Global Frictional Equilibrium via Stochastic, Local Coulomb Frictional Slips

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Key Points:

- A simple quasi-static 2D model is introduced, quantifying and extending the classic notion of frictional equilibrium of the brittle crust
- We investigate the global scale stress evolution due to stochastic, local scale frictional slips in the crustal rock masses
- Frictional equilibrium of a stochastic system is greatly affected by its intrinsic friction heterogeneity

Abstract

Natural variability of fault friction and slip uncertainty exist in the Earth's crust. To what extent it influences crustal stress and its evolution is intriguing. We established a quasi-static, 2D model to simulate the stress evolution due to Coulomb frictional slips in the brittle crust. The model simply features randomly-oriented fractures with heterogeneous frictional coefficients. We emphasized the global stress response by summing the contribution of cascades of local frictional slip under specific boundary conditions. We illustrated that the decrease in stress difference manifests as a self-organized process that ultimately leads to frictional equilibrium. The model informs that the frictional equilibrium of a stochastic system can depart substantially from a deterministic estimation. Although the model quantitatively corroborates the notion of frictional equilibrium in places where fracture slip is the dominant mechanism for stress release, it reveals far more profound influence of system heterogeneity on the local and global stress evolution.

Plain Language Summary

Knowledge of crustal stress and its uncertainty is of fundamental importance to a wide range of problems. It is recognized that the intra-plate continental crust is generally in a state of frictional failure, the stress magnitudes of which usually cannot accumulate beyond the frictional strength. As a conventional practice, Coulomb theory is adopted together with laboratory-derived frictional coefficients for crustal stress estimations. Although it is able to attain a first-order agreement, such a practice has been primarily employed in a deterministic sense, which overlooks the fact that stress distribution is highly complex and spatially heterogeneous at different scales in the Earth's crust. In addition, how the upper crust keeps its stress magnitudes at its frictional strength is yet well understood. To this end, we proposed a simple quasi-static 2D model with distributed frictional coefficient as a proxy of the intrinsic system heterogeneity. By quantitatively investigating the global-scale stress evolution due to stochastic, local-scale frictional slips, this study shows that the magnitudes and uncertainties of both local- and global-scale stresses of the system can be greatly controlled by its friction heterogeneity. This model is believed to quantify and extend the classic notion of frictional equilibrium within the brittle crust.

1 Introduction

Fault slip is one of the dominant mechanisms for stress release in the Earth's upper crust. The stress of the fractured crust is often considered under 'frictional equilibrium', a dynamic status induced by ongoing tectonic/gravity loading and resulting fault slips (Zoback and Townend, 2001). Via the simple Coulomb frictional failure theory, the limiting state of stress can be conveniently expressed as:

$$\sigma_1/\sigma_3=\left(\sqrt{\mu^2+1}+\mu\right)^2 \tag{1}$$

where σ_1 and σ_3 are the effective major and minor principal stress, respectively, and μ is the frictional coefficient. Adopting laboratory-derived frictional coefficient values (μ = 0.6-1.0) (Byerlee, 1978), Eq.(1) has enabled the estimation of in situ stress and vice versa, the analysis of fault criticality (Brace and Kohlstedt, 1980; Townend and Zoback, 2000). However, to what spatial and temporal scale a deterministic use of Eq.(1) applies to is questionable, and has been often misused and mis-interpreted. Evidently, the value of μ varies spatially and temporally in the Earth's crust (Dieterich, 1979; Rivera and Kanamori, 2002), which underscores that such variability in a natural system must be considered.

To reflect such variability, recent attempts in stress estimation and/or fault slip analysis incorporated uncertainties in geomechanical parameters with a probabilistic approach (e.g., Walsh and Zoback, 2016; Hosseini et al., 2018; Luo and Ampuero, 2018), which offers more insights than a pure deterministic application of Eq.(1). However, one aspect still missing from existing stress models is how such system variability and heterogeneity influence the evolution of the in situ stress. How fault slip leads to frictional equilibrium, if possible, and whether it is attained is intriguing. The understanding of this evolution requires not only the influence of the far-field stress on the local fault slip, but also the feedback from the local slip to the global stress release. In this paper, we present a quasi-static, 2D model to simulate the stress evolution due to Coulomb frictional slip in the crustal rock masses. We explicitly consider frictional coefficient heterogeneity, as a proxy of the combined system uncertainties and variabilities, and emphasize the connection between the global and local stress response.

2 Methods

2.1 Model Configuration

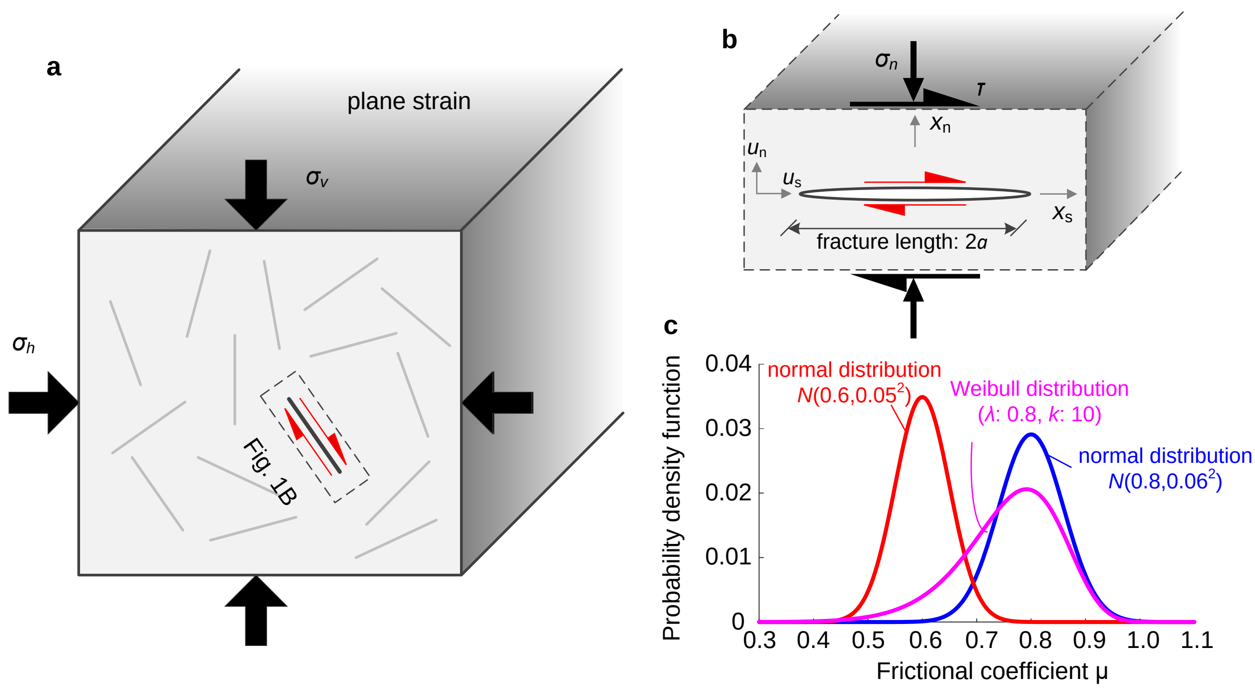

Figure 1. a Schematics of the plane strain model: randomly-distributed fractures in an elastic matrix subject to uniform stresses at the boundary. **b** Close-up of a fracture with its geometrical and mechanical features. **c** Distributions of frictional coefficient (μ) of fractures adopted to in the model.

The model we present is a fractured, elastic matrix configured under plane strain condition (Figure 1A). The embedded fractures are linear, planar, and cohesion-less. They are perpendicular to the plane section and through-going. The fractures are spatially characterized only by their orientations and their actual positions in the plane are irrelevant, see Text S1. This treatment follows Wiebols and Cook (1968) and other work on effective medium (Kachanov, 1992; Davy et al., 2018). Deemed essential to our model, the embedded fractures differ in their frictional coefficient μ, which can follow any arbitrary distribution, e.g., Figure 1C, as a proxy for the inherent heterogeneity in the system. The elastic matrix is simply characterized by its shear modulus G and Poisson's ratio ν. As a quasi-static model for stress relaxation of much longer time scales, complex dynamic issues such as fracture initiation, propagation and termination are not addressed.

2.2 Local Slip – Shear Displacement

Given a remotely applied effective stress tensor σ at the model boundary, local shear and normal stresses (σ_n and τ) acting on individual fractures are mathematically expressed via the unit normal and shear vector, n and s, of each fracture. We are cognizant of stress perturbation near fractures, but considered it trivial in the context of upscaling (see Text S1). We simply adopt the classic Coulomb frictional failure criterion to determine whether slip occurs on a fracture. If $\tau > \mu\cdot\sigma_n$, the fracture is identified as critical and frictional slip occurs, otherwise the fracture stays perfectly bonded, behaving as part of the elastic matrix with no relative displacement occurring between opposite fracture sides. We assume that the shear stress on the fracture will drop to its frictional resistance after the slip, so that the shear stress difference $\Delta\tau = \tau - \mu\cdot\sigma_n$ drives the relative displacement across the fracture.

Based on elastic crack theory (Pollard and Segall, 1987), the normal and shear displacements (u_n and u_s) on opposite sides of a fracture associated with the slip can be analyzed conveniently in the local fracture coordinates (x_n, x_s) (Figure 1B). Specifically, they are:

$$u_n = \Delta\tau \frac{1-2\nu}{2G} x_s \qquad (2a)$$

$$u_s^{\pm} = \pm\Delta\tau \frac{1-\nu}{G} \sqrt{a^2 - x_s^2} \qquad (2b)$$

where $x_s \in [-a, a]$, a is the fracture half-length, and the superscript '±' of u_s refers to displacement along the upper and lower fracture side ($x_n = \pm 0$), respectively. The average relative shear displacement between opposite sides $\bar{\mathbf{d}}_s$ is from integrating the relative shear displacement ($u_s^+ - u_s^-$) across the fracture length:

$$\bar{\mathbf{d}}_s = \left(\frac{1}{a}\int_0^a \Delta\tau \frac{2(1-\nu)}{G}\sqrt{a^2 - x_s^2}\, \mathrm{d}x_s\right)\mathbf{s} = \left(\Delta\tau \frac{a\pi(1-\nu)}{2G}\right)\mathbf{s} \qquad (3a)$$

To reflect shear-induced dilatancy commonly observed in the brittle rock mass (Scholz, 1974; Fielding et al., 2009), we utilize dilatancy factor β to relate $\bar{\mathbf{d}}_s$ to the average relative normal displacement $\bar{\mathbf{d}}_n$:

$$\bar{\mathbf{d}}_n = \beta|\bar{\mathbf{d}}_s|\mathbf{n} = \beta\left(\Delta\tau \frac{a\pi(1-\nu)}{2G}\right)\mathbf{n} \qquad (3b)$$

2.3 Upscaling Local Slips

We invoke Gaussian theorem (Hill, 1963; Kachanov, 1992) to relate the contribution of local displacement incurred by individual frictional slip to the global strain at the model boundary $\Delta\boldsymbol{\varepsilon}$:

$$\Delta\boldsymbol{\varepsilon}=\frac{a}{A}(\bar{\mathbf{d}}\otimes\mathbf{n}+\mathbf{n}\otimes\bar{\mathbf{d}}) \quad (4)$$

where A is the cross-section area of the model and $\bar{\mathbf{d}}=\bar{\mathbf{d}}_s+\bar{\mathbf{d}}_n$.

The total global strain at the model boundary comprises the strain of the intact elastic matrix $\boldsymbol{\varepsilon}^m$ and, if critical fractures exist, the summed strain $\Delta\boldsymbol{\varepsilon}$ induced by successive, individual slips of a cascade of critical fractures:

$$\boldsymbol{\varepsilon}_i=\boldsymbol{\varepsilon}^m+\sum_i\frac{a_i}{A}(\bar{\mathbf{d}}_i\otimes\mathbf{n}_i+\mathbf{n}_i\otimes\bar{\mathbf{d}}_i) \quad (5)$$

where subscript i denotes the ith fracture in a cascade of slips. The intact elastic matrix strain $\boldsymbol{\varepsilon}^m$ is simply regulated by Hooke's law under plane strain. Details on the slip cascades and upscaling are expanded in Text S2.

2.4 Slip Iterations, Time Steps

If stress and/or strain is mandated constant at the model boundary, the contribution of local fracture slips requires adjustment of stress/strain in the intact elastic matrix. This entails global stress adjustment at the model boundary and further modifies the criticality of individual fractures locally, necessitating a frequent re-evaluation of the fracture criticality. To this end, we impose an iterative process to accommodate the interplay between local fracture criticality, global stress and strain, and fractured matrix, tailored to the model experimentation below of specific boundary conditions (Text S2). We predicate the termination of the iteration when the shear stress difference $\Delta\tau$ of the most critical fracture is below 0.01 MPa. Constant slip rate depending on the global effective properties and local stress is assumed for each critical fracture within a time step, at the end of which the contribution of multiple slips is summed by the non-interaction approximation (Bristow, 1960) (expanded in Text S2).

3 Results

3.1 Model Experimentation in the Context of Normal Faulting Stress Regime

We start experimenting our model by simulating the simple scenario of stable intra-plate region with normal faulting stress environment ($\sigma_v = \sigma_1 > \sigma_h = \sigma_3$). The boundary condition is set with constant vertical stress and constant lateral strain. We assign the model with size A of 100×100 (in unit length), and 10,000 randomly-oriented fractures with equal length ($a_i \equiv a = 1$, unit length). The frictional coefficients of all fractures are normally distributed: the mean and the standard deviation of the distribution are 0.6 and 0.05, respectively, i.e., $N(0.6, 0.05^2)$ shown in Figure 1C. The dilatancy factor of fractures β is 0.05. The (intact) elastic matrix is assigned shear modulus G = 20 GPa and Poisson's ratio v = 0.3.

We arbitrarily apply an effective stress tensor $\boldsymbol{\sigma}$ ($\sigma_{3,0}$ = 20 MPa, and $\sigma_{1,0}$ = 100 MPa) at the model boundary instantaneously at initial time t_0. It will become clear later in the text that the starting stress difference hardly matters to the final frictional equilibrium. Since no fracture slip occurs at t_0, $\varepsilon_{1,0}$ and $\varepsilon_{3,0}$ at the model boundary are only related to the elastic matrix response, i.e., $\varepsilon_{1,0} = \varepsilon_1^m$ and $\varepsilon_{3,0} = \varepsilon_3^m$. An initial evaluation of fracture criticality allows the iterative slip process to begin. Mohr diagrams are used to illustrate the first two time steps as an example (Figure 2). The slip of critical fractures reduces the shear stresses on themselves to their frictional resistance, revealing local stress heterogeneities in the system. Upon the end of a time step, i.e., a cascade of slips, boundary stress σ_3 is increased to maintain constant lateral strain and fracture criticality evaluation re-iterates. With the starting stress difference substantially above the possible equilibrium state, the stress evolution undergoes multiple time steps (1, 2, …, j, …) before it terminates (see details in Text S2). As the vertical stress σ_1 is held constant, the horizontal stress $\sigma_{3,j}$ increases, or, the stress difference ($\sigma_{1,j}$ - $\sigma_{3,j}$) relaxes monotonically. The stress evolution at the model boundary manifests itself as a series of contracting Mohr diagrams (Figure 3A). The number of fracture slips and the amount of stress relaxation of each time step diminishes significantly as the iteration continues. Numerous fracture slips induce the accumulation of vertical strain and the reduction of system stiffness, as illustrated in Figure 3B. Such response is characteristic of the absence of tectonic loading.

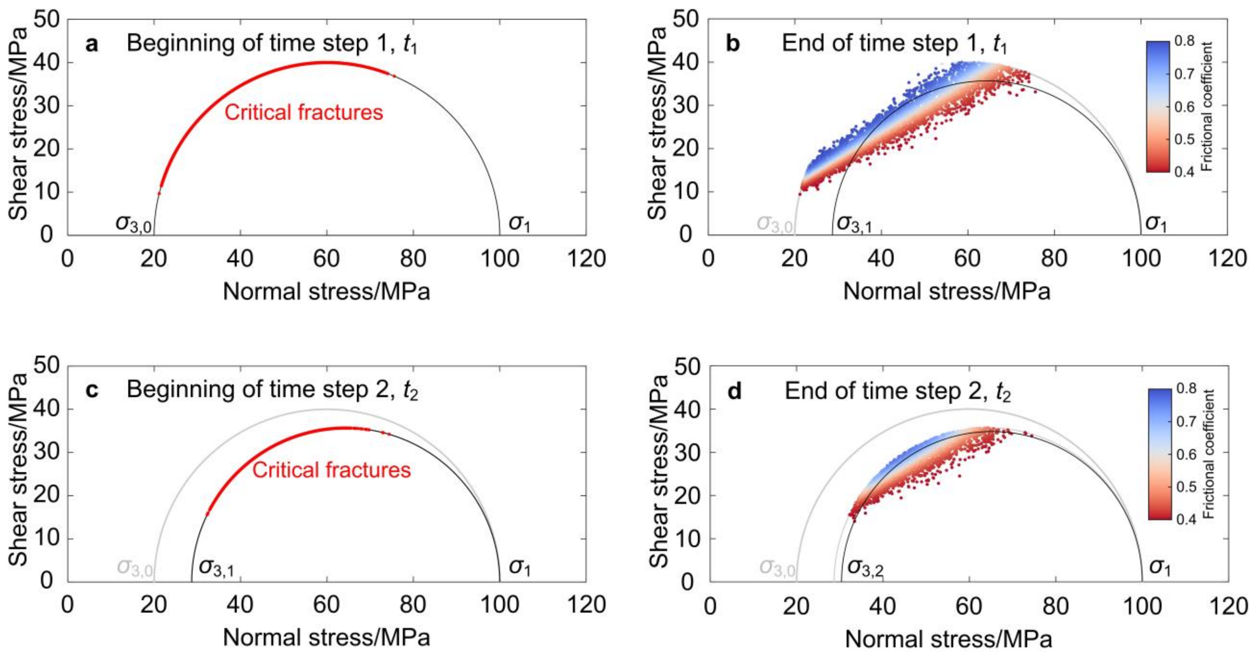

Figure 2. Mohr diagrams illustrating the first two iterations of stress evolution from an arbitrary, initial stress condition within the normal faulting stress regime. The fractures in the system follow a normally distributed frictional coefficient $N(0.6, 0.05^2)$. **a**, **c**: Beginning each time step, critical fractures (red) are identified. **b**, **d**: After a cascade of local fracture slips within the time step, shear stress of each critical fracture drops to its frictional resistance, which results in the global stress update, i.e., σ_3 increase.

The model's final stress state, or frictional equilibrium, is attained when iterations terminate. The most critical fracture in the system can be located. Retrospectively, the stress state and frictional resistance of the most critical fracture through the iterations can be traced, as illustrated in Figure 3A. Evidently, the fracture keeps slipping as long as its shear stress is larger than but converges towards its frictional resistance. The linear trace of the fracture frictional resistance can be interpreted as the equivalent frictional strength of the system, that is, μ = 0.43.

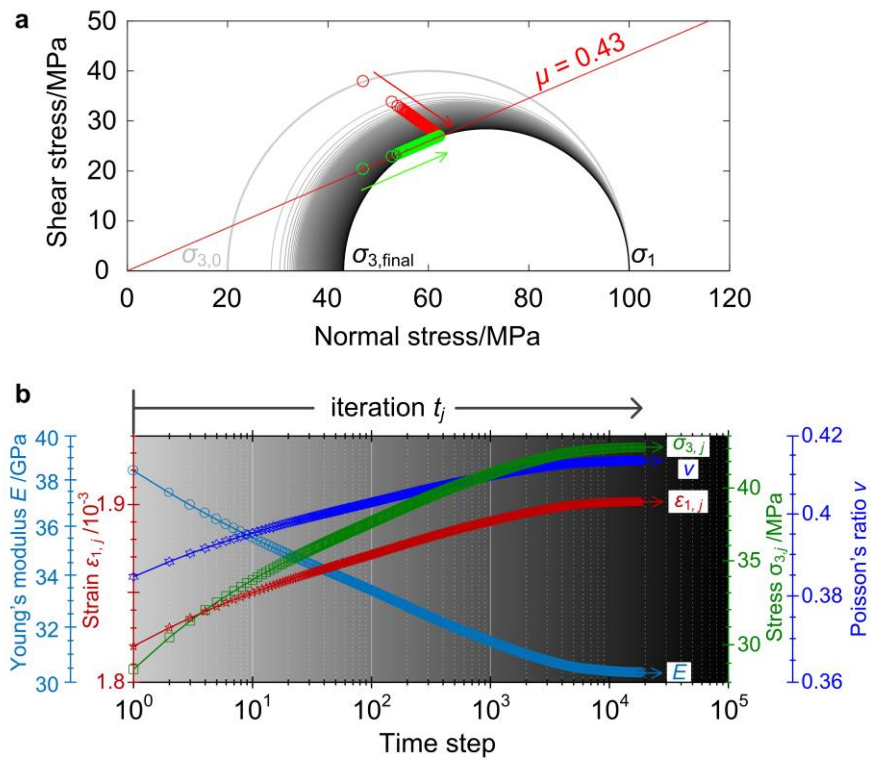

Figure 3. Overview of the temporal evolution of the model specified in Figure 2. **a** The stress relaxation due to cascades of fracture slips is illustrated by contracting Mohr diagrams. The final stress state rests on the frictional envelop (μ = 0.43), which is in fact controlled by the most critical fracture in the system. Red and green circles represent the resolved stress state and frictional resistance on the most critical fracture at each time step. **b** Evolving stress (σ), strain (ε), Young's modulus (E), and Poisson's ratio (v) through iterations. Note the rate of change in these parameters gradually diminishes towards the final stress state. Gray scale color scheme in **a** and **b** corresponds to iterations (logarithmic).

3.2 More on Heterogeneous Frictional Coefficient

Comparing this stochastic treatment with the deterministic case in which the frictional coefficients of all fractures are homogeneous (μ = 0.6, see Text S3), the final σ_3 upon equilibrium of the former is smaller than that of the latter, intuitively indicated in Figure 4A. Apparently, the maximum stress difference that can be sustained by the stochastic system does not depend on the mean or the upper bound of the frictional coefficient distribution. This is further demonstrated by two additional distributions, i.e., normal distribution $N(0.8, 0.06^2)$ and Weibull distribution with scale parameter λ = 0.8 and shape parameter k = 10, as illustrated in Figure 4B and 4C, respectively.

Further reviewing the stress evolution of each distribution, we identified that the frictional coefficient of the most critical fracture does not necessarily correspond to the lowest value, as one would assume, but it is located close to the lower bound of the distribution. Monte Carlo simulations shows such an observation is of high probability (Text S4). This suggests that the most critical fracture is determined jointly by its frictional coefficient and orientation with respect to the global stress. That said, the equivalent frictional strength of the fractured matrix is dependent on the combination of frictional coefficient distribution and orientations of all fractures. As indicated by Figure 4, the uncertainty of the equivalent frictional strength becomes more evident when the μ distribution departs further from uniformity, which also implies the degree of heterogeneity of global stress in such a stochastic system. Therefore, when inferring the state of stress in situ assuming frictional equilibrium, the practical value of frictional coefficient to be adopted is of utter importance.

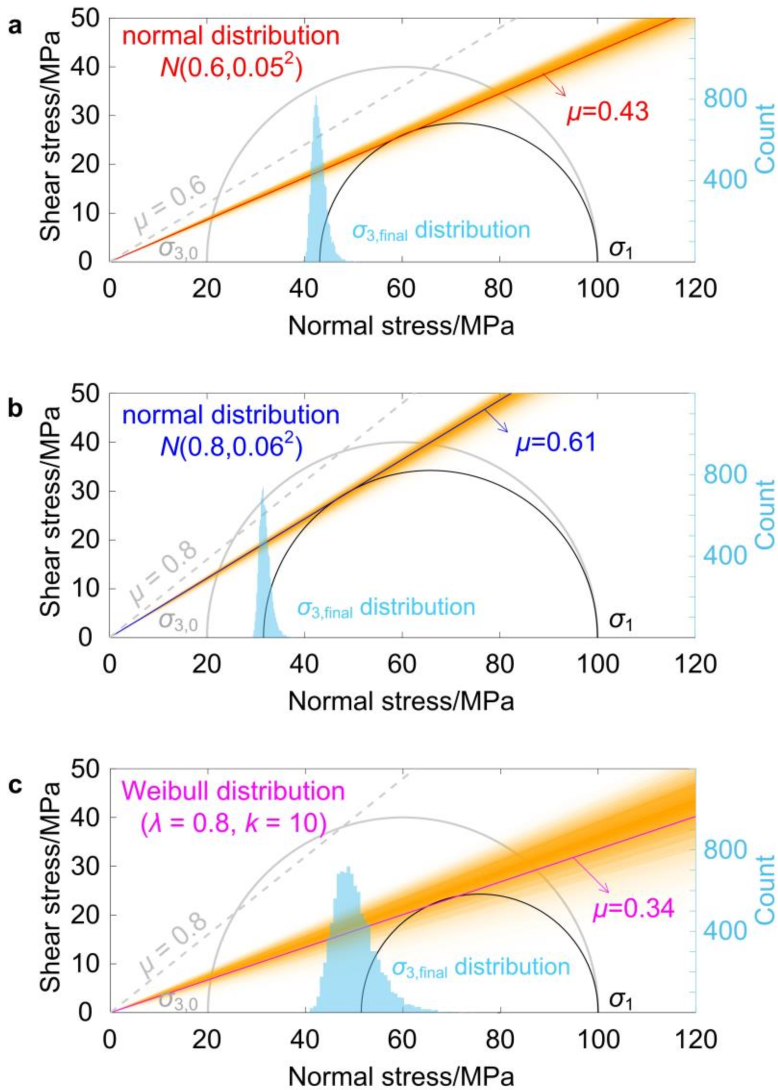

Figure 4. Initial (gray) and final (black) stress state for the same model configuration but different frictional coefficient distributions. The most probable final stress state, bounded by the corresponding equivalent frictional coefficient of the system, is associated with uncertainties (fuzzy lines or the distribution of σ_3, see Figure S6 for more details).

4 Conclusions

The model extends the notion of frictional equilibrium. For the deterministic interpretation in which the frictional coefficient is homogeneous, the frictional equilibrium refers to a state prescribed by Eq. (1). In a heterogeneous system, the frictional equilibrium can be understood as a dynamic process as illustrated in the model experimentation, spanning from the very first frictional slip to the final one possibly allowed by the prevailing in situ stress difference. The use of Eq. (1) in this instance therefore incurs great uncertainty. Informed by the numerous time steps leading to the final frictional equilibrium, the most critical fracture or fault stays critical while the rest of the crust experiences practically few slips, i.e., little global stress reduction. The apparent discrepancy between the local and global response reflects the stress heterogeneity within the system, which is jointly regulated by the variability of frictional coefficient and orientation of the fractures. To this end, the controversy over whether the upper crust is critically stressed is plausibly resolved. Again, we emphasize that the dominant mechanism of stress release in this context is frictional slip. Other mechanisms that may further lower the stress difference below frictional equilibrium in certain lithologies, such as viscoplastic deformation in shales (Sone and Zoback, 2014; Ma and Zoback, 2017), pressure solution in carbonates (Gunzburger and Cornet 2007; Gratier et al., 2013; Brantut et al., 2014), are not addressed here.

The evolution of stress reduction raises questions about the stage at which the current state of stress is with respect to the equilibrium. This is informative to stress estimation and fault slip tendency analysis. Note that in our model, the evolution iterates through 'pseudo' time steps and is not calibrated against real time. This is a compromise for computational feasibility and efficiency, so the interpretation in a temporal sense should be executed with caution. Nonetheless, the time-dependent stress reduction and matrix response appears to be reasonable and is deemed of first-order importance. Because of the difficulty to impose real time in the model, we were unable to experiment boundary conditions with prescribed strain or stress rate, which is more realistic in tectonically active regions. It is worth noting that no fracture interaction, extension and matrix damage was allowed in the model. If that was the case, the expected stress reduction will be more significant due to increased number and length of fractures and lowered equivalent matrix stiffness. The equivalent frictional strength of the crust will be even lower, in other words, the difference between the reality and the deterministic model will be more substantial.

Acknowledgments

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