on the horizontal scales. This can be attributed to the surface boundary conditions not including buoyancy anomalies (i.e., $\mathrm{d}\Phi/\mathrm{d}z = 0$ at $z = 0$)—an assumption shared by both of the flat- and rough-bottom setups that is inconsistent with the SST observations exhibiting warm- and cold-core eddies (Lapeyre 2009; Hausmann and Czaja 2012; Frenger et al. 2015).

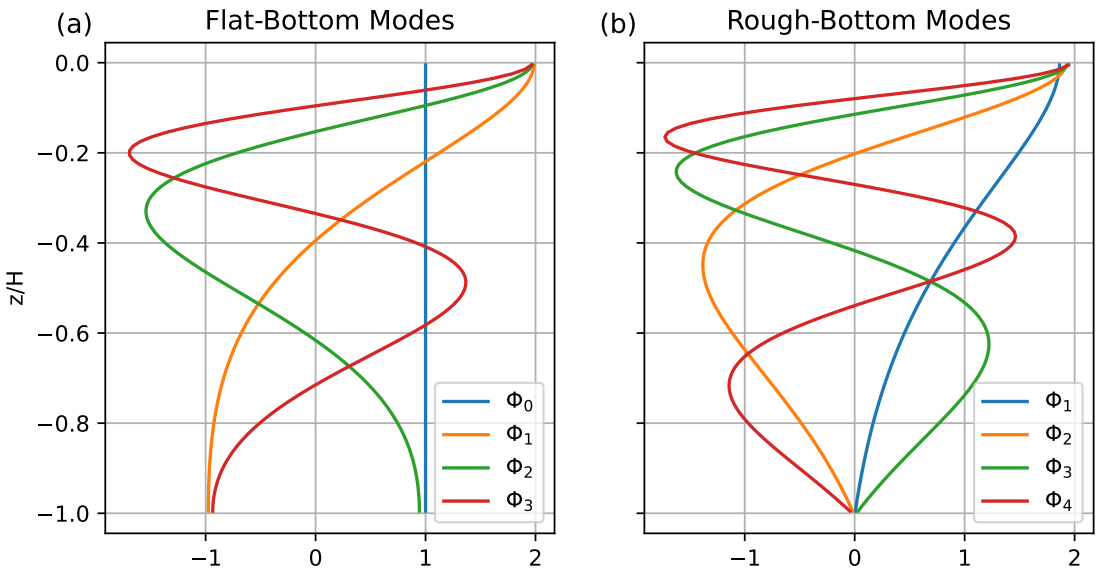


FIG. 1. The first four vertical normal modes solved from the eigenvalue problem (3) with (a) flat bottom and rigid lid boundaries, and (b) rough bottom and rigid lid boundaries. The stratification decays exponentially with depth following an e-folding scale of a third of the water depth in both cases. Each profile of Φ_m is normalized so that $\int_{-H}^{0} \Phi_m^2 \mathrm{d}z/H = 1$. Blue, orange, green, and red lines indicate the first, second, third, and fourth modes. The first mode, Φ_0 in (a) is depth independent and called the barotropic mode, while the other modes are baroclinic (BC) modes. The Φ_0 mode in (b) is represented by bottom-trapped topographic waves, which are not plotted here. The first mode, Φ_1, in (b) is often called the equivalent barotropic (EBT) mode.

b. Surface-Trapped Modes

To incorporate surface buoyancy anomalies, a surface-aware formulation is proposed to include the horizontal buoyancy gradient at the surface (and bottom), as briefly outlined here following Smith and Vanneste (2013) as well as Yassin and Griffies (2022a).

In this modified approach, the eigenvalue problem consists of the same differential equation (3) but with a set of more general BCs

$$\begin{aligned} \frac{f^2}{N^2 H}\frac{\mathrm{d}\Phi}{\mathrm{d}z} &= \alpha(0)(k^2+\lambda^2)\Phi, \text{ at } z=0, \\ \frac{f^2}{N^2 H}\frac{\mathrm{d}\Phi}{\mathrm{d}z} &= -\alpha(-H)(k^2+\lambda^2)\Phi, \text{ at } z=-H, \end{aligned} \tag{6}$$

with a non-dimensional function

$$\alpha(z) \equiv \frac{f}{N^2 \beta k_x}\left(k_x \frac{\partial B}{\partial y} - k_y \frac{\partial B}{\partial x}\right), \tag{7}$$

$k = (k_x^2 + k_y^2)^{1/2}$, and B the large-scale buoyancy. The resulting system differs from the standard Sturm-Liouville problem in that the wavenumbers k, k_x, k_y and eigenvalue λ^2 are now part of the BCs. Following Smith and Vanneste (2013), the most relevant case for our discussion here is the limit where $\alpha(-H) \to 0$ (i.e., the buoyancy anomaly vanishes at the bottom) and $\alpha(0) \to \infty$ (strong horizontal buoyancy gradient and/or weak vertical stratification at the surface). The full set of normal modes in this case consists of a surface-trapped mode $\Phi_s(z;k)$ satisfying

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{f^2}{N^2}\frac{\mathrm{d}\Phi_s}{\mathrm{d}z}\right) &= k^2\Phi_s, \\ \Phi_s|_{z=0} &= 1, \\ \left.\frac{\mathrm{d}\Phi_s}{\mathrm{d}z}\right|_{z=-H} &= 0, \end{aligned} \tag{8}$$

and the interior modes $\Phi_m(z)$ (for integral index $m \geq 1$)

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{f^2}{N^2}\frac{\mathrm{d}\Phi_m}{\mathrm{d}z}\right) &= -\lambda_m^2 \Phi_m, \\ \Phi_m|_{z=0} &= 0, \\ \left.\frac{\mathrm{d}\Phi_m}{\mathrm{d}z}\right|_{z=-H} &= 0. \end{aligned} \tag{9}$$

A few examples of these modes are illustrated in Fig. 2. Together, the admissible Φ_s and Φ_m form an orthogonal basis, which is different from the decomposition of eddy flow into surface and interior solutions by Lapeyre and Klein (2006). See Appendix A for detailed comparison.

Note the similarity between equation (9) and the earlier rough-bottom setup (the surface and bottom BCs are switched here). Especially, the vertical structure of the interior modes does not depend on horizontal scales (see Fig. 2b).

In contrast, equation (8) explicitly includes the horizontal wavenumber, yielding an outstanding feature of the surface-trapped mode: Larger eddies (with smaller horizontal wavenumber k) tend to be associated with deeper modes whose vertical response decays slower with depth; see Fig. 2a. Because Φ_s has a zero interior PV anomaly and is driven by a surface buoyancy anomaly—consistent with the surface QG (SQG) dynamics (Blumen 1978; Held et al. 1995)—it is also referred to as the "SQG mode." The SQG mode is relevant to the oceanic cases where horizontal buoyancy gradient intensifies near the surface (Smith 2007; Capet et al. 2016), and is central for this paper.

Both of the surface-trapped and interior modes described above assume a flat-bottom condition, i.e., vanishing buoyancy anomaly; recall the bottom BCs in equations (8) and (9). These are the limiting cases of equation (6) through which more complicated topographic effects can be included (Lacasce 2017; Yassin and Griffies 2022a) and will be explored in future work.

3. Methods

a. Model Configuration

The numerical model used in this study is an idealized configuration of the Modular Ocean Model version 6 (MOM6). This configuration is named "Neverworld2" by Marques et al. (2022) and has been used to study mesoscale eddy parameterizations (e.g., Loose et al. 2022; Yankovsky

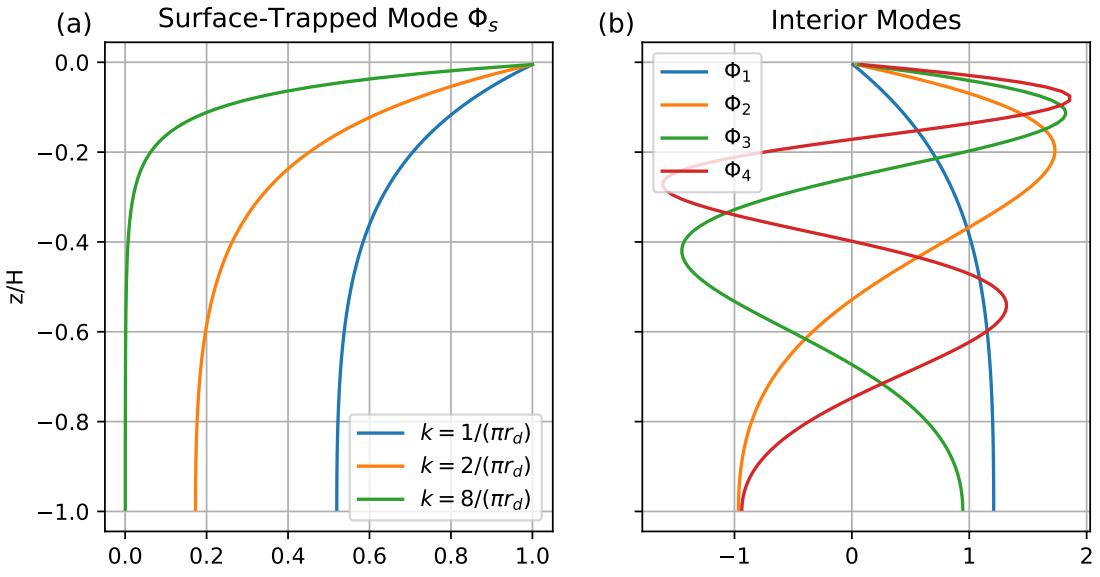


FIG. 2. Vertical profiles of the (a) surface-trapped mode and (b) interior modes, determined by solving equations (8), and (9), respectively. The stratification profile is an exponential function of z, as in Fig. 1. The surface mode is calculated for three different horizontal wavenumbers, $1/(\pi r_d)$, $2/(\pi r_d)$, and $8/(\pi r_d)$, where r_d is the Rossby deformation radius of the first flat-bottom baroclinic mode (orange line in Fig. 1a). Each surface-trapped mode profile is normalized by its surface value. The lines in panel (b) indicate the first four interior modes, each of which is normalized in the same way as the modes in Fig. 1.

et al. 2022). The model domain is a single basin with two hemispheres using spherical coordinates. The domain extends from $-70°$S to $70°$N in the meridional direction, $0-60°$ in the zonal direction, and has a maximum depth of 4000 m. The domain is bounded by a 200-m continental shelf along all side boundaries, except for a zonally reentrant channel located at $60°$S to $40°$S, mimicking the Southern Ocean. A ridge topography orienting meridionally spans the entire meridional extent of the domain. The ridge has a width of $40°$ and maximum height of 2000 m. There is a semi-circular ridge with radius of $10°$ spanning $0°$ to $10°$ longitude and $60°$S to $40°$S latitude that mimics the Scotia Arc across the Drake Passage.

The hydrostatic primitive equations are discretized in the vertical using isopycnal coordinates, with a total of 15 isopycnal layers with finer spacing near the surface in the initial setup. The horizontal grid spacing is $1/32°$. The model is driven solely via mechanical forcing from a zonally uniform zonal wind stress that varies meridionally and is fixed in time. There is no surface buoyancy

forcing, no diabatic mixing, nor is there a mixed layer parameterization (i.e., it is a wind-driven stacked shallow water model). Dissipation arises from a background kinematic vertical viscosity ($A_v = 1.0 \times 10^{-4}\,\mathrm{m^2\,s^{-1}}$), a dimensionless bottom drag ($C_d = 0.003$), and the horizontal friction given by a biharmonic Smagorinsky viscosity (Griffies and Hallberg 2000). More details about the model setup and spinup are given in Marques et al. (2022).

Fig. 3a shows a snapshot of the surface specific kinetic energy (KE). The black lines correspond to the 500-day mean sea surface height (SSH) contours, representing the streamlines of surface geostrophic currents outside the tropics. The circulation pattern is a caricature of the Atlantic Ocean, with subtropical gyres and western boundary currents in both hemispheres. In the northern hemisphere, there is a subpolar gyre, while in the southern hemisphere, a circumpolar current prevails in the reentrant channel. The KE illustrates the prevalence of mesoscale eddies throughout the domain. The Rossby deformation radius is resolved over most regions of the model, with the exception being the continental shelf and very high latitudes near the boundaries (Yankovsky et al. 2022).

b. Solution for the SQG Mode

The SQG mode, Φ_s, described by equations (8), is diagnosed in the stacked shallow water model introduced above. In this model, the dynamical impact of surface buoyancy is represented by the surface-layer PV, as described by the layered QG dynamics detailed in Appendix B.

The SQG mode can be determined numerically for realistic stratification profiles. We can also derive an analytical solution using the WKB approximation. To get a simple analytical form, we assume $\Phi_s \to 0$ if $z \to -\infty$. For each horizontal wavenumber k, the WKB solution, Φ_s^{WKB}, is given to the first order by,

$$\Phi_s^{\mathrm{WKB}} = e^{k z_s}, \tag{10}$$

where

$$z_s = -\int_z^0 \frac{N}{|f|}\, dz, \tag{11}$$

is the stretched vertical coordinate with $z < 0$ and $z_s < 0$ in the ocean interior.

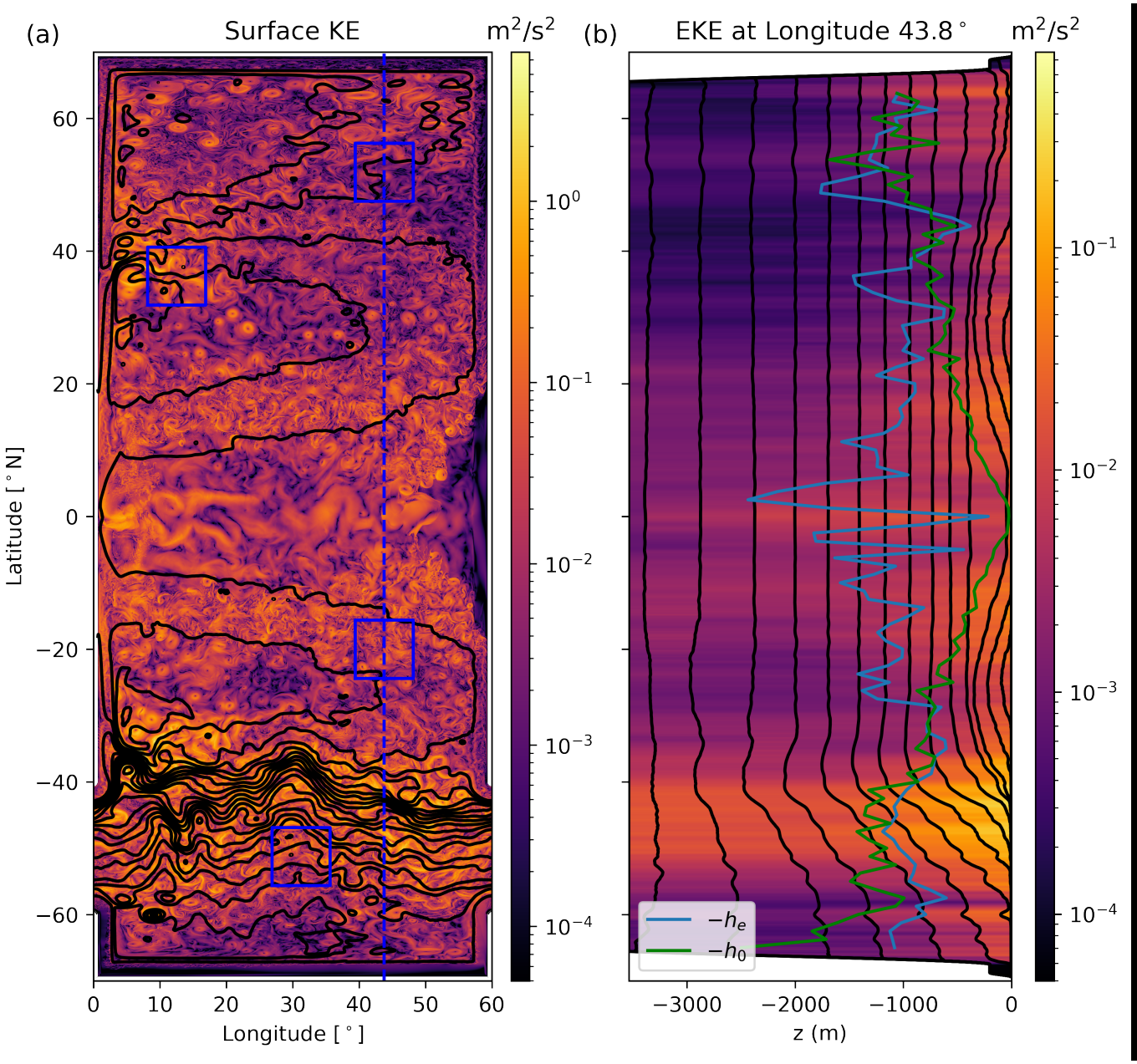


FIG. 3. (a) Snapshot of horizontal kinetic energy at the surface. Black lines are contours of the 500-day mean sea surface height (SSH). The four blue boxes indicate the $8.7° \times 8.7°$ regions where the vertical structure of eddies is analyzed in later figures. Panel (b) shows a meridional section of the eddy kinetic energy per unit mass along the longitude shown by the blue dashed line in panel (a). Black lines show all the isopycnals in the model. Blue lines indicate the e-folding depth, h_e, of the rms eddy velocity (square root of EKE). Green line is the deformation depth, h_0, of the energy-containing scale, introduced in section 4b.

The Fourier transform of the streamfunction field associated with the SQG mode, $\hat{\psi}_s$, is

$$\hat{\psi}_s^{\mathrm{WKB}} = \hat{\psi}_0 e^{k z_s}, \tag{12}$$

where $\hat{\psi}_0$ is the same as that in equation (2), which is the Fourier transform of the streamfunction at the surface.

Equation (10) reveals important characteristics of the SQG mode, namely, its exponential decay with z_s and its dependence on the horizontal scale of eddies. Smaller eddies with larger wavenumber, k, decay faster with depth than larger eddies with smaller wavenumber. The scale dependence of the SQG mode provides valuable insights for parameterizating the vertical structure of eddies, as we discuss in Section 6.

Note that equation (10) differs from the widely used "eSQG" method in ocean studies (Lapeyre and Klein 2006). The eSQG method assumes a constant N with depth, determined empirically to account for the impact of an internal PV gradient (Lapeyre and Klein 2006). In contrast, our formula accounts for the vertical variation of N within the stretched vertical coordinate z_s, allowing for a more accurate representation of the SQG mode itself. See also Yassin and Griffies (2022b) for more on SQG theory with vertically varying stratification.

4. Vertical Structure of EKE

We analyze the vertical structure of EKE, defined as

$$\mathrm{EKE} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2}\right), \tag{13}$$

where $\bar{\cdot}$ indicates a 500-day mean, and $\cdot'$ is the anomaly from the mean and is saved in snapshots in 5-day intervals. Fig. 3b shows the EKE along a meridional section indicated by the blue dashed line in Fig. 3a. The EKE tends to be surface-intensified. It maximizes at the surface and rapidly decays with depth in the gyre and low latitudes, while it remains significant in the deep ocean in the circumpolar current. To examine the detailed vertical structure, we select four distinct dynamical regions, highlighted by the four blue boxes in Fig. 3a, that are located in the circumpolar current, subtropical gyre, western boundary current extension, and subpolar gyre. By examining

the vertical profile of EKE in these different regions, we aim to identify the dynamics that controls the eddy vertical structure.

a. Comparison to the Vertical Modes

Vertical profiles of EKE in the four selected regions are shown in Fig .4. The EKE decays rapidly with depth in the upper ocean and gradually approaches a constant in the deep ocean. The barotropic component (i.e., the depth-independent flat-bottom mode) accounts for about 78% of the total EKE in the circumpolar current region and about 50% in the other three regions.

The vertical structure of EKE is compared to the structure predicted by the EBT mode (Section 2a),

$$\mathrm{E_{EBT}} = \mathrm{E_0}\,\Phi^2_{\mathrm{EBT}}, \tag{14}$$

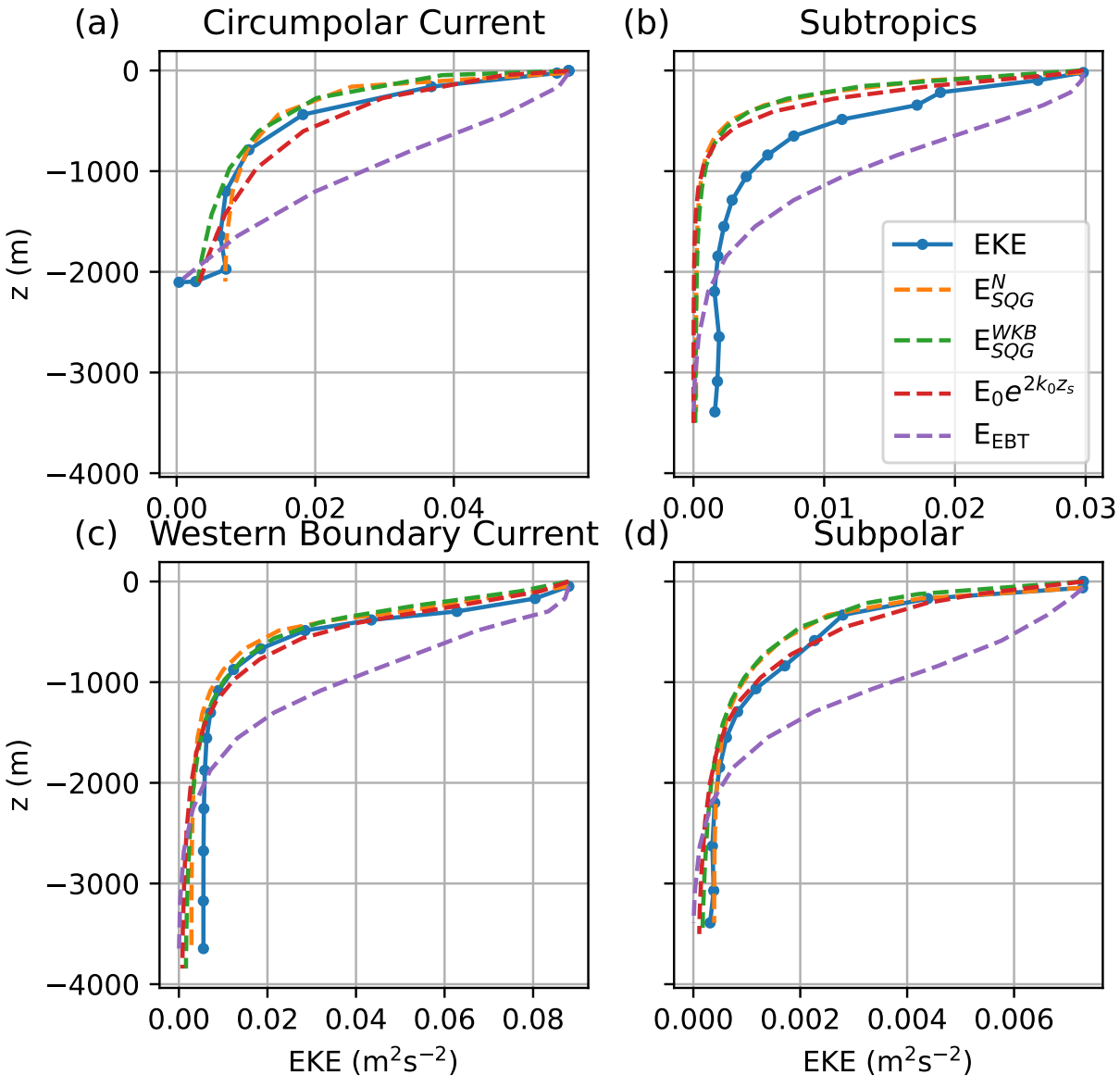
where $\mathrm{E_0}$ is the surface EKE. The square is taken because Φ_{EBT} describes the vertical structure of eddy velocity [see equation (2)].

The profile of $\mathrm{E_{EBT}}$ is shown by the purple dashed line in Fig. 4. The EBT mode is surface-intensified, but it decays more slowly with depth than the diagnosed EKE. Furthermore, the EBT mode has zero vertical gradient at the surface due to its boundary condition in equation (5), whereas the EKE has a strong vertical gradient near the surface. Another limitation is that the magnitude of the EBT mode is zero at the bottom, but the EKE can be significant at the bottom even with strong gradients in the bottom topography.

The vertical gradient of eddy velocity near the surface indicates that the surface-layer PV plays an important role in the vertical structure of EKE, thus motivating us to compare the vertical structure of EKE to the SQG mode. The WKB solution of the SQG mode equation (10) gives the vertical structure of the SQG mode at each wavenumber, $\hat{\psi}_s^{\mathrm{WKB}}$. If we know the surface streamfunction spectrum, $\hat{\psi}_0$, the vertical structure of EKE reproduced by the SQG mode, $\mathrm{E^{WKB}_{SQG}}$, is

$$\mathrm{E^{WKB}_{SQG}} = \frac{1}{2}\sum_k k^2 |\hat{\psi}_s^{\mathrm{WKB}}|^2 = \frac{1}{2}\sum_k k^2 |\hat{\psi}_0|^2 e^{2kz_s}. \tag{15}$$

The green dashed lines in Fig. 4 show the WKB solution of the SQG mode, $\mathrm{E^{WKB}_{SQG}}$, where $\hat{\psi}_0 = g\hat{\eta}/f$ is estimated from Fourier transform of the SSH field, η, within the four regions indicated by the blue boxes in Fig. 3a. To check the accuracy of the WKB solution, we also compute the



287 FIG. 4. Vertical profiles of diagnosed EKE (solid blue lines with dots) in the four regions shown in figure 3.
288 The four regions are located in the (a) circumpolar current, (b) subtropical gyre, (c) western boundary current,
289 and (d) subpolar regions. Dashed orange and green lines are the numerical and WKB solutions of the SQG
290 mode, respectively, summed over horizontal wavenumbers of the surface EKE spectrum. Red dashed line is the
291 WKB solution of the SQG mode at the energy-containing scale. Purple dashed line is the vertical structure of
292 the EBT mode.

315 numerical solution of $\hat{\psi}_s$. The numerical solution of SQG EKE, $\mathrm{E}^{\mathrm{N}}_{\mathrm{SQG}}$, is plotted as the orange
316 dashed line, which is close to the WKB solution. The SQG mode captures the near exponential

decay of EKE well in all the regions, with the exception of the subtropics, where EKE decays slower than the SQG mode (though faster than the EBT mode). The discrepancy in the subtropics indicates a nonnegligible role of interior modes, equation (9). Overall, the SQG mode matches the vertical structure of EKE better than the EBT mode. This result suggests that the vertical structure of EKE is mainly controlled by the surface PV, in which case the vertical structure can be reconstructed using the SQG mode as described by equations (8). This result is consistent with previous studies showing how the eddy vertical structure can be reconstructed by the SQG mode in realistic ocean simulations (Klein et al. 2009; Isern-Fontanet et al. 2008; Qiu et al. 2016, 2020).

b. Relating to the Energy-Containing Scale

Reconstruction of the vertical structure based on the SQG mode, equation (15), requires knowledge of the surface energy spectrum, $k^2|\hat{\psi}_0|^2$, which is unavailable in coarse-resolution simulations. The surface energy spectrum is usually dominated by a peak, which indicates the energy-containing scale (surface energy spectra in the four regions are shown in Fig. 5). This feature suggests that the eddy vertical structure may also be dominated by the energy-containing eddy. The energy-containing wavenumber, k_0 can be estimated following Thompson and Young (2006) and Zhang and Wolfe (2022),

$$k_0 = \sqrt{\frac{\overline{\left\langle |\nabla \eta'|^2 \right\rangle}}{\overline{\left\langle \eta'^2 \right\rangle}}}, \tag{16}$$

where η' is the spatial SSH anomaly, and $\langle\rangle$ indicates a spatial average over each of the four box regions in Fig. 3.

The k_0 estimated by equation (16) is shown by the black dashed line in Fig.5. This estimate is generally close to the peak of the energy spectrum. We then estimate the SQG mode structure at the energy-containing scale,

$$\mathrm{E}_{\mathrm{SQG}}^{k_0} = \mathrm{E}_0 e^{2k_0 z_s}, \tag{17}$$

which can be calculated at each grid point without Fourier analysis.

The vertical structure given by equation (17) is shown by the red dashed line in Fig. 4. It works similarly well as the SQG solution based on the full energy spectrum, meaning that the vertical structure of EKE can be represented by the SQG mode structure at the energy-containing scale.

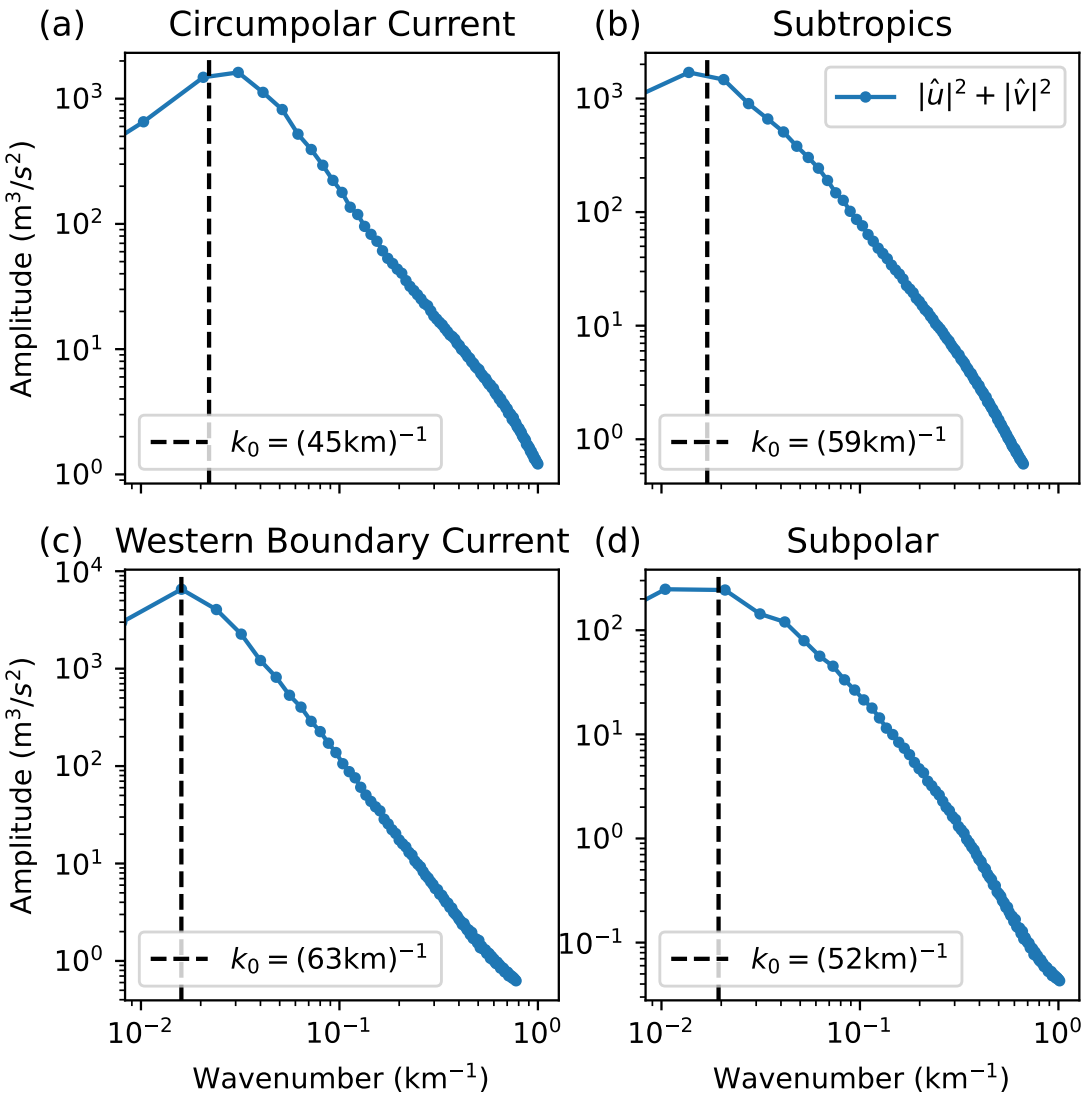


FIG. 5. (a)-(d) 1D surface kinetic energy spectrum (blue lines with dots) in the four regions shown in figure 3. The spectrum, $|\hat{u}|^2+|\hat{v}|^2$ is calculated by Fourier transforming the 2D instantaneous velocity fields in $8.7^\circ \times 8.7^\circ$ windows and then averaged over 500 days. The 2D spectrum is then azimuthally integrated to obtain the 1D spectrum. Black dashed line indicates the energy-containing wavenumber k_0, estimated from (16) at the center of the four regions.

Equation (17) also indicates a relation between the horizontal and vertical scales of EKE. Here we measure the vertical scale as the e-folding depth, h_e, of the eddy velocity (i.e., the square root

of EKE). From equation (17), we can also estimate the e-folding depth of the SQG mode, h_0,

$$\int_{-h_0}^{0} \frac{N}{|f|} dz = k_0^{-1}, \tag{18}$$

which is the deformation depth of the energy-containing wavenumber k_0 (Pierrehumbert and Swanson 1995). If equation (17) is a good approximation of the EKE vertical structure, h_0 should be equal to h_e.

The e-folding depth, h_e, and equivalent energy-containing depth, h_0, are compared in the meridional section in Fig. 3b. They compare well to each other in the extra-tropical regions (poleward of 20°), which is consistent with the good comparison between the EKE vertical structure and SQG mode in these regions in Fig. 4. In the tropics, h_0 decreases to zero toward the equator due to the decreasing magnitude of Coriolis parameter, $|f|$, in equation (18), while h_e remains around 1000-2000m. The large fluctuations of h_e in the tropics might be due to the spatial variability of strong undercurrent and equatorial waves.

The comparison between h_e and h_0 at other meridional sections is similar to that in Fig. 3b. The SQG mode captures the vertical structure of EKE well in the extra-tropical regions, where the eddy horizontal and vertical scales are coupled in the way described by equation (18). The e-folding depth for the eddy velocity is generally shallower than a half of the water depth in the extra tropics, indicating that eddies are shallow. The SQG mode does not capture the vertical structure of EKE well in the tropics. Fig. 3a shows that the kinetic energy pattern is wave-like in the tropics, indicating that EKE is dominated by linear waves there, while in the extra tropics, the flow field contains abundant coherent vortices, indicating the dominance of nonlinear eddies. Previous studies have shown that the eddy flux is dominated by linear waves and nonlinear eddies in the tropics and midlatitude ocean, respectively, leading to different scalings of eddy mixing in these regions (Klocker and Abernathey 2014; Zhang and Wolfe 2022). We expect that equation (17) is effective in describing the vertical structure of EKE in the midlatitude ocean.

5. Why is the Vertical Structure SQG-like?

In section 4, we have seen that the EKE structure tends to be SQG-like. Here, we discuss the underlying physics by showing: (i) the EKE profile is consistent with the distributions of the eddy PV and large-scale meridional PV gradient; (ii) the surface meridional PV gradient yields

baroclinic instability that produces surface-trapped unstable modes; (iii) the kinetic energy created by baroclinic instability at small scales cascades toward the energy-containing scale in the form of surface-trapped mode, resulting in an SQG-like EKE vertical structure.

a. PV Structure

SQG prescribes zero interior PV anomaly. To examine this assumption, the rms eddy PV, q_{rms}, is computed as

$$q_{\mathrm{rms}} = \sqrt{\overline{q'^2}}, \tag{19}$$

where q is the QG PV, estimated by equation (B1) in Appendix B. Fig. 6a shows the distribution of the rms eddy PV at the same meridional section as Fig. 3b. The eddy PV peaks at the surface layer and becomes about an order of magnitude smaller in the interior layers, which is consistent with the SQG-like EKE structure in the midlatitudes. In the subtropics ($10^\circ - 25^\circ$), the PV also intensifies at about 400 m, which might be why the EKE structure is less SQG-like in these regions.

The distribution of eddy PV is associated with the large-scale meridional PV gradient, Q_y, structure, shown in Fig. 6b. The meridional PV gradient is estimated from equation (C3) in Appendix C for the layered shallow water model. The magnitude of meridional PV gradient peaks at the surface due to the isopycnal outcropping. The interior PV gradient is weak, especially in the upper 1000 m in the mid- and high-latitudes (latitude poleward of 25°). The weak interior PV gradient is likely due to the isopycnal mixing by eddies, which tend to homogenize the PV in the interior (Holland and Rhines 1980; Rhines and Young 1982). In the circumpolar current, the magnitude of PV gradient also intensifies in the near-bottom layers, which is consistent with the intensification of eddy PV in these regions. In the subtropics, the PV gradient also peaks at about 400 m due to the flattening of isopycnals at the interior pycnocline. This pronounced interior PV gradient is likely the cause of the strong subsurface eddy PV and the deviation of EKE profile from SQG in the low latitudes. Quantitative analyses of the impact of PV gradient on the eddy vertical structure is given in the next section.

In addition to variations of magnitude in the vertical, the meridional PV gradient also tends to change sign from surface to the interior. In the mid- and high-latitudes, the meridional PV gradient is positive at the surface layer, negative at layers below surface, and positive again in the deep ocean. In the subtropics, the PV gradient is negative at the surface layer and positive in the

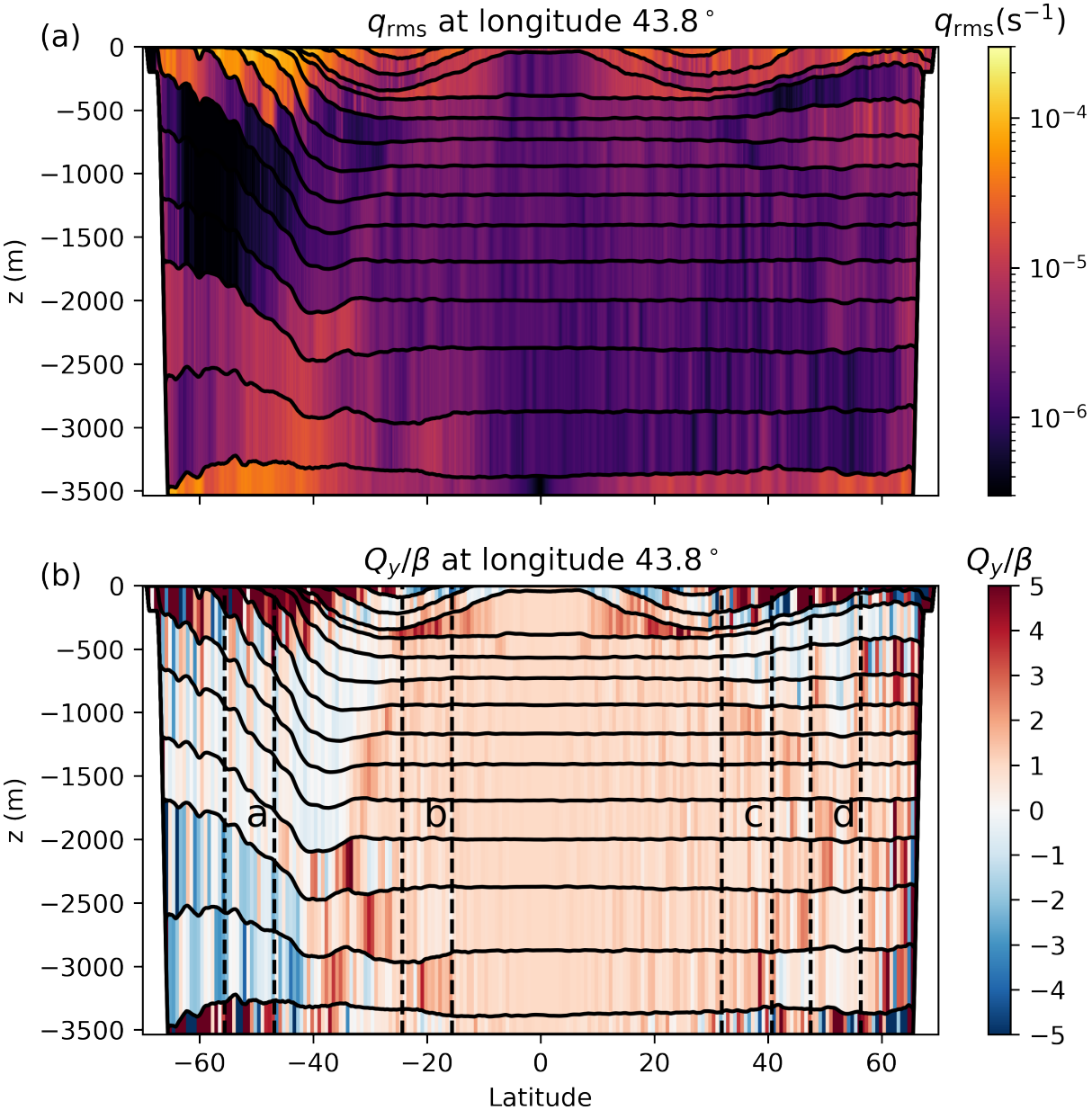


FIG. 6. Meridional and vertical distribution of (a) the rms eddy PV and (b) the meridional PV gradient normalized by β at the longitude indicated by the blue dashed line in Fig. 3a. Black solid lines are the isopycnals in the model. Black dashed lines with characters a,b,c,d indicate the latitude range of the four regions shown in Fig. 3. Note that a and c are not in the same longitude as this meridional section.

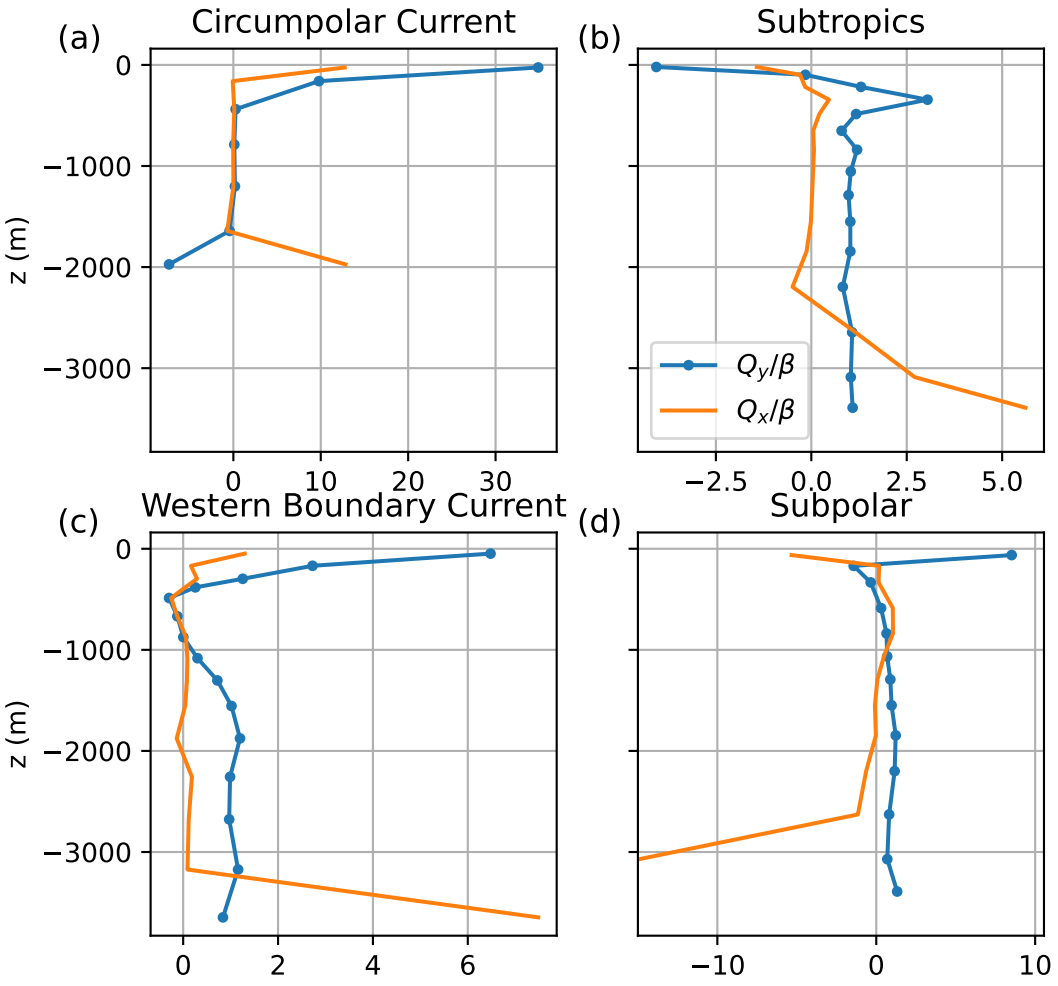
interior. A similar distribution of the meridional PV gradient is also found by Tulloch et al. (2011) in the schematic of their figure 2, which is based on reanalysis climatology. However, the positive

PV gradient is more confined near the surface in the midlatitude regions in our simulation than in Tulloch et al. (2011). This feature is found to be common in the mode water regions (e.g., western boundary current extension and Southern Ocean) by Capet et al. (2016) using observations. These regions tend to have weak stratification layers right above the main thermocline, which squeezes the positive PV gradient to the surface and creates negative meridional PV gradient below it, consistent with the midlatitude PV gradient distribution in the simulation.

b. Linear Stability

The meridional PV gradient is often found to play a crucial role in oceanic baroclinic instability, which is an important energy source for mesoscale eddies (Smith 2007; Tulloch et al. 2011; Capet et al. 2016). Baroclinic instability occurs when the background horizontal PV gradient changes sign in the vertical, which is a typical feature of the meridional PV gradient shown in Fig. 6b.

We analyze baroclinic stability in the four regions discussed in section 4. The vertical profiles of the meridional and zonal PV gradients in the four regions are shown in Fig. 7. Fig. 8 shows the distribution of the growth rate, σ, of unstable modes, which are solved from the eigenvalue problem described by equation (C6) in Appendix C. Fig. 9 shows the vertical structure of the most unstable mode, indicated by the cyan point in Fig. 8. In the circumpolar current, the interior PV gradients are about ten times smaller than the surface and bottom gradients. The surface and bottom meridional PV gradients are opposite to each other, which gives rise to unstable modes at scales larger than the deformation radius. The vertical structure of the most unstable mode intensifies at both surface and bottom, which is a typical feature of the Eady-type instability arising from the interaction between the surface and bottom edge waves (Eady 1949). In the other three regions, both the meridional and zonal PV gradient changes sign near the surface, and the meridional PV gradient is generally stronger than the zonal gradient. The most unstable mode occurs at scales smaller than the deformation radius in these three regions. The vertical structure of the most unstable mode is surface-intensified, indicating a Charney-type instability arising from the interaction between surface edge waves and interior Rossby waves (Charney 1947; Tulloch et al. 2011; Capet et al. 2016). In the subtropics, the unstable mode also intensifies at around 400m, which is associated with the subsurface local maximum of the meridional PV gradient. This PV gradient maximum



421 FIG. 7. (a)-(d) Vertical profiles of meridional (blue lines with dots at the layer centers) and zonal (orange lines)
422 PV gradient normalized by β in the four regions shown in Fig. 3.

440 occurs at the top of the pycnocline where isopycnals start to become flat at about 400m depth in
441 the column labeled by b in Fig. 6.

442 In the western boundary current and subpolar regions, although the meridional PV gradient also
443 changes sign at 500–1000 m depth, nearly all unstable modes are surface-intensified in these two
444 regions (not shown), indicating that the instability is always associated with surface edge waves.
445 The Rossby wave propagates eastward in the negative PV gradient layers and westward in the
446 positive PV gradient layers below (the zonal mean flow is eastward above 1000 m and becomes
447 weak below), so they cannot couple to each other due to the mismatch between their zonal phase

speeds. Consequently, the sign change of PV gradient at 500–1000 m does not lead to the Phillips type instability in the interior (Phillips 1954).

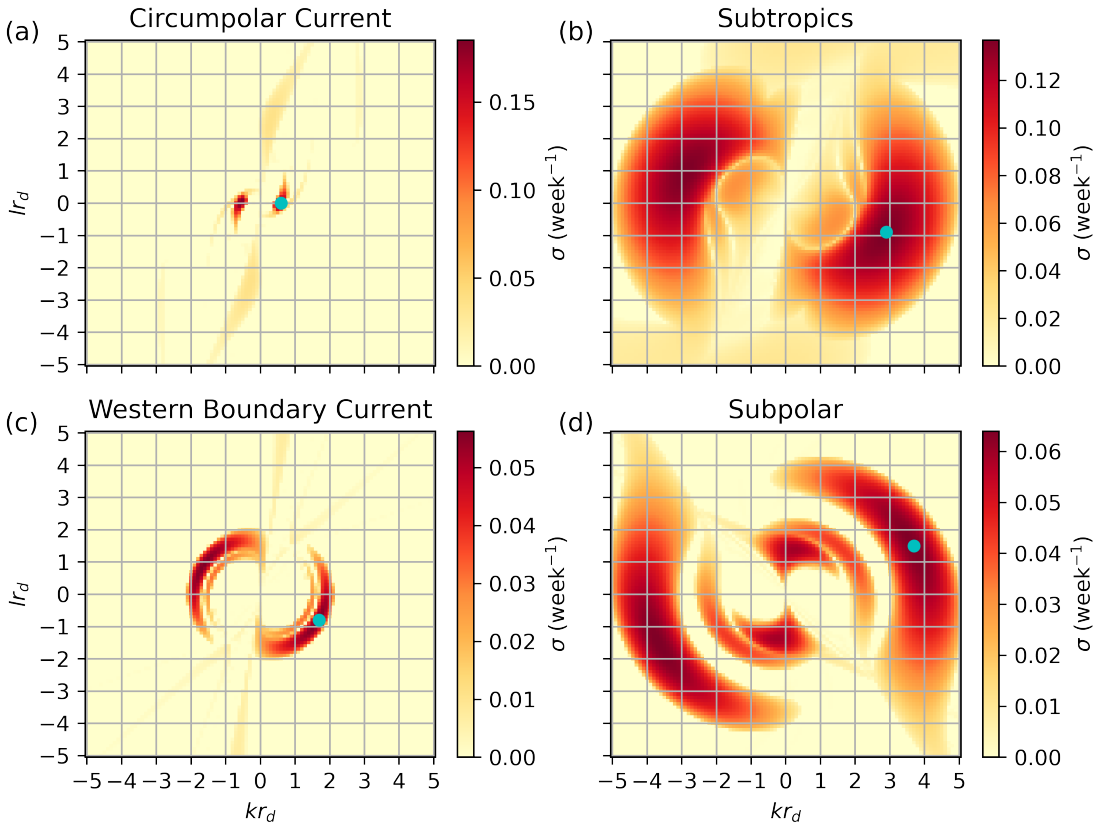


FIG. 8. (a)-(d) Growth rate of unstable modes as a function of zonal and meridional wavenumbers, k_x and k_y, respectively, that are normalized by the deformation radius in the same four regions as in Fig. 3. Cyan point indicates the most unstable mode.

The vertical structure of the most unstable mode is compared to the SQG mode at the same horizontal wavenumber in Fig. 9. A resemblance between the unstable mode and the SQG mode is found in the western boundary current and subpolar regions, consistent with the comparison between the SQG mode and the vertical structure of EKE. In the subtropics, the unstable mode is similar to the SQG mode, with the exception of its subsurface intensification, likely attributed to the intensified meridional PV gradient at around 400 m due to the pycnocline. Such significant subsurface PV gradient might also contribute to the deviation of the vertical structure of EKE from the SQG mode in this region (Fig. 4b). In the circumpolar current, the SQG mode captures the vertical structure of unstable mode in the upper ocean, but fails to capture the bottom intensification

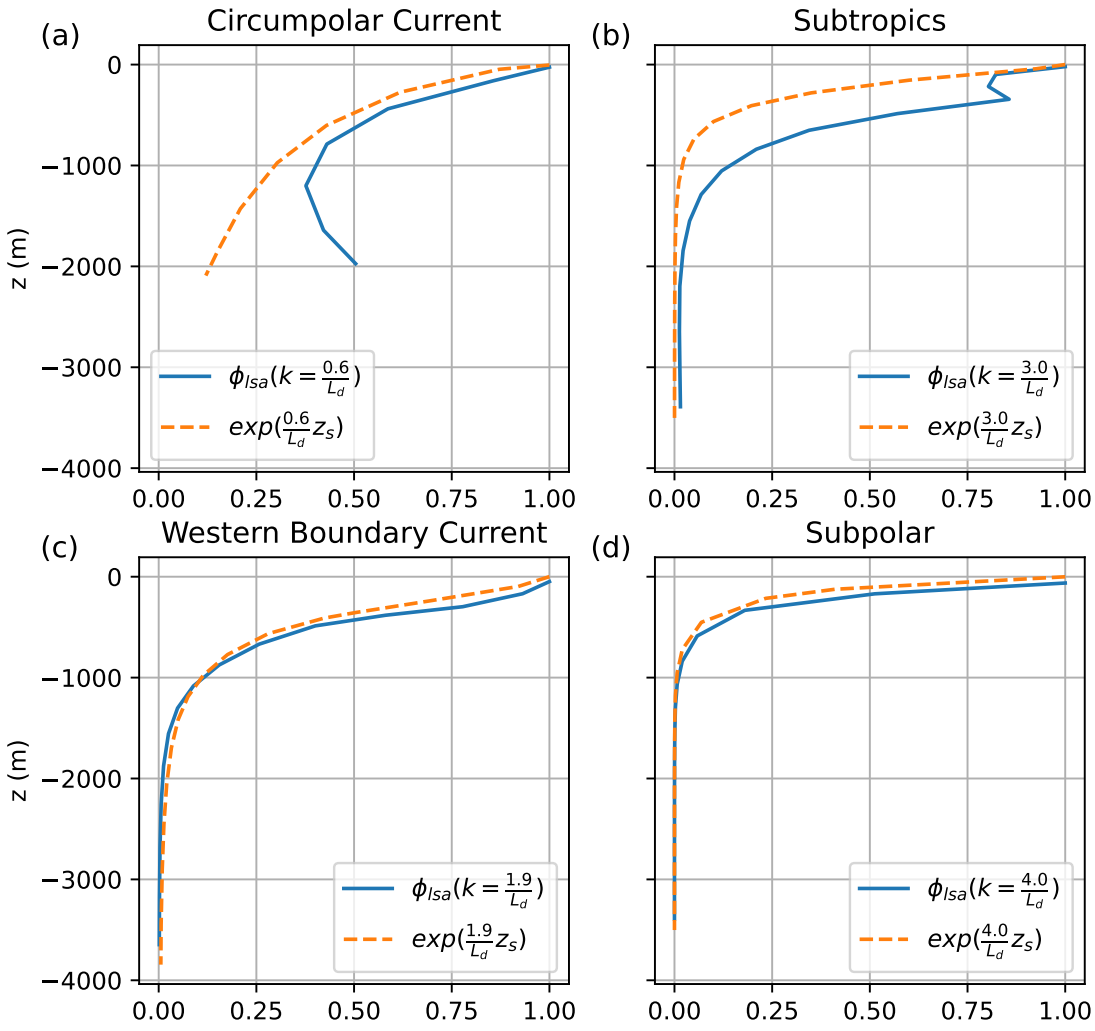


FIG. 9. (a)-(d) Vertical structure of the most unstable mode (blue solid line) indicated by the cyan point in Fig. 8. Orange dashed line is the normalized vertical structure of the SQG mode at the same wavenumber as the unstable mode.

of the unstable mode. Conversely, the vertical structure of EKE only shows a slight intensification near the bottom and appears more like the SQG mode than the unstable mode in this region (Fig. 4a). This discrepancy is likely due to the strong damping of EKE by friction at the bottom, thereby reducing the bottom intensification of EKE.

The resemblance between the unstable modes and SQG mode is attributed to the impact of the strong surface meridional PV gradient, which gives rise to strong surface edge waves that play a more important role in the vertical structure of unstable modes than the interior Rossby

waves. While the stability analysis is based on linear equations and local background states, it offers insights into the vertical structure of EKE. The relationship between the EKE and baroclinic instability will be further discussed in the subsequent section.

c. Role of the Energy Cascade

The linear stability analysis does not account for the energy cascade due to nonlinear interactions. We here diagnose the kinetic energy budget to identify the role of nonlinear advection in the eddy vertical structure. The layer-wise KE equation is

$$\frac{\partial \mathrm{KE}_n}{\partial t} = -\mathbf{u}_n \cdot (\mathbf{u}_n \cdot \nabla \mathbf{u}_n) - \mathbf{u}_n \cdot \nabla M_n + \mathbf{u}_n \cdot \mathbf{F}_n, \tag{20}$$

where M_n is the Montgomery potential, $\mathbf{F}_n$ contains the wind stress and bottom friction, as well as vertical and horizontal viscous friction.

The first term on the RHS of equation (20) is the KE tendency due to nonlinear advection, which is not accounted for in the linear stability analysis. The role of nonlinear advection is examined by decomposing it in spectral space. The cospectrum of the nonlinear advection, T_n, is calculated as

$$T_n = -\mathrm{Re}\left[\overline{\widehat{\mathbf{u}_n}^* \cdot (\widehat{\mathbf{u}_n \cdot \nabla \mathbf{u}_n})}\right], \tag{21}$$

where $\mathrm{Re}(\cdot)$ indicates taking the real part, $(\cdot)^*$ is the complex conjugate, $\bar{\cdot}$ indicates a 500-day average.

The advection term, T_n, quantifies the spectral tendency of KE due to the KE cascade between horizontal wavenumbers. The positive and negative values of $T_n(k)$ indicate that energy cascade deposits and removes KE, respectively, for eddies at the corresponding wavenumber k. The time tendency term on the LHS of equation (20) is close to zero when taking a long-time average. In this case, the energy cascade balances the net energy source or sink due to the work by the horizontal pressure gradient [second term on the RHS of equation (20)], forcing and dissipation [third term on the RHS of equation (20)] at each wavenumber. The work by the horizontal pressure gradient includes the contribution from the divergence of 3D energy transport by pressure and the conversion of potential energy (PE) to KE due to baroclinic instability (Capet et al. 2008).

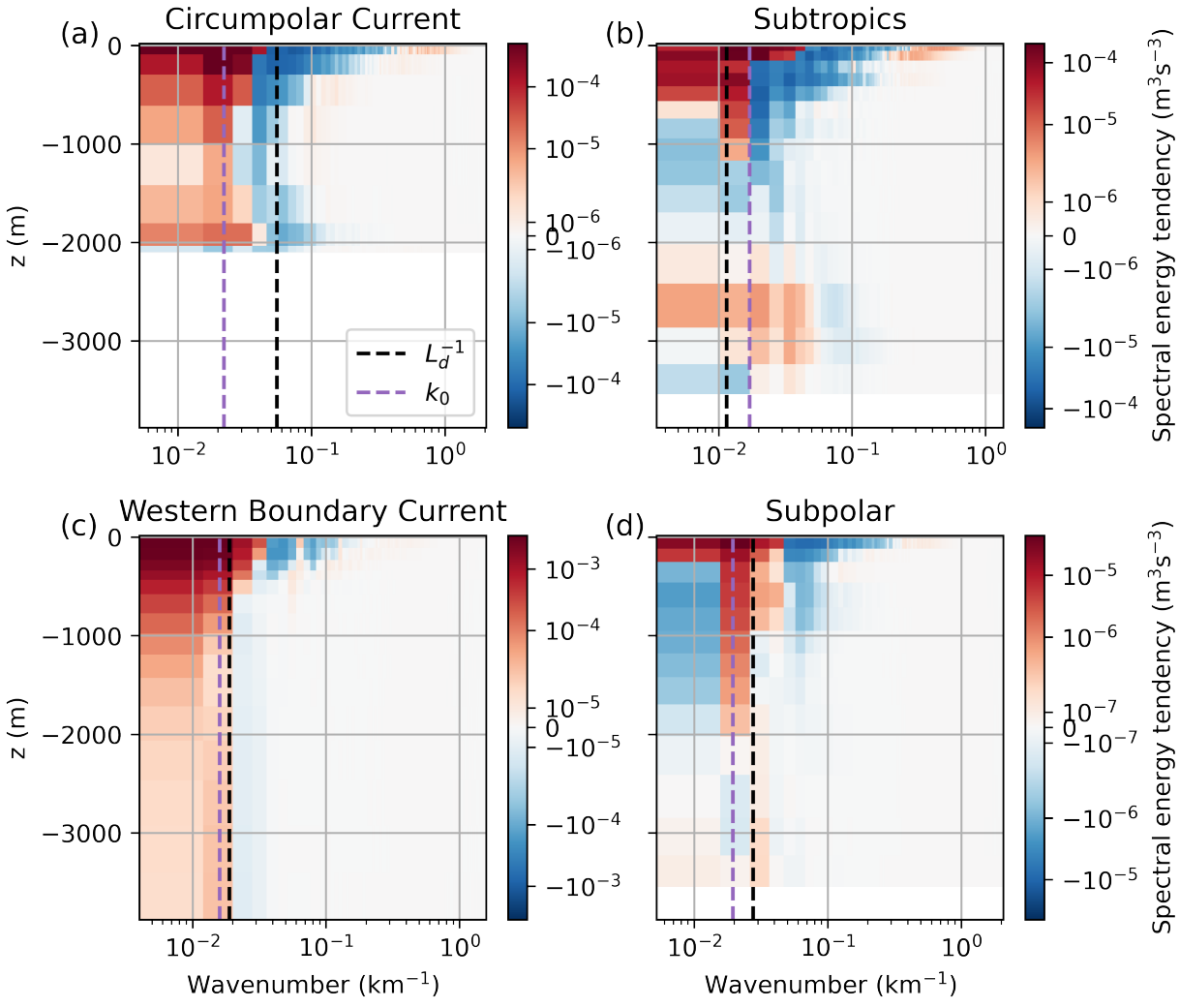


FIG. 10. Cospectrum of the advection term of KE, T_n, as a function of horizontal wavenumber and depth in the four regions shown in Fig. 3. Blue color indicates that nonlinear advection removes KE from the corresponding wavenumber, which balances the KE source by other terms on the RHS of the KE equation (20), while red color means that nonlinear advection supplies KE to balance the KE sink by the other terms. Black dashed line indicates the inverse of the Rossby deformation radius. Purple dashed line indicates the energy-containing wavenumber, k_0. Note the different color range for each panel.

Fig. 10 shows the distribution of T_n at horizontal wavenumber and depth in the four regions. The red and blue colors indicate that energy cascade deposits and removes KE, respectively, for eddies at the corresponding wavenumber. The vertical structure of T_n is generally surface intensified and decays faster with depth at smaller scales (larger wavenumbers), which is consistent with

characteristics of the SQG mode. In the circumpolar current, the energy cascade removes energy at around the deformation radius. This energy sink due to energy cascade intensifies at both the surface and bottom, which is consistent with the unstable mode of the Eady instability at this region (section 5b), indicating that the energy cascade balances the baroclinic instability at these wavenumbers. At the other three regions, the energy cascade mainly removes energy from scales smaller than the deformation radius. This energy sink concentrates near the surface, which is likely to balance the energy source due to the Charney baroclinic instability in these regions. There is also an energy sink at small wavenumbers at around 1000 m in the subtropical and subpolar regions, though it is about an order smaller than the value of T_n at surface. This sink is likely to balance the baroclinic instability at small wavenumbers in these two regions (see the growth rate distribution in Fig. 8).

The energy cascade deposits energy at scales larger than the deformation radius in all the four regions. This energy source due to energy cascade maximizes at the energy-containing scale (purple dashed line in Fig. 10). This source is balanced by energy conversion from KE to PE and friction (not shown). These results show that the energy cascade transfers energy generated by baroclinic instability at scales close to or smaller than the deformation radius to the energy-containing eddies, where kinetic energy is consumed by conversion to potential energy and friction.

The energy cycle shown by Fig. 10 is illustrated by the schematic in Fig. 11, which is motivated by a similar schematic in Roullet et al. (2012). The forcing inputs PE to circulation at the large scales. The PE is converted to KE at scales that are close to or smaller than the deformation radius through baroclinic instability. The baroclinic instability is generally dominated by the surface PV gradient, which gives rise to surface-trapped unstable modes. These unstable modes cascade KE from small scales to large scales through their nonlinear interactions until the cascade is halted at the energy-containing scale. Consequently, the vertical structure of EKE is dominated by the mode structure at the energy-containing scale, which is deeper than the original unstable modes but still remains surface-trapped.

6. A Scale-Aware Parameterization

Section 4 shows that SQG modes play an important role in setting the vertical structure of EKE. How can this finding inform the parameterization of the eddy vertical structure? A crucial property

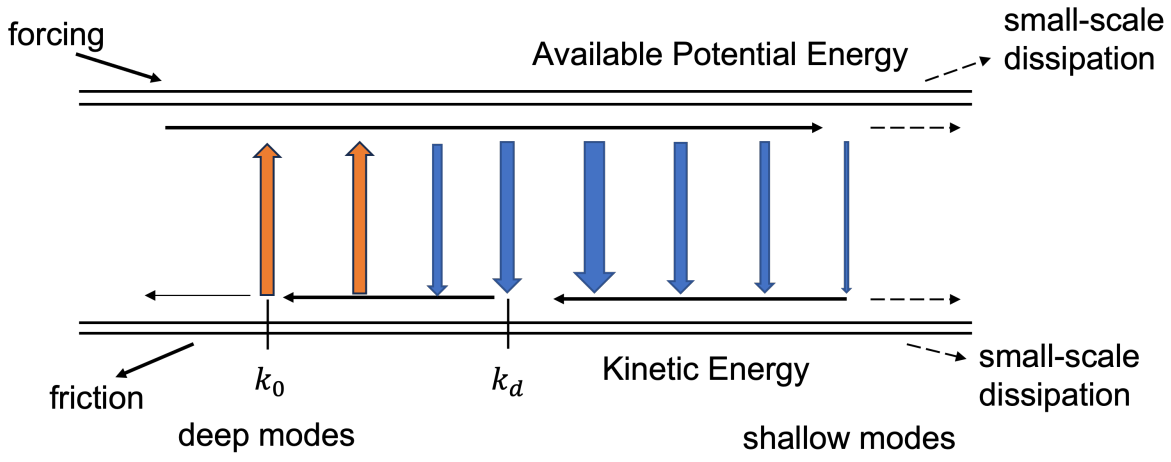


FIG. 11. Energy schematic modified from Roullet et al. (2012). The forcing supplies available potential energy (upper horizontal lines). The available potential energy cascades to smaller scales and is then converted to kinetic energy (lower horizontal lines) through baroclinic instability (blue arrows) at a broad range of horizontal scales that are close to or smaller than the deformation radius, the inverse of k_d. The unstable modes arising from the instability are surface-trapped, which decay faster with depth at smaller scales. The kinetic energy of these unstable modes cascades from small scales (shallow modes) to large scales (deep modes) until the cascade is halted at the energy-containing scale, the inverse of k_0. Part of the kinetic energy is dissipated by the friction and the other part is converted back to available potential energy (orange arrows) at around the energy-containing scale.

of the SQG mode is the coupling of its vertical and horizontal scales. As is shown in the analytical expression (10), smaller horizontal scale eddies, which have a larger horizontal wavenumber, decay more rapidly with depth than larger eddies. In the context of eddy parameterization, the focus is on representing the subgrid EKE that is unresolved by the model. As the model grid spacing is refined, the scale of unresolved eddies becomes smaller, resulting in a shallower vertical structure of subgrid EKE. This result suggests that the parameterization of the vertical structure of subgrid EKE should depend explicitly on the model grid spacing.

To examine the scale dependence of the eddy vertical structure, we employ high-pass spatial filters on the eddy velocity fields to compare the vertical structure at different horizontal scales. Fig. 12 shows the normalized vertical structures for eddies smaller than $2\pi/0.25°$, $2\pi/0.5°$, and $2\pi/1°$, represented by the solid blue, orange, and green lines, respectively. The vertical structure

of EKE becomes shallower as the horizontal scale becomes smaller. This finding is consistent with the prediction by SQG, highlighting the scale dependence of the eddy vertical structure.

To represent the scale-dependent eddy vertical structure at different model resolutions, we propose a scale-aware parameterization of the normalized EKE profile, E_p,

$$\mathrm{E}_p = e^{2k_g z_s}, \tag{22}$$

with

$$k_g = \max\left(k_0,\ c/\Delta\right), \tag{23}$$

where Δ is the model grid spacing, and c is a dimensionless constant, which will be a tuning parameter. The ratio, c/Δ, is proportional to the largest wavenumber resolved by the model with grid spacing Δ. The constant c is on the order of 1 and determined by the minimum number of grid points needed to resolve a wavelength ($c = 1$ means that the model needs about 6 grid points to resolve a wavelength since $2\pi/6\Delta \approx 1/\Delta$). If $k_0 > c/\Delta$, that means the energy containing scale is unresolved by the model, so the vertical structure of the subgrid EKE is dominated by the energy-containing wavenumber k_0. If $k_0 < c/\Delta$, the energy-containing scale is resolved, so the vertical structure of subgrid EKE will be dominated by the largest resolved wavenumber (i.e., c/Δ) instead.

To evaluate the effectiveness of the parameterization (23), we conduct tests by varying Δ to 0.25°, 0.5°, and 1° and assume $c = 1$, which means we assume a minimum of 6 grid points is required to resolve an eddy. The parameterization profiles (22) with 0.25°, 0.5°, and 1° are shown by the dashed blue, orange, and green lines in Fig.12, respectively. Compared with the solid blue, orange, and green lines in Fig.12, which represent the vertical structure of subgrid EKE in 0.25°, 0.5°, and 1° model grid spacings, respectively, the parameterization well captures the scale dependence of the eddy vertical structure. The parameterization profiles look similar to the vertical structure of subgrid EKE at different resolutions, with the exception for the subtropics where the parameterization tends to decay faster than the subgrid EKE. This result is consistent with the comparison between the SQG mode and total EKE in Fig. 4.

The parameterization based on equation (22) requires the prediction of the energy-containing scale. Studies have estimated the energy-containing scale as the Rossby deformation radius (Stone

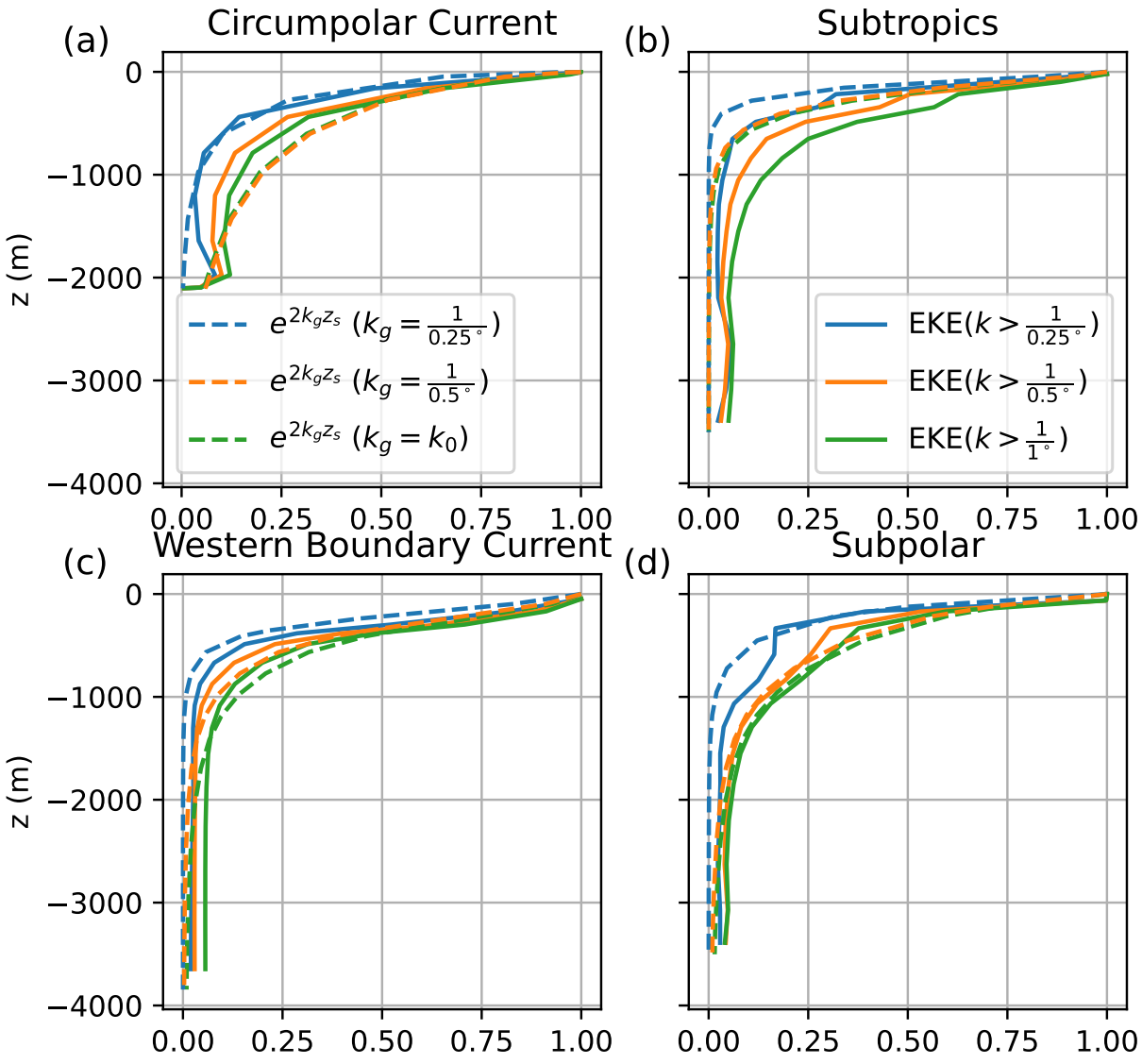


FIG. 12. The vertical structure of spatially filtered EKE in the four regions shown in Fig.4. Blue, orange, and green solid lines are the high-pass filtered vertical structure of EKE with cutoff wavenumber as $1/0.25°$, $1/0.5°$, and $1/1°$, respectively. They represent the vertical structures of subgrid EKE that is unresolved by $0.25°$, $0.5°$, and $1°$ models, assuming a minimum of six grid points is required to resolve an eddy. All EKE profiles are normalized by their surface values. Blue, orange, and green dashed lines are the parameterized vertical structure $\mathrm{E}_p(z, \Delta)$. E_p depends on the model grid spacing Δ and the energy-containing wavenumber k_0. The vertical structure of subgrid EKE is determined by Δ at $0.25°$ and $0.5°$ resolutions, where the energy-containing eddy is resolved, and by k_0 at $1°$ resolution. The parameterization captures the scale dependence reasonably well, except for an underestimation in the subtropical region.

1972), width of the baroclinic zone (Visbeck et al. 1997), Eady length scale (Larichev and Held 1995; Jansen et al. 2015), and Rhines scale (Jansen et al. 2015, 2019). Examination of these theories for predicting the eddy scale is beyond the scope of this study and will be pursued in forthcoming work.

7. Discussion and Conclusions

This study addresses the question of why ocean mesoscale eddies are surface intensified—at least those realized within our idealized stacked shallow water model—with an aim to parameterize the eddy vertical structure. Unlike previous studies that explain the eddy vertical structure from the perspective of baroclinic modes (Wunsch 1997; de La Lama et al. 2016; Lacasce 2017; Brink and Pedlosky 2020; Quan et al. 2023), we attribute the eddy vertical structure to baroclinic instability and the energy cascade. The vertical structure of EKE is found to be well-represented by a single SQG mode of the energy-containing scale in the extra tropics, indicating that the surface PV plays a dominant role in the eddy vertical structure.

a. Baroclinic instability and the surface-trapped mode

The prevalence of the SQG mode is attributed to the surface-trapped baroclinic instability and energy cascade. Although the model used in this study does not have surface buoyancy forcing, the wind forcing induces isopycnal outcropping, leading to pronounced surface meridional PV gradients. The interior PV gradient is much weaker compared with the surface PV gradient due to the interior PV homogenization by eddies. Linear stability analysis shows that the surface PV gradient plays a dominant role in baroclinic instability, giving rise to surface-trapped unstable modes. These unstable modes are often smaller than the Rossby deformation radius, except in the circumpolar current where they are close to the deformation radius. Through nonlinear interactions, unstable modes transfer their kinetic energy upscale to energize mesoscale eddies, which have an energy-containing scale larger than the deformation radius but retain the surface-trapped vertical structure.

Early geostrophic turbulence theory shows that baroclinic eddies tend to transfer their energy to the barotropic (depth independent) mode, and then barotropic eddies cascade their energy upscale (Charney 1971; Salmon 1980). Later studies have found that ocean eddies are not fully barotropic,

and as a consequence, the inverse energy cascade can occur in both the barotropic and baroclinic modes (Smith and Vallis 2001; Scott and Arbic 2007). In this study, we propose a different interpretation of the eddy energy cycle. Instead of decomposing the energy into the barotropic and baroclinic modes, we find that the energy cascade can directly occur among surface-trapped modes. The vertical and horizontal scales of these surface-trapped modes are coupled—smaller eddies decay faster with depth. Consequently, as eddies transfer energy to larger scales, they also grow deeper, in alignment with the barotropization tendency of geostrophic turbulence (Charney 1971; Salmon 1980). Although eddies can become nearly barotropic if their horizontal scales are an order of magnitude larger than the deformation radius, the inverse energy cascade tends to be halted at a scale that is comparable to the deformation radius. As a result, the vertical structure of energy-containing eddies is surface intensified, similar to the SQG mode.

b. Parameterization of vertical structure based on SQG

The vertical structure of EKE depends on the horizontal scale, with smaller eddies decaying faster with depth. This feature suggests that the parameterization of eddy vertical structure should account for the horizontal scale of unresolved eddies. As the model grid spacing becomes finer, the unresolved eddies will be smaller and have a shallower vertical structure. Based on the WKB solution of the SQG mode, we propose a scale-aware parameterization of the vertical structure of EKE [equations (22) and (23)], building upon previous work using the SQG mode for diagnostic studies of the ocean's eddy vertical structure (e.g., Lapeyre and Klein 2006; Isern-Fontanet et al. 2008; Klein et al. 2009; Qiu et al. 2016, 2020). This parameterization requires the specification of an eddy horizontal scale that depends on the model grid spacing. If the energy-containing scale is not resolved, the vertical structure parameterization is determined by the energy-containing scale itself. On the other hand, if the energy-containing scale is resolved—which may be the case in eddy-permitting models now being used for climate studies (e.g., Adcroft et al. 2019)—the vertical structure parameterization is determined by the largest unresolved scale that is proportional to the horizontal grid spacing. This parameterization is shown to capture the vertical structure of EKE filtered with different horizontal scales, so that it can be useful for both non-eddying and eddy-permitting simulations.

Full closure of the vertical structure parameterization requires a prediction of the horizontal energy-containing scale. The energy-containing scale is associated with the mechanism that halts the inverse energy cascade; for example, bottom friction and the planetary vorticity gradient (Rhines 1975; Larichev and Held 1995). Prediction of such an eddy length scale has been the focus of many studies (Larichev and Held 1995; Held and Larichev 1996; Thompson and Young 2006; Jansen et al. 2015; Kong and Jansen 2017; Chang and Held 2019; Gallet and Ferrari 2020; Chang and Held 2021; Gallet and Ferrari 2021).

The SQG mode does not fully capture the vertical structure of EKE in the lower latitudes and tropics. Previous studies reveal that the scaling for eddy mixing is different between the tropics and midlatitude ocean due to the dominance of linear waves (tropics) versus nonlinear eddies (extra tropics) (Klocker and Abernathey 2014; Zhang and Wolfe 2022). Parameterization of the eddy vertical structure in the tropics requires additional work, though eddies there are mostly large enough to be well resolved by models targeted for climate simulations in the near future such as Adcroft et al. (2019).

c. Prospects and future work

Many previous studies consider the importance of bottom topography in causing the surface intensification of ocean eddies (Aoki et al. 2009; de La Lama et al. 2016; Lacasce 2017; Quan et al. 2023). Bottom topography can decouple abyssal eddies from upper-ocean eddies, leading to baroclinic modes with zero horizontal velocity at the bottom (Rhines 1970; Samelson 1992; Hallberg 1997; Bobrovich and Reznik 1999; Tailleux and McWilliams 2001). We find that the first rough-bottom mode (the EBT mode), with zero horizontal velocity at the bottom, does not describe the vertical structure of EKE well in our simulation. This limitation arises from the EBT mode's ignorance of the surface buoyancy anomaly, which we find to be essential for reconstructing the eddy vertical structure. In fact, both the surface buoyancy and bottom topography can be incorporated in the general boundary conditions, described by equation (6), of vertical modes (Lacasce 2017; Yassin and Griffies 2022a). The vertical mode accounting for both effects will be explored in future work.

The model used in this study does not have a mixed layer, which can change the quantitative analyses of the instability and eddy energetics but, we expect, will not qualitatively change the

role of surface PV in the vertical structure. Mixed layer baroclinic instability provides another important energy source for submesoscale eddies, which can cascade their energy upscale to energize mesoscale eddies (e.g., Fox-Kemper et al. 2008b; Sasaki et al. 2014; Callies et al. 2016; Schubert et al. 2020; Dong et al. 2020; Khatri et al. 2021). Callies et al. (2016) shows that the energy generated by mixed layer instability resides in the SQG mode, where the energy cascades upscale to the largest, most energetic eddies. This energy cascade is similar to our finding, with the difference that the SQG mode is mainly excited by the surface Charney instability in our model. The mixed-layer instability thus may play a similar role to the Charney instability in enhancing the surface-trapped mode. In a more realistic numerical setup, the mixed layer and Charney instabilities can coexist, with the relative importance between them depending on the depth of the mixed layer, stratification, and lateral buoyancy gradient in the upper ocean (Capet et al. 2016; Zhang et al. 2023).

This study serves as a step forward to understand and parameterize the eddy vertical structure by emphasizing its dependence on the horizontal scale, resulting from the impact of surface PV. The parameterization, described by equation (22), is ripe to be tested in non-eddying and eddy-permitting simulations after combining it with a closure of the horizontal energy-containing scale (e.g., Jansen et al. 2015, 2019). We are planing to apply this vertical structure to the kinetic energy backscatter parameterization that has recently been used in both idealized and realistic eddy-permitting simulations (Jansen et al. 2019; Juricke et al. 2019, 2020; Yankovsky et al. 2023; Chang et al. 2023). Other aspects, including the impacts of bottom topography and mixed layer, will also be incorporated in this scheme to predict the eddy vertical structure in more realistic situations. Such scheme will be validated by comparing to observations of full-depth eddy profile (Ni et al. 2023).

Acknowledgments. We thank Alistair Adcroft, Laure Zanna, Elizabeth Yankovsky, Malte Jansen, Shafer Smith and Baylor Fox-Kemper for helpful discussions. We also thank Chiung-Yin Chang, Jacob Steinberg, and Matthew Lobo for comments on drafts of this manuscript. This work was supported by award NA18OAR4320123 from the National Oceanic and Atmospheric Administration, U.S. Department of Commerce. The statements, findings, conclusions, and recommendations are those of the author(s) and do not necessarily reflect the views of the National Oceanic and Atmospheric Administration, or the U.S. Department of Commerce. W.Z. was also supported by the NSF (OCE-2048826).

Data availability statement. The model, NeverWorld2, data set and detailed information on its configuration are available at https://doi.org/10.26024/f130-ev71 (Marques et al. 2022).

APPENDIX A

Comparison of the Surface-Trapped Mode to the Solution in Lapeyre and Klein (2006)

The orthogonal basis derived from equations (8) and (9) is different from that in Lapeyre and Klein (2006). They proposed a decomposition of the eddy streamfunction ψ into a surface component ψ_{sur} satisfying

$$\begin{aligned} \nabla^2\psi_{\text{sur}} + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial\psi_{\text{sur}}}{\partial z}\right) &= 0, \\ \left.\frac{\partial\psi_{\text{sur}}}{\partial z}\right|_{z=0} &= \frac{b_s}{f}, \\ \left.\frac{\partial\psi_{\text{sur}}}{\partial z}\right|_{z=-H} &= 0, \end{aligned} \tag{A1}$$

and an interior component ψ_{int} satisfying

$$\begin{aligned} \nabla^2\psi_{\text{int}} + \frac{\partial}{\partial z}\left(\frac{f^2}{N^2}\frac{\partial\psi_{\text{int}}}{\partial z}\right) &= q, \\ \left.\frac{\partial\psi_{\text{int}}}{\partial z}\right|_{z=0} &= 0, \\ \left.\frac{\partial\psi_{\text{int}}}{\partial z}\right|_{z=-H} &= 0. \end{aligned} \tag{A2}$$

Here b_s is the surface buoyancy anomaly and q the PV, with the bottom boundary condition specified following Lapeyre (2009).

The interior solution, $\psi_{\rm int}$, is driven by interior PV anomalies and can be projected onto the flat-bottom modes, described by equations (2) and (4). The surface solution, $\psi_{\rm sur}$, is driven by surface buoyancy anomalies. Although $\psi_{\rm sur}$ and the surface-trapped mode Φ_s [described by equation (8)] share certain properties (i.e., zero interior PV anomaly and horizontal wavenumber dependence), $\psi_{\rm sur}$ satisfies a Neumann surface boundary condition (i.e., surface buoyancy anomaly is given) while Φ_s a Dirichlet surface boundary condition (i.e., surface pressure anomaly is given). Consequently, $\psi_{\rm sur}$ is not orthogonal to the interior solution $\psi_{\rm int}$. In contrast, Φ_s is orthogonal to the interior modes Φ_m described by equation (9). Later studies have shown that the surface solution $\psi_{\rm sur}$—modified to incorporate surface pressure anomaly—is better for reproducing the vertical structures of EKE and vertical velocity than the original formulation based on surface buoyancy anomaly (Isern-Fontanet et al. 2008; Klein et al. 2009; Isern-Fontanet et al. 2014; González-Haro and Isern-Fontanet 2014).

APPENDIX B

SQG Mode in Layered Quasigeostrophic Models

Eddies in the shallow water model, described in Section 3a, are analyzed based on layered quasigeostrophic (QG) dynamics. The eddy streamfunction ψ can be obtained from the distribution of the QG PV, q, by inverting the relation

$$\begin{aligned} q_1 &= \nabla^2\psi_1 + \frac{f^2}{H_1}\frac{\psi_2-\psi_1}{g'_1}, \\ q_n &= \nabla^2\psi_n + \frac{f^2}{H_n}\left(\frac{\psi_{n-1}-\psi_n}{g'_{n-1}} - \frac{\psi_n-\psi_{n+1}}{g'_n}\right),\ n\in[2,n_{\rm max}-1], \\ q_{n_{\rm max}} &= \nabla^2\psi_{n_{\rm max}} + \frac{f^2}{H_{n_{\rm max}}}\frac{\psi_{n_{\rm max}-1}-\psi_{n_{\rm max}}}{g'_{n_{\rm max}-1}}, \end{aligned} \tag{B1}$$

where $(\cdot)_n$ indicates the property on the n-th layer, H_n is the layer thickness, f is the Coriolis parameter, $n_{\rm max} = 15$ for our simulations, and g'_n is the reduced gravity, computed as

$$g'_n = g\,\frac{\rho_{n+1}-\rho_n}{\rho_0}, \tag{B2}$$

where $g = 10\ \mathrm{m^2\ s^{-1}}$, ρ_n is the uniform layer density, and $\rho_0 = 1000\ \mathrm{kg/m^3}$ is the reference density. We emphasize that n in this paper is used to label the discrete layer number in the stacked shallow water model, whereas m was introduced earlier and denotes the vertical eigenmode.

In matrix form, the relationship between $\mathbf{q} = \left[q_1,\ \cdots,\ q_{n_{\max}}\right]^{\mathrm{T}}$ and $\boldsymbol{\psi} = \left[\psi_1,\ \cdots,\ \psi_{n_{\max}}\right]^{\mathrm{T}}$ can be written as

$$\mathbf{q} = \left(\mathbf{S} + \mathbf{I}\nabla^2\right)\boldsymbol{\psi}, \tag{B3}$$

where $\mathbf{I}$ is the $N \times N$ identity matrix, and $\mathbf{S}$ is the stretching matrix written as

$$\mathbf{S} = f^2 \begin{bmatrix} -\frac{1}{H_1 g_1'} & \frac{1}{H_1 g_1'} & 0 & 0 & \cdots \\ & \vdots & & \vdots & \\ \cdots & \frac{1}{H_n g_{n-1}'} & -\frac{1}{H_n g_{n-1}'} - \frac{1}{H_n g_n'} & \frac{1}{H_n g_n'} & \cdots \\ & \vdots & & \vdots & \\ \cdots & 0 & 0 & \frac{1}{H_{n_{\max}} g_{n_{\max}-1}'} & -\frac{1}{H_{n_{\max}} g_{n_{\max}-1}'} \end{bmatrix}$$

Surface buoyancy is not explicitly simulated in a stacked shallow water model. Even so, its dynamical impact is similar to that of the surface-layer PV, q_1. The streamfunction due to SQG dynamics, $\boldsymbol{\psi}_s$, is equivalent to the streamfunction induced by the surface-layer PV,

$$\boldsymbol{\psi}_s = (\mathbf{S} + \nabla^2\mathbf{I})^{-1}\mathbf{q}_s, \tag{B4}$$

where $\mathbf{q}_s = [q_1, 0, \cdots, 0]^{\mathrm{T}}$.

APPENDIX C

Linear Stability Analysis

The QG PV evolution equation in a layered QG model (e.g., Vallis 2017) is

$$\frac{\partial q_n}{\partial t} + U_n\frac{\partial q_n}{\partial x} + V_n\frac{\partial q_n}{\partial x} + \mathrm{J}(\psi_n, q_n) + \mathrm{J}(\psi_n, Q_n) = 0, \tag{C1}$$

where J is the Jacobian, U and V are the background zonal and meridional velocities, and Q is the background PV estimated as

$$\mathbf{Q} = \mathbf{SV}x + (\beta - \mathbf{SU})\, y, \tag{C2}$$

where $\mathbf{Q} = \left[Q_1,\ \cdots,\ Q_{n_{\max}}\right]^{\mathrm{T}}$, $\mathbf{U} = \left[U_1,\ \cdots,\ U_{n_{\max}}\right]^{\mathrm{T}}$, and $\mathbf{V} = \left[V_1,\ \cdots,\ V_{n_{\max}}\right]^{\mathrm{T}}$. Zonal and meridional background PV gradients, Q_x and Q_y, respectively, are

$$\begin{aligned} \mathbf{Q}_x &= \mathbf{SV}, \\ \mathbf{Q}_y &= \beta - \mathbf{SU}. \end{aligned} \tag{C3}$$

Inserting a wave ansatz, $\psi_n = Re\left[\hat{\psi}_n e^{i(k_x x + k_y y - \omega t)}\right]$, into the linearized PV equation (C1), we get

$$i(k_x U_n + k_y V_n - \omega)\hat{q}_n + (ik_x Q_{ny} - ik_y Q_{nx})\hat{\psi}_n = 0, \tag{C4}$$

where the relationship between $\hat{\mathbf{q}} = \left[\hat{q}_1,\ \cdots,\ \hat{q}_{n_{\max}}\right]^{\mathrm{T}}$ and $\hat{\boldsymbol{\psi}} = \left[\hat{\psi}_1,\ \cdots,\ \hat{\psi}_{n_{\max}}\right]^{\mathrm{T}}$ is given by equation (B3),

$$\hat{\mathbf{q}} = \left(\mathbf{S} - k^2\mathbf{I}\right)\hat{\boldsymbol{\psi}}, \tag{C5}$$

Equation (C4) forms a generalized eigenvalue problem for the mode $\hat{\boldsymbol{\psi}}$ (eigenvectors) and the frequency ω (eigenvalues),

$$\omega\mathbf{A}\hat{\boldsymbol{\psi}} = \left[(k_x\mathbf{U} + k_y\mathbf{V})\mathbf{A} + k_x\mathbf{Q}_y - k_y\mathbf{Q}_x\right]\hat{\boldsymbol{\psi}}, \tag{C6}$$

where $\mathbf{A} = \mathbf{S} - k^2\mathbf{I}$. For $\omega = \omega_r + i\sigma$, if the imaginary component, $\sigma > 0$, then the mode will grow exponentially, which signals a linear instability.

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