Plasma waves in space: the importance of properly accounting for the measuring device

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Key Points:

- Measured plasma wave spectra are determined by the antenna geometry in a non-intuitive way
- We correct several recent papers on plasma spontaneous fluctuations or quasi-thermal noise
- We consider various space applications including Van Allen Probes and Parker Solar Probe

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Abstract

Electric fields are generally measured or calculated using two intuitive assumptions: (1) the electric field equals the voltage divided by the antenna length when the antenna is electromagnetically short, (2) the antenna responds best to electric field along its length. Both assumptions are often incorrect for electrostatic fields because they scale as the Debye length or as the electron gyroradius, which may be smaller than the antenna length. Taking into account this little-known fact enables us to complete or correct several recent papers on plasma spontaneous fluctuations in various solar system environments.

Plain Language Summary

Electric fields are measured in space by detecting the voltage across an electric antenna, and dividing this voltage by the antenna length (or an effective length taking into account geometry and receiver gain). The antenna is also assumed to respond best to electric field along its length. Both intuitive assumptions are correct for antennas shorter than the scale of variation of the field, and therefore for measuring electromagnetic waves with electromagnetically short antennas. However, in space, the measured power often stems from electrostatic waves, scaling as the plasma Debye length or the electron gyroradius, which are generally much smaller than the electromagnetic wavelength. In that case, the electric antenna may not be short compared to the scale of the electric field, so that the voltage is no longer proportional to the antenna length, nor maximum when the antenna lies along the field. This fact is generally ignored, producing incorrect results. We complete and correct several recent papers and discuss the adequate antenna response and directivity in various space environments.

1 Introduction

Most wave instruments in space use electric antennas of length exceeding several meters or tenth of meters in order to maximise sensitivity. Notable examples are ISEE-3 (length $L = 45$ m) (Knoll et al., 1978), Ulysses/URAP (35 m) (Stone et al., 1992), WIND/WAVES (50 m) (Bougeret et al., 1995), Cassini/RPWS (10 m) (Gurnett et al., 2004), Van Allen Probes wave instrument (50 m) (Wygant et al., 2013), Bepicolombo/MMO/SORBET (15 m) (Moncuquet et al., 2006), Parker Solar Probe/FIELDS (2 m) (Bale et al., 2016), and Solar Orbiter/RPW (6.5 m) (Maksimovic et al., 2005).

Electric power spectra are generally deduced by dividing the voltage power spectrum by the square of the antenna length L (or an effective length taking into account geometry and spacecraft structure as well as the receiver gain (e.g., Fischer et al., 2001), Pulupa et al. (2019)) when the antennas are shorter than the electromagnetic wavelength. Unfortunately, at frequencies of the order of magnitude of the plasma characteristic frequencies, the measured power generally stems from electrostatic waves, which scale as the plasma Debye length L_D or the electron gyroradius r_g. These latter scales are in practice much smaller than the electromagnetic scale c/ω (ω is the angular frequency and c the velocity of light) and are often smaller than L. In that case, even though the antennas are electromagnetically short ($\omega L/c \ll 1$), they are not smaller than the scale of electrostatic waves or fluctuations, so that the voltage is not proportional to L (nor to any effective length). This occurs for instance in the solar wind ($L_D \simeq 10$ m at 1 AU to less than 1 m at Parker Solar Probe (PSP) perihelion (Bale et al., 2016)), in cometary plasma tails ($L_D \simeq 1$ m (Meyer-Vernet et al., 1986)), and in planetary environments such as the plasma torus of Jupiter's satellite Io explored by Ulysses ($r_g \simeq 10$ m $> L_D$ (Meyer-Vernet, 2001)), the plasma torus of Saturn's satellite Enceladus explored by Cassini ($r_g \simeq 7$ m $> L_D$ (Moncuquet et al., 2005)), or the

Earth plasmasphere explored by WIND (Issautier et al., 2001), by IMAGE (Reinisch et al., 2000) or by the Van Allen Probes (e.g., Kurth et al., 2015)).

The antenna response must therefore be carefully taken into account, especially when calculations of spontaneous plasma fluctuations are used to measure plasma properties via quasi-thermal noise (QTN) spectroscopy (Meyer-Vernet et al., 2017, and references therein). Nevertheless, a common practice is to deduce the measured electric field by merely dividing the voltage by the antenna length, whereas on the theoretical side, a number of recent interesting calculations of spontaneous plasma fluctuations or QTN approximated the antenna response inadequately (e.g., Yoon, 2014; Yoon et al., 2017). A related question concerns the antenna directivity. Contrary to common belief, and in particular to a recent claim (Yoon et al., 2019), an electric antenna shorter than the EM wavelength does not always respond best to electric field along its length. When the power is dominated by electrostatic waves or fluctuations, this intuitive response does not generally hold for a wire antenna longer than the plasma Debye length or the electron gyroradius.

2 Antenna response and directivity

For electrostatic fields ($\omega/kc \ll 1$) with longitudinal fluctuations ($\mathbf{E} \parallel \mathbf{k}$, the wave vector) given in Fourier space by $\langle E^2(\mathbf{k},\omega)\rangle$, the voltage power spectrum is

$$V_f^2 = \frac{2}{(2\pi)^3}\int d^3k\ \frac{|\mathbf{k}.\mathbf{J}|^2}{k^2}\ \langle E^2(\mathbf{k},\omega)\rangle \tag{1}$$

where

$$|\mathbf{k}.\mathbf{J}| = \Big|\Big[\frac{4\sin^2(kL\cos\alpha/2)}{kL\cos\alpha}\Big]J_0(ka\sin\alpha)\Big| \tag{2}$$

for a cylindrical antenna made of two aligned wires of length L and radius a, making the angle α with $\mathbf{k}$ (Figure 1).

Since the spontaneous electrostatic fluctuations have $k \sim L_D^{-1}$ or r_g^{-1} (of typical orders of magnitude given in Section 1), the antenna radius, generally a few millimetres or smaller, satisfies $ka \ll 1$ so that $J_0(ka\sin\alpha) \simeq 1$, except in very dense and cold plasmas such as planetary ionospheres not considered here. Hence the antenna response (2) is determined by the factor in brackets. Therefore, whereas an antenna satisfying $kL \ll 1$, whence $|\mathbf{k}.\mathbf{J}| \simeq kL\cos\alpha$, responds best to $\mathbf{k}$ parallel to the antenna direction ($\alpha = 0$) in agreement with intuition, a long antenna ($kL \gg 1$) instead responds best to wave vectors satisfying $\cos\alpha \simeq \pi/kL \ll 1$, whence $\alpha \simeq \pi/2$ (Meyer-Vernet, 1994) (Figure 1, top).

If the fluctuations are isotropic, the 3-D integral (1) simplifies to

$$V_f^2 = \frac{8}{\pi^2}\int_0^\infty dk\ F(kL)\langle E^2(k,\omega)\rangle \tag{3}$$

where the function $F(x)$, given by Meyer-Vernet and Perche (1989), has a maximum for $x \sim \pi$, whereas $F(x) \propto x^2$ for $x \ll 1$ and $F(x) \propto 1/x$ for $x \gg 1$.

In the solar wind, where the electrostatic fluctuations scale as L_D, which is generally a few times smaller than L as mentioned in Section 1, the response $F(kL)$ is close to maximum, and properties of the electron velocity distribution can be accurately deduced from the observed spectrum by inversion of (3). In that case Meyer-Vernet et al. (2017) have shown that V_f^2 depends weakly on L for $f < f_p$, has a peak at the plasma frequency f_p of shape depending on L, and is proportional to $1/L$ for $f \gg f_p$. Such a dependence illustrates the inadequacy of the usual paradigm according to which E_f^2 can be obtained by dividing V_f^2 by L^2.

This inadequacy appears still more clearly when $L \ll L_D$. In that case, the QTN power $V_f^2 \propto (L/L_D)^2 \ln(L_D/L)$ (Meyer-Vernet & Perche, 1989), so that $V_f^2/L^2 \to \infty$ for $L \to 0$. The origin of this surprising result is that the above formula does not hold for $L \to 0$ because in a plasma at temperature T the antenna length should satisfy $L > r_L = e^2/(4\pi\epsilon_0 k_B T)$, since the usual plasma approximations do not hold at scales $< r_L$ (e.g., Meyer-Vernet, 1984). Since $r_L \simeq 1.4/T_{\rm eV}$ nanometers ($T_{\rm eV}$ being the temperature in eV), the case $L < r_L$ never occurs in practice, except formally in theoretical calculations of E_f^2 that assume implicitly $L \to 0$.

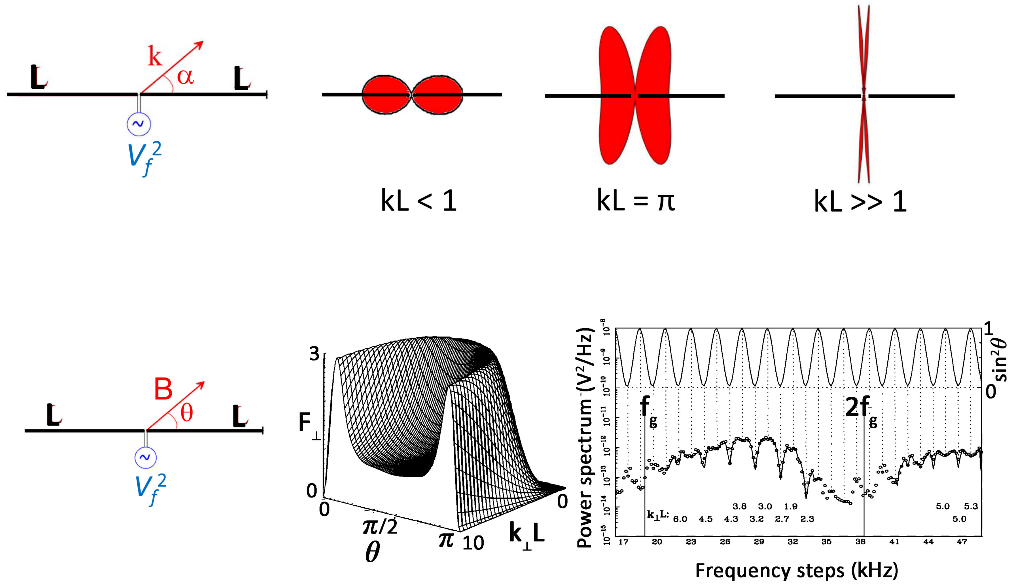

Figure 1. Top. Reception pattern (2) of a short (left), half-wave (middle), and long antenna (right), showing that whereas a short antenna responds best to electric field along its length, the opposite is true for a long antenna. Bottom. Antenna response $F_\perp$ for $\mathbf{k}$ nearly perpendicular to the magnetic field $\mathbf{B}$ plotted as a function of $k_\perp L$ and θ, showing the minimum at $\theta = \pi/2$ when $k_\perp L \gg 1$ (left); example of power spectrum recorded by Ulysses/URAP as the spacecraft spins, making the angle θ between the antenna and $\mathbf{B}$ vary, with the corresponding values of $k_\perp L$ indicated (right); from Moncuquet et al. (1995).

In magnetized low-β plasmas where transverse and longitudinal modes decouple, the quasi-thermal electrostatic fluctuations near the upper-hybrid frequency and between gyroharmonics nf_g propagate nearly perpendicular to the magnetic field $\mathbf{B}$, i.e. $|k_\|| \ll |k_\perp|$ where subscripts relate to the direction of $\mathbf{B}$ (Sentman, 1982). In that case Meyer-Vernet et al. (1993) has shown that V_f^2 varies with the angle θ between the antenna and the magnetic field as $F_\perp(k_\perp L \sin\theta)$, where $F_\perp$ (Figure 1, bottom middle) varies qualitatively as $F(x)$ defined above. Hence $V_f^2 \propto L^2 \sin^2\theta$ when $k_\perp L \ll 1$, in agreement with intuition. In contrast, when $k_\perp L \gg 1$, the voltage has minima at $\theta = \pi/2$, since in that case $\mathbf{k}$ lies nearly along the antenna ($\alpha \simeq 0$). Figure 1 (bottom right) shows an observed example of this anti intuitive behavior, around and between the first two gyroharmonics; the predicted minima at $\theta = \pi/2$ appear clearly in the lower end of the harmonic bands, where $k_\perp r_g \gg 1$ (Moncuquet et al., 1995).

3 When theory and practice differ and how to make them agree

We show below two examples of calculations of spontaneous fluctuations or QTN spectrum assuming inadequate antenna responses. Yoon (2014) used an equation equivalent to putting in (3) a function $F(kL) = \pi^2/(4k^2)$, instead of the correct response decreasing as k^2L^2 when $kL \ll 1$. This function amplifies so much the small k fluctuations that it would produce an infinite QTN spectrum for $f \lesssim f_p$ in a stable isotropic unmagnetized plasma (Meyer-Vernet et al., 2017). Yoon et al. (2017) used an equation equivalent to putting $F(kL) = \pi^2/4$ in (3), which amplifies arbitrarily the fluctuations at both small and large k. Both functions, which do not represent the response of any physical electric antenna, prevent any quantitative comparison between theory and observations to be made.

Consider now the variation of the antenna response with its orientation. Yoon et al. (2019) aimed at complementing the paper by Meyer-Vernet et al. (1993), that they inadequately cited as (Moncuquet et al., 1993), by determining the direction of $\mathbf{k}$ to be chosen in their QTN calculations. They asserted that the antenna response (2) is maximized when the antenna is oriented along $\mathbf{k}$ because of the factor J_0. However, as shown in section 2, the antenna response (2) is in general (and in the application they consider) determined instead by the factor in brackets, so that the antenna responds best to electric field along its length only when $kL < 1$ (Figure 1).

Yoon et al. (2019) analyzed an interesting observation of gyroharmonic bands by the Van Allen Probes (VAP) wave instrument (Wygant et al., 2013). These bands were observed when the spin axis made a small angle with $\mathbf{B}$, so that the spinning antennas lied approximately in a plane perpendicular to $\mathbf{B}$, i.e. $\theta \simeq \pi/2$. Since the quasi-thermal fluctuations between gyroharmonics have $|k_\parallel| \ll k_\perp$ (Sentman, 1982), the antennas were adequately oriented for detection if $F_\perp$ was maximum for $\theta \simeq \pi/2$, which requires that $k_\perp L \lesssim 3$ from Figure 1. With $L = 50$ m, the gyrofrequency $f_g \simeq 4$ kHz, whence $r_g \simeq 20$ m (assuming $T \simeq 1$ eV), this inequality is equivalent to $k_\perp r_g \lesssim 1.5$. This latter range lies in the upper part of the harmonic bands (Bernstein, 1958), which explains the spectrogram shown by Yoon et al. (2019). Indeed, although these bands are called $(n+1/2)f_h$ bands, it is only for short antennas and $\theta \simeq \pi/2$ that the emissions are observed in the middle of the bands (Moncuquet et al., 1997). Our calculations (Meyer-Vernet et al., 1993) could also be used to deduce the temperature of energetic electrons from the observed amplitudes. Note that the applicability of (2), used by Yoon et al. (2019), is uncertain because of the geometry of the VAP antennas which are made of spheres mounted on wires (Wygant et al., 2013).

4 Final remarks

Taking into account the antenna response is essential in implementing QTN spectroscopy on PSP/FIELDS (Moncuquet et al., 2019). When the probe will reach its perihelion near 10 solar radii, one expects $f_g \simeq 60$ kHz, $f_p \simeq 750$ kHz, $L_D \simeq 0.8$ m and $r_g \simeq 15$ m, with considerable fluctuations (Bale et al., 2016). The spacecraft carries two $L = 2$ m electric dipoles perpendicular to the spacecraft axis oriented towards the Sun, so that the antennas should be perpendicular to the average direction of $\mathbf{B}$, and therefore in general oriented best to detect the QTN between gyroharmonics if $k_\perp L < 3$, whence $k_\perp r_g \lesssim 20$ from the parameters sketched above. The first gyroharmonic bands should be difficult to detect because they should be hidden by the shot noise (Meyer-Vernet et al., 2017); however, the upper ones might possibly be detected in the presence of enough suprathermal electrons if the ratio f_g/f_p is not too small, in order that the upper gyroharmonic bands below f_p have $k_\perp r_g$ in the range defined above.

In the present paper, we have shown the importance of carefully taking into account the response of the electric antennas to study plasma spontaneous fluctuations or electrostatic waves, or implementing QTN spectroscopy. Since the scale of these fluctuations or waves is often smaller than the antenna length, the antenna response depends on the scale, so that the electric field cannot be deduced by merely dividing the voltage by the antenna length, nor by any effective length, contrary to common practice.

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References

Bale, S. D., Goetz, K., Harvey, P. R., Turin, P., Bonnell, J. W., Dudok de Wit, T., ... Wygant, J. R. (2016, Dec). The FIELDS Instrument Suite for Solar Probe Plus. Measuring the Coronal Plasma and Magnetic Field, Plasma Waves and Turbulence, and Radio Signatures of Solar Transients. *Space Sci. Rev.*, *204*(1-4), 49-82. doi: 10.1007/s11214-016-0244-5

Bernstein, I. B. (1958, Jan). Waves in a Plasma in a Magnetic Field. *Physical Review*, *109*(1), 10-21. doi: 10.1103/PhysRev.109.10

Bougeret, J. L., Kaiser, M. L., Kellogg, P. J., Manning, R., Goetz, K., Monson, S. J., ... Hoang, S. (1995, Feb). Waves: The Radio and Plasma Wave Investigation on the Wind Spacecraft. *Space Sci. Rev.*, *71*(1-4), 231-263. doi: 10.1007/BF00751331

Fischer, G., Macher, W., Rucker, H. O., Ladreiter, H. P., Vogl, D. F., & Cassini/RPWS Team. (2001, Jan). Wire-grid Modeling of Cassini Spacecraft for the Determination of Effective Antenna Length Vectors of the RPWS Antennas. In *Planetary radio emissions v* (p. 347-356).

Gurnett, D. A., Kurth, W. S., Kirchner, D. L., Hospodarsky, G. B., Averkamp, T. F., Zarka, P., ... Pedersen, A. (2004, Sep). The Cassini Radio and Plasma Wave Investigation. *Space Sci. Rev.*, *114*(1-4), 395-463. doi: 10.1007/s11214-004-1434-0

Issautier, K., Moncuquet, M., Meyer-Vernet, N., Hoang, S., & Manning, R. (2001, Jun). Quasi-Thermal Noise Diagnostics in Space Plasmas. *Astrophys. Space Sci.*, *277*, 309-311. doi: 10.1023/A:1012281730151

Knoll, R., Epstein, G., Hoang, S., Huntzinger, G., Steinberg, J. L., Fainberg, J., ... Stone, R. G. (1978, Jul). The 3-dimensional radio mapping experiment (SBH) on ISEE-C. *IEEE Transactions on Geoscience Electronics*, *16*, 199-204. doi: 10.1109/TGE.1978.294546

Kurth, W. S., De Pascuale, S., Faden, J. B., Kletzing, C. A., Hospodarsky, G. B., Thaller, S., & Wygant, J. R. (2015, Feb). Electron densities inferred from plasma wave spectra obtained by the Waves instrument on Van Allen Probes. *J. Geophys. Res. Space Phys.*, *120*(2), 904-914. doi: 10.1002/2014JA020857

Maksimovic, M., Issautier, K., Meyer-Vernet, N., Perche, C., Moncuquet, M., Zouganelis, I., ... Bougeret, J. L. (2005, Jan). Solar wind electron temperature and density measurements on the Solar Orbiter with thermal noise spectroscopy. *Adv. Space Res.*, *36*(8), 1471-1473. doi: 10.1016/j.asr.2005.01.088

Meyer-Vernet, N. (1984, Jul). Surprises in classical physics: radiation problems in stable and linear plasmas. *Eur. J. Phys.*, *5*(3), 150-156. doi: 10.1088/0143-0807/5/3/005

Meyer-Vernet, N. (1994, Mar). On the thermal noise "temperature" in an anisotropic plasma. *Geophys. Res. Lett.*, *21*(6), 397-400. doi: 10.1029/94GL00197

Meyer-Vernet, N. (2001, Mar). Large scale structure of planetary environments: the importance of not being Maxwellian. *Planet. Space Sci.*, *49*(3-4), 247-260. doi: 10.1016/S0032-0633(00)00146-X

Meyer-Vernet, N., Couturier, P., Hoang, S., Perche, C., & Steinberg, J. L. (1986, Mar).

Physical parameters for hot and cold electron populations in comet Giacobini-Zinner with the ICE Radio Experiment. *Geophys. Res. Lett.*, *13*(3), 279-282. doi: 10.1029/GL013i003p00279

Meyer-Vernet, N., Hoang, S., & Moncuquet, M. (1993, Dec). Bernstein waves in the Io plasma torus: A novel kind of electron temperature sensor. *J. Geophys. Res.*, *98*(A12), 21163-21176. doi: 10.1029/93JA02587

Meyer-Vernet, N., Issautier, K., & Moncuquet, M. (2017, Aug). Quasi-thermal noise spectroscopy: The art and the practice. *J. Geophys. Res. Space Phys.*, *122*(8), 7925-7945. doi: 10.1002/2017JA024449

Meyer-Vernet, N., & Perche, C. (1989, Mar). Tool kit for antennae and thermal noise near the plasma frequency. *J. Geophys. Res.*, *94*, 2405-2415. doi: 10.1029/JA094iA03p02405

Moncuquet, M., Lecacheux, A., Meyer-Vernet, N., Cecconi, B., & Kurth, W. S. (2005, Jun). Quasi thermal noise spectroscopy in the inner magnetosphere of Saturn with Cassini/RPWS: Electron temperatures and density. *Geophys. Res. Lett.*, *32*(20), L20S02. doi: 10.1029/2005GL022508

Moncuquet, M., Matsumoto, H., Bougeret, J. L., Blomberg, L. G., Issautier, K., Kasaba, Y., … Zarka, P. (2006, Jan). The radio waves and thermal electrostatic noise spectroscopy (SORBET) experiment on BEPICOLOMBO/MMO/PWI: Scientific objectives and performance. *Adv. Space Res.*, *38*(4), 680-685. doi: 10.1016/j.asr.2006.01.020

Moncuquet, M., Meyer-Vernet, N., & Hoang, S. (1995, Nov). Dispersion of electrostatic waves in the Io plasma torus and derived electron temperature. *J. Geophys. Res.*, *100*(A11), 21697-21708. doi: 10.1029/95JA02299

Moncuquet, M., Meyer-Vernet, N., Hoang, S., Forsyth, R. J., & Canu, P. (1997, Feb). Detection of Bernstein wave forbidden bands in the Jovian magnetosphere: A new way to measure the electron density. *J. Geophys. Res.*, *102*(A2), 2373-2380. doi: 10.1029/96JA03313

Moncuquet, M., Meyer-Vernet, N., Issautier, K., Pulupa, M., Bonnell, J. W., Bale, S. D., … Malaspina, D. M. (2019, Dec). First in-situ Measurements of Electron Density and Temperature from Quasi-Thermal Noise Spectroscopy with Parker Solar Probe/FIELDS. *ApJS*, arXiv:1912.02518.

Reinisch, B. W., Haines, D. M., Bibl, K., Cheney, G., Galkin, I. A., Huang, X., … Reiff, P. (2000, Jan). The Radio Plasma Imager investigation on the IMAGE spacecraft. *Space Sci. Rev.*, *91*, 319-359.

Sentman, D. D. (1982, Mar). Thermal fluctuations and the diffuse electrostatic emissions. *J. Geophys. Res.*, *87*(A3), 1455-1472. doi: 10.1029/JA087iA03p01455

Stone, R. G., Bougeret, J. L., Caldwell, J., Canu, P., de Conchy, Y., Cornilleau-Wehrlin, N., … Goldstein, M. L. (1992, Jan). The Unified Radio and Plasma wave investigation. *Astron. Astrophys. Suppl. Ser.*, *92*(2), 291-316.

Wygant, J. R., Bonnell, J. W., Goetz, K., Ergun, R. E., Mozer, F. S., Bale, S. D., … Tao, J. B. (2013, Nov). The Electric Field and Waves Instruments on the Radiation Belt Storm Probes Mission. *Space Sci. Rev.*, *179*(1-4), 183-220. doi: 10.1007/s11214-013-0013-7

Yoon, P. H. (2014, Sep). Electron kappa distribution and quasi-thermal noise. *J. Geophys. Res. Space Phys.*, *119*(9), 7074-7087. doi: 10.1002/2014JA020353

Yoon, P. H., Hwang, J., Kim, H., & Seough, J. (2019, Apr). Quasi Thermal Noise Spectroscopy for Van Allen Probes. *J. Geophys. Res. Space Phys.*, *124*(4), 2811-2818. doi: 10.1029/2019JA026460

Yoon, P. H., Kim, S., Hwang, J., & Shin, D.-K. (2017, May). Upper hybrid waves and energetic electrons in the radiation belt. *J. Geophys. Res. Space Phys.*, *122*(5), 5365-5376. doi: 10.1002/2016JA023321