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Supporting Information for

A hybrid mechanism for enhanced core-mantle chemical interaction

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Additional Supporting Information (Files uploaded separately)

N/A

Introduction

This supporting information contains the description of equations used in the numerical models and gravitational collapse of the mushy layer.

Text S1.

For an isoviscous mantle that is approximated as an incompressible, Boussinesq fluid with an infinite Prandtl number, the governing equations of motion in Cartesian geometry are (e.g., Schubert et al., 2001):

$$\eta \nabla^2 \boldsymbol{v} - \nabla P_v = \rho_0 [1 - \alpha (T - T_0)] g \hat{\boldsymbol{z}} \tag{S1}$$

$$\nabla \cdot \boldsymbol{v} = 0 \tag{S2}$$

where η is the reference viscosity of the mantle, $\boldsymbol{v}$ is the velocity field due to buoyancy variations, P_v is the pressure field, ρ_0 is the reference density of the mantle, α is the thermal expansion coefficient, T is the temperature, T_0 is the reference temperature, and g is the acceleration due to gravity. As mentioned in the main text, convective flows exert normal stresses σ_{zz} on the CMB which creates dynamic topography h. In our calculations, we assume that the mantle follows a Newtonian rheology such that

$$\sigma_{zz} = 2\eta \frac{\partial v_z}{\partial z} \tag{S3}$$

This viscous stress deforms the CMB to produce topography variations according to Equation (1) in the main text.

The rate of change of the mushy-layer thickness $\frac{\partial h}{\partial t}$ in Equation (3) in the main text has contributions from both the silicate and metallic components, and we let the combined vertical velocity of both components at the mantle-mushy layer interface be $\overline{U}_z(x, y)$ such that $\frac{\partial h}{\partial t} = \overline{U}_z(x, y)$. Since only the silicate component is exchanged across this interface, the metallic component has to be subtracted from the total velocity such that $\overline{U}_z - \phi \overline{U}_z = (1 - \phi)\overline{U}_z$ where $\phi \overline{U}_z$ is the contribution from the liquid metal. Let u_{z+} be the velocity of the silicate component. Hence, $u_{z+} = (1 - \phi)\overline{U}_z$. Plugging this result back into Equation (3), the velocity of the silicate component is

$$u_{z+} = \frac{\Delta \rho g (1 - \phi)}{12\mu} \nabla_H^2 h^4 \tag{S4}$$

Gravitational collapse of the mushy layer creates a secondary flow in the overlying mantle $\boldsymbol{u}$ that superimposes on the existing buoyancy-driven flow. This velocity field can be obtained by solving the Stokes' and continuity equations:

$$\eta \nabla^2 \boldsymbol{u} - \nabla P_u = \rho_0 g \hat{\boldsymbol{z}} \tag{S5}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{S6}$$

where P_u is the pressure field arising from the flow $\boldsymbol{u}$. These two equations have to be solved with the inhomogeneous boundary condition given by Equation (S4) at each timestep. The two velocity fields $\boldsymbol{v}$ and $\boldsymbol{u}$ are summed together to advect any relevant scalar fields in the domain. In the instance of temperature, the temperature field T will evolve according to the following equation:

$$\frac{\partial T}{\partial t} + \boldsymbol{v}_{eff} \cdot \nabla T = \kappa \nabla^2 T \tag{S7}$$

where $\boldsymbol{v}_{eff} = \boldsymbol{v} + \boldsymbol{u}$.

To compute the material flux due to buoyancy-driven flows F_{bd}, we first only approximate $v_z(x, y, z)$ about the point $z = 0$ using the Maclaurin series

$$v_z(x, y, z) \approx v_z(x, y, 0) + \left.\frac{\partial v_z}{\partial z}\right|_{z=0} z + \left.\frac{\partial^2 v_z}{\partial z^2}\right|_{z=0} \frac{z^2}{2!} + \cdots \tag{S8}$$

Since the boundaries are impenetrable $v_z(x, y, 0) = 0$, the first term on the right side disappears. Ignoring the terms z^2 and larger, we can approximate $\bar{V}_z(x, y) = v_z(x, y, h)$ as

$$\bar{V}_z \approx \left.\frac{\partial v_z}{\partial z}\right|_{z=0} h \tag{S9}$$

Since $\bar{V}_z$ contains contributions from both silicate and metallic components, the latter has to be subtracted from $\bar{V}_z$ to obtain the velocity of the silicate component v_{z+}. Hence,

$$v_{z+} \approx (1 - \phi) \left.\frac{\partial v_z}{\partial z}\right|_{z=0} h \tag{S10}$$

This allows us to compute F_{bd} using the following equation

$$F_{bd} = \int_S \rho_r \frac{|v_{z+}| - v_{z+}}{2} dS \tag{S11}$$

where S is the area of the interface between the mushy layer and overlying mantle. Equation (S11) is written with a "rectified" velocity because the flux into the layer is only non-zero when the v_{z+} is less than 0.

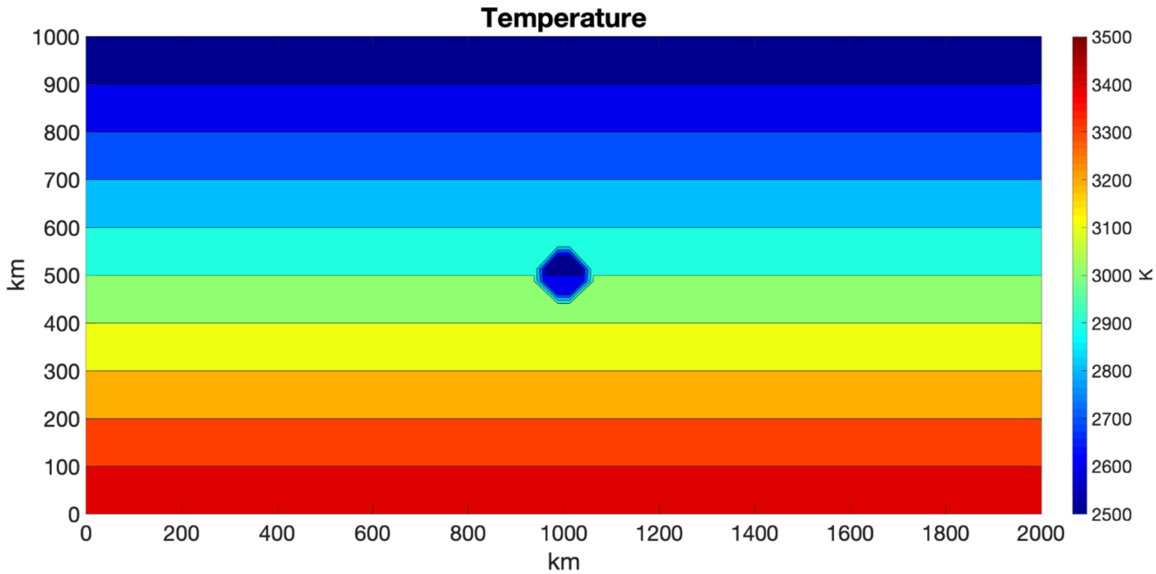

Figure S1. Plot of the initial temperature field during each calculation. In our illustrative models, buoyancy-driven convection is run to steady-state and then collapse-driven flow is initiated. This eliminates transient effects associated with particular choices for initial conditions.

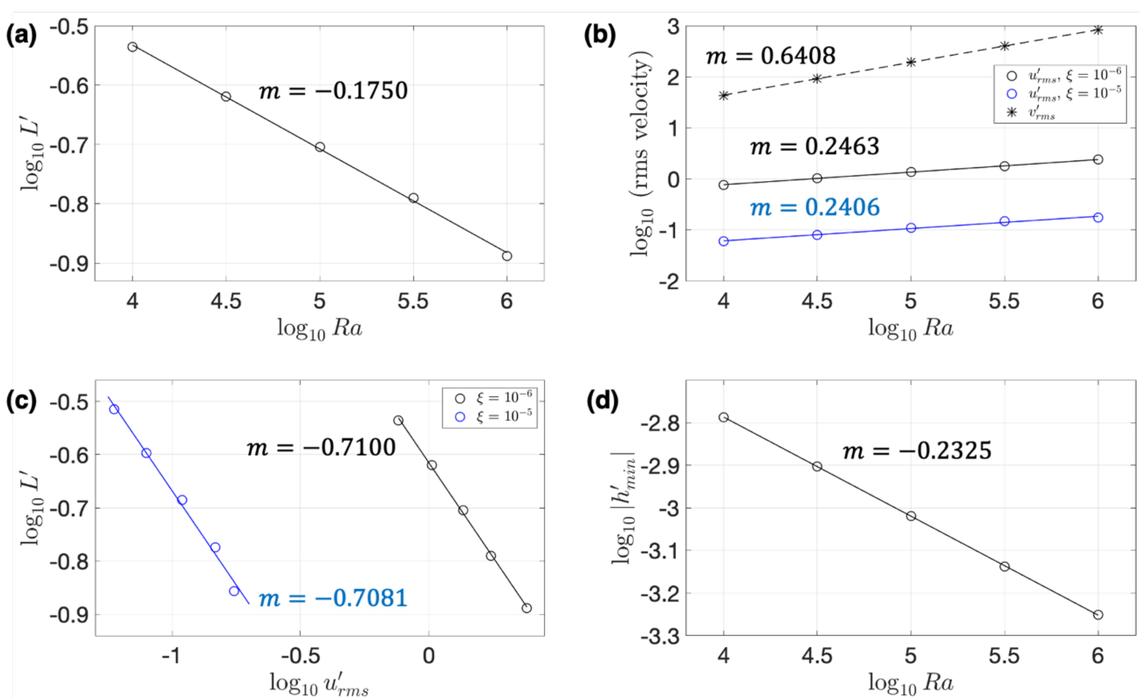

Figure S2. In our numerical calculations, the topographic depressions produced by buoyancy forces in the mantle are symmetric about the mid-point (see Figure 2b in the main text). We use

L to quantify the value of the half width at half the minimum amplitude of the topography. Prime notation indicates nondimensional variables. (a) A plot of $\log_{10} L'$ against $\log_{10} Ra$ where $L' = L/H$. The results indicate that $L' \propto Ra^{-0.1750}$. (b) Plot of the logarithmic root-mean-square (rms) velocities against $\log_{10} Ra$. The velocities are multiplied with a normalization factor H/κ. The best-fit line for $\log_{10} v'_{rms}$ has a slope close to 2/3 which agrees with boundary layer theory. The slope of $\log_{10} u'_{rms}$ against $\log_{10} Ra$ is gentler and does not seems to vary significantly with different ξ. (c) Plot showing the relationship between the width of the topography with the collapse-driven flow. (d) Plot of maximum amplitude of dynamic topography with Rayleigh number. The amplitude is normalized by H.

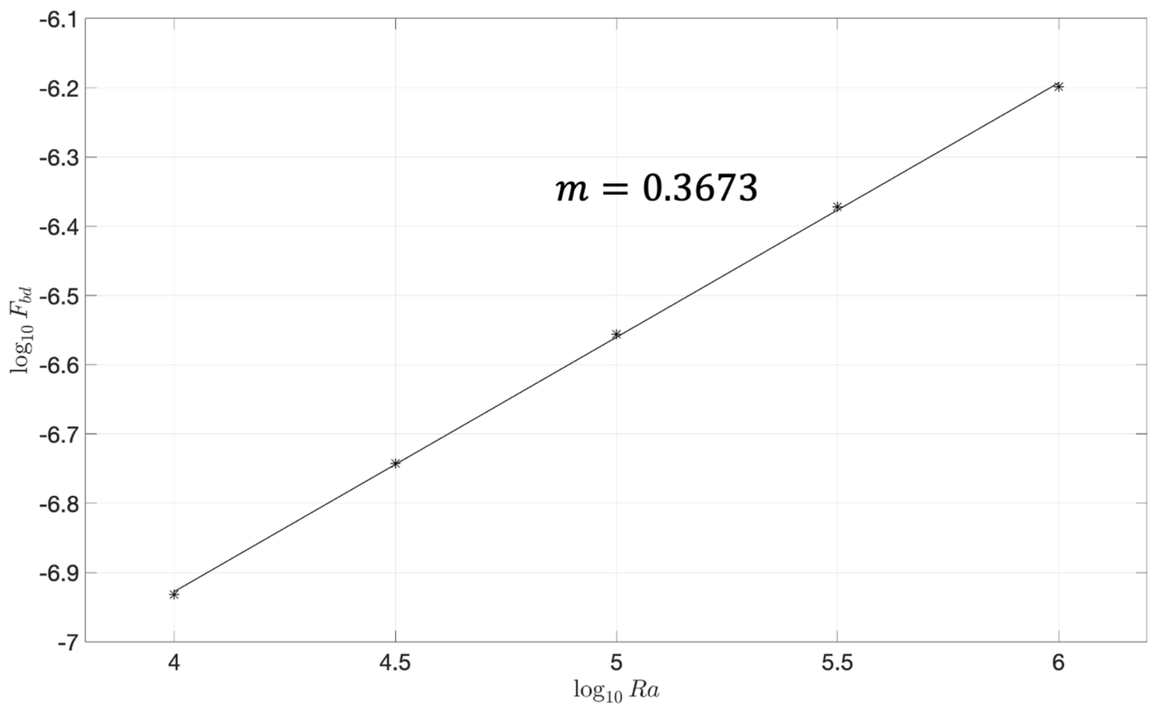

Figure S3. A plot of $\log_{10} F_{bd}$ against $\log_{10} Ra$ showing that $F_{bd} \propto Ra^{0.3673}$.

Parameter	Value	Unit
ρ_r	5500	kg m^{-3}
ρ_m	9900	kg m^{-3}
ϕ	0.2	-
α	2.5×10^{-5}	K^{-1}
T_0	2500	K
g	10	m s^{-2}
κ	10^{-6}	m^2 s^{-1}
η	$1.375 \times 10^{21} - 1.375 \times 10^{23}$	Pa s
$\xi \left(=\frac{\mu}{\eta}\right)$	$10^{-6} - 10^{-5}$	-

Table S1. List of parameters and values used in the numerical calculations.

References

Schubert, G., Turcotte, D. L., & Olson P. (2001). *Mantle Convection in the Earth and Planets*. Cambridge University Press. doi: 10.1017/CBO9780511612879