

916 *Journal of Geophysical Research: Machine Learning and Computation*

917 Supporting Information for

918 **Air Quality Estimation and Forecasting via Data Fusion with Uncertainty**
919 **Quantification: Theoretical Framework and Preliminary Results**

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929 **Contents of this file**

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935 **Introduction**

936 This document provides supplemental supporting information for the manuscript indicated
937 above. This includes a section (S1) detailing the handling of data from low-cost air quality
938 sensors (LCS), as alluded to in Section 2.2.3. Additional results to supplement those presented in
939 Section 3 are provided in Figure S5 through Figure S9. Diagrams of the various phases of the
940 data fusion process are also illustrated in Figure S1 through Figure S4.

941 Note also that the data used to generate the results and figures presented here and are
942 available in an [online Zenodo archive](#) (Malings, 2024), governed under a [CC BY-NC](#) License.

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944 **Text S1. Details of the supplemental New York City case study example**

945 For the supplemental study area of interest is the region surrounding New York City, New
 946 York, USA (defined as between 40°N and 42°N and between 73°W and 75°W). Data sources
 947 were the same as indicated in the paper for the San Francisco study area. Data from calendar
 948 year 2019 were included as potential inputs for calibration purposes.

949 **Text S2. Handling less reliable in-situ data from low-cost monitors**

950 In the case of data from LCS, there are typically concerns associated with using the raw
 951 output data from these sensors. It is preferred that these data be calibrated to nearby RGM, with
 952 these calibrations usually being regionally specific, i.e., a single calibration approach is typically
 953 unsuitable beyond the region where it was developed (Giordano et al., 2021; McFarlane, Raheja,
 954 et al., 2021). Wherever possible, such regionally specific calibrations should be applied to LCS
 955 data before they are considered in this data fusion approach. However, due to the relative lack
 956 of RGM for conducting such calibration (a major motivation for data fusion approaches in the
 957 first place), such a local calibration may be lacking. In that case, the data fusion approach itself
 958 could be used to provide necessary data to conduct a crude regional calibration.

959 To address data from LCS with lower reliability and potentially large biases, we propose to
 960 apply a linear calibration approach, where data collected by LCS, $\mathbf{G}_{LCS}(x, t)$, provide the
 961 independent variable. The phase 3 estimates, $E_3(x, t)$, which include any RGM information in the
 962 area but not LCS information, provide the dependent variable. In regions lacking any RGM, the
 963 phase 2 estimate $E_2(x, t)$ may be used instead. As a vector quantity, $\mathbf{G}_{LCS}(x, t)$ may include
 964 important ancillary data such as temperature and humidity measurements, which are often
 965 important in calibrating LCS, together with measurements of the target pollutant. Regression is
 966 conducted considering a time interval T_c and the set of discrete surface monitoring sites with
 967 LCS in the region X_{LCS} :

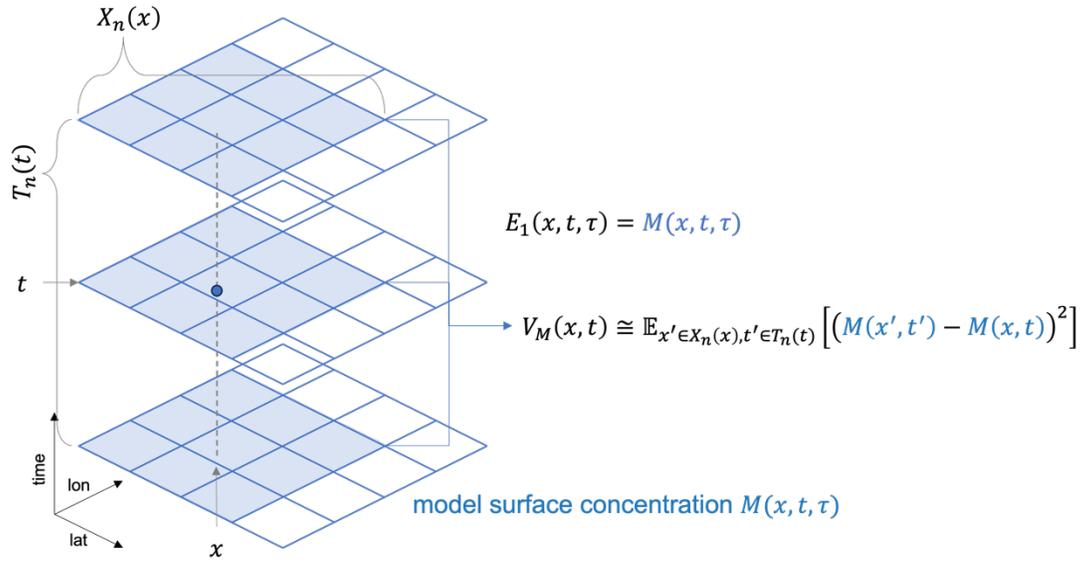
$$968 \quad \boldsymbol{\zeta}, \xi, \mathbf{V}_{\boldsymbol{\zeta}}, V_{\xi}, \mathbf{V}_{\boldsymbol{\zeta}\xi}, V_{R,LCS} = \mathbb{L}\mathbb{R}_{t' \in T_c(t), x' \in X_{LCS}} [E_3(x', t') \sim \mathbf{G}_{LCS}(x', t')]. \quad (S1)$$

969 The linear regression is then applied to the raw LCS data:

$$970 \quad G_{LCS,calibrated}(x, t) = \boldsymbol{\zeta} \cdot \mathbf{G}_{LCS}(x, t) + \xi, \quad (S2)$$

971 where \cdot denotes a dot product. The calibrated LCS data are then used in phase 4 to provide
 972 information for local updating of the estimates in their vicinities. In doing so, the relatively
 973 higher measurement uncertainties of these LCS should be considered when evaluating
 974 $K(x, x', t, t')$. These uncertainties can be quantified using the regression residual variance $V_{R,LCS}$.
 975 Note that since this calibration approach seeks to match, on a regional basis and for an
 976 extended calibration period, the LCS data to the phase 3 data fusion estimates, including these
 977 calibrated data back into the phase 3 estimation would be redundant. Once calibrated, however,
 978 individual LCS can provide valuable local and near-real-time information, and so including these
 979 data in phase 4 is potentially beneficial.

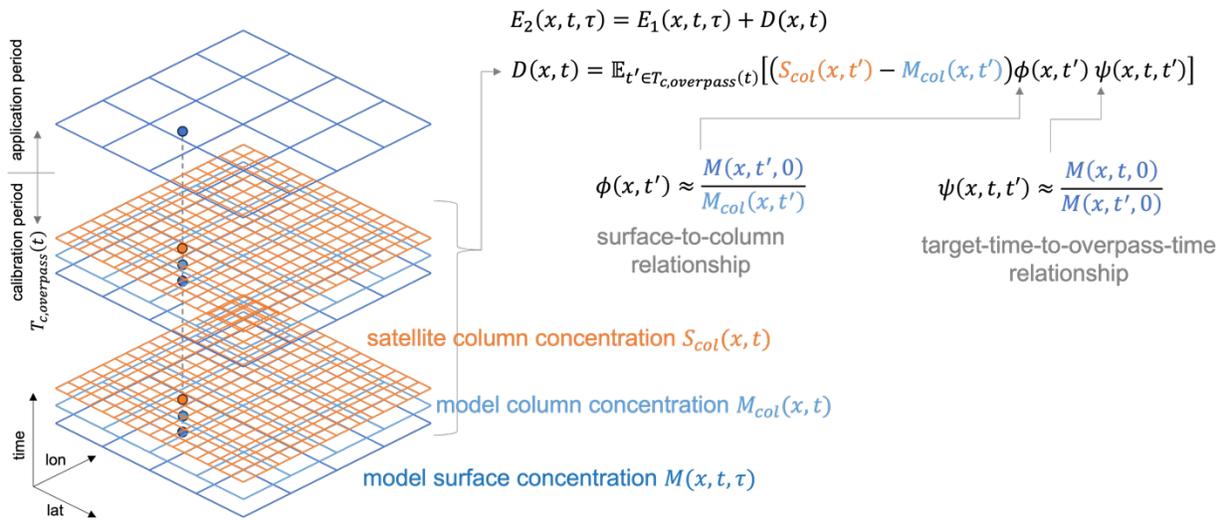
980 This approach is most suited to networks of LCS containing multiple devices with high
 981 inter-sensor precision and where the network is broadly distributed at a representative set of
 982 locations over the region of interest. In situations where inter-sensor precision is low, few LCS
 983 and no RGM are available, and/or where LCS deployments over-represent specific environments,
 984 especially near-source environments, this approach is likely to perform poorly.



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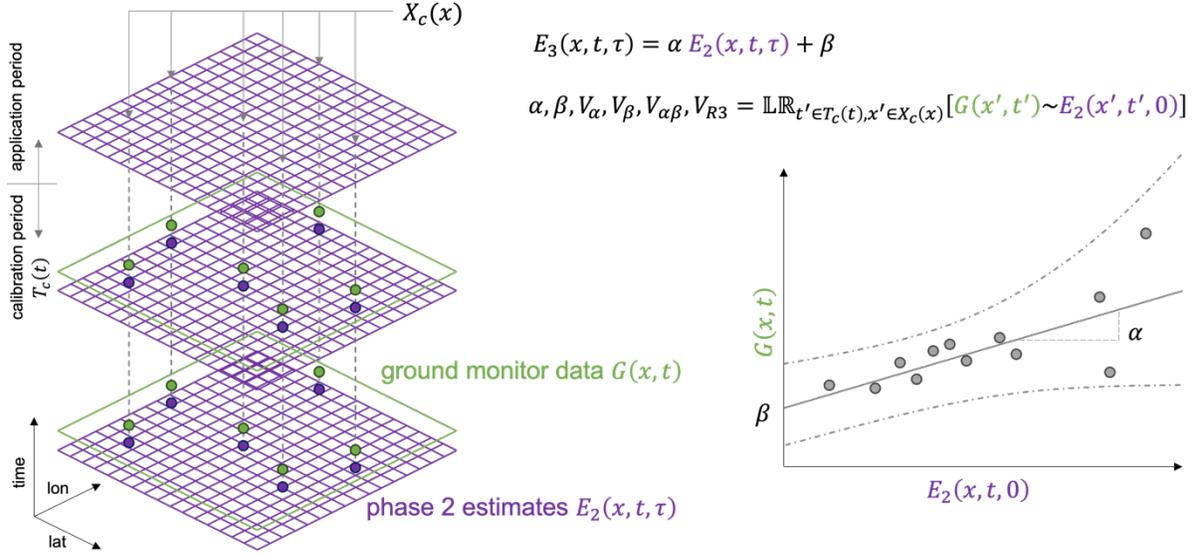
986 **Figure S1. Diagram of phase 1 of the data fusion process. Blue grids denote model grids in**
 987 **space, with different layers denoting different timesteps. Shaded grids indicate the**
 988 **neighborhood of the grid cell corresponding to location x and time t , used for estimation**
 989 **of model variability.**

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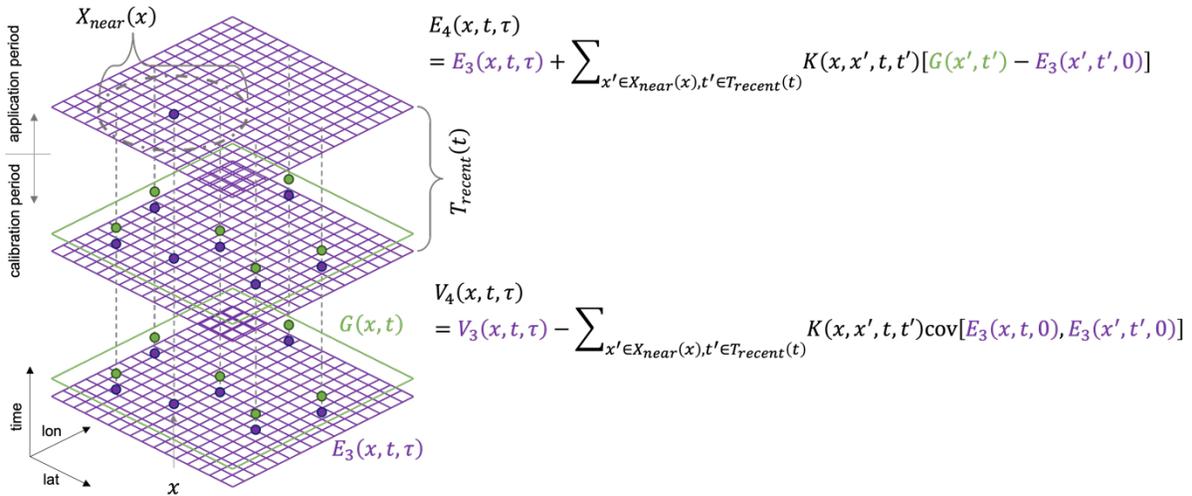
992 **Figure S2. Diagram of phase 2 of the data fusion process. Orange grids denote satellite**
 993 **remote sensing data, with light blue grids corresponding to the analogous modeled column**
 994 **quantity.**



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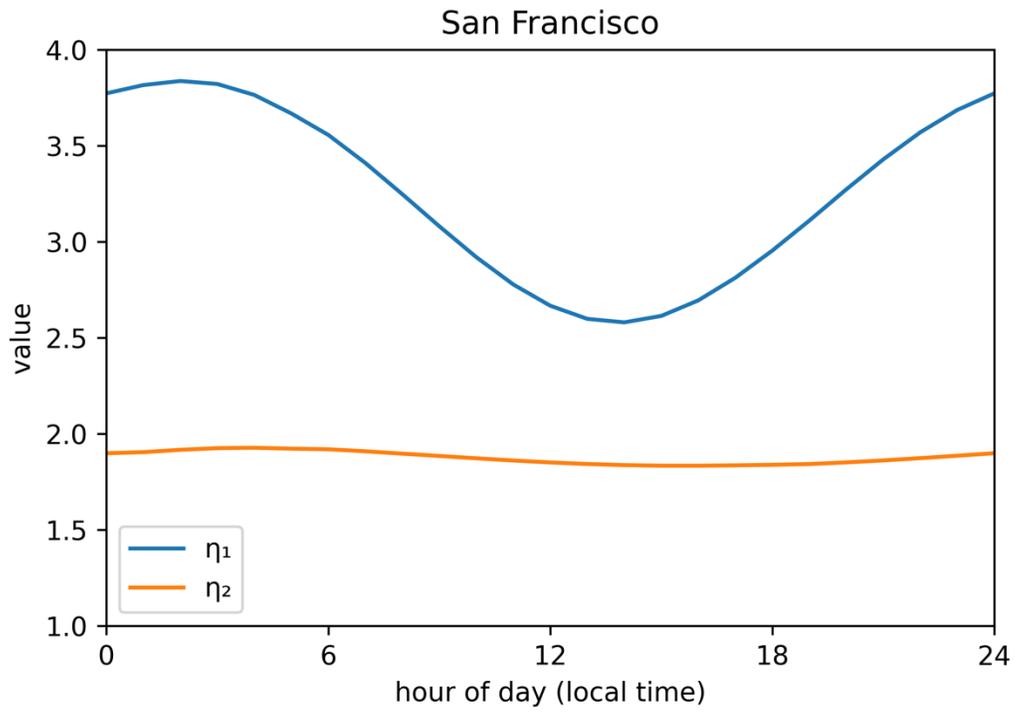
996 **Figure S3. Diagram of phase 3 of the data fusion process. Purple grids correspond to the**
 997 **phase 2 estimates. Green points indicate ground measurements at monitor sites $X_c(x)$**
 998 **collected during calibration period $T_c(t)$. A conceptual illustration of the linear regression**
 999 **process is provided on the right.**

1000



1001

1002 **Figure S4. Diagram of phase 4 of the data fusion process. The nearby region used for this**
 1003 **phase, $X_{near}(x)$, is denoted with a grey ring. Recent times $T_{recent}(t)$ are considered to be the**
 1004 **last timestep in the calibration period.**

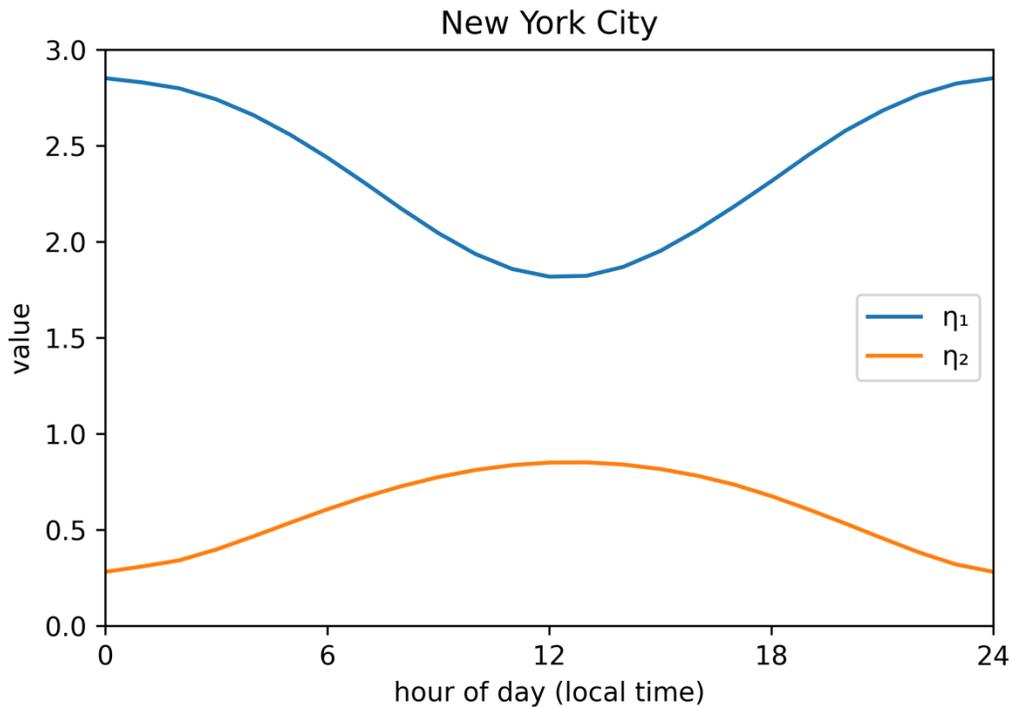


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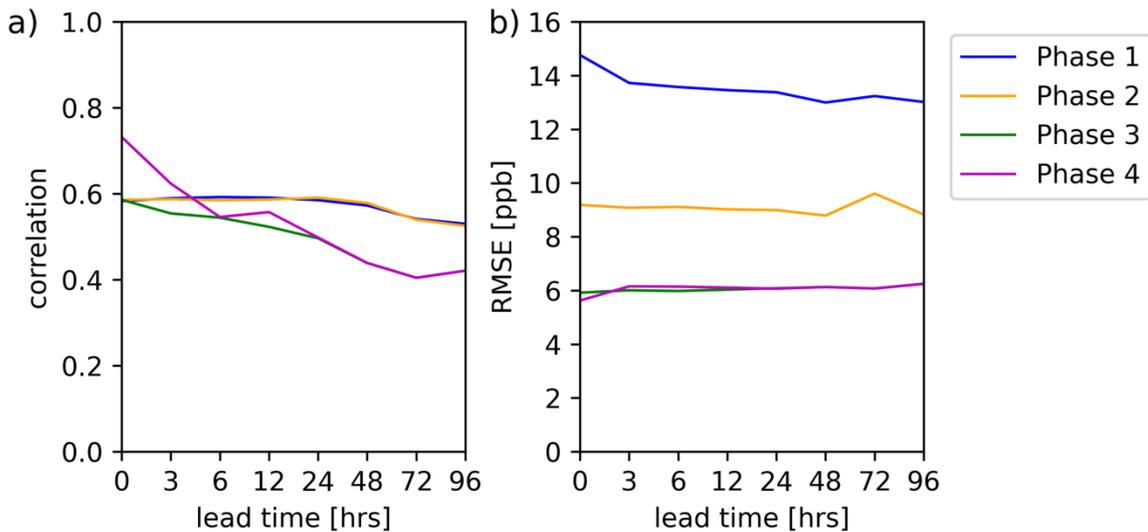
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Figure S5. Empirically determined values for η_1 and η_2 used for San Francisco in this paper, as a function of hour of the day (presented in local time).



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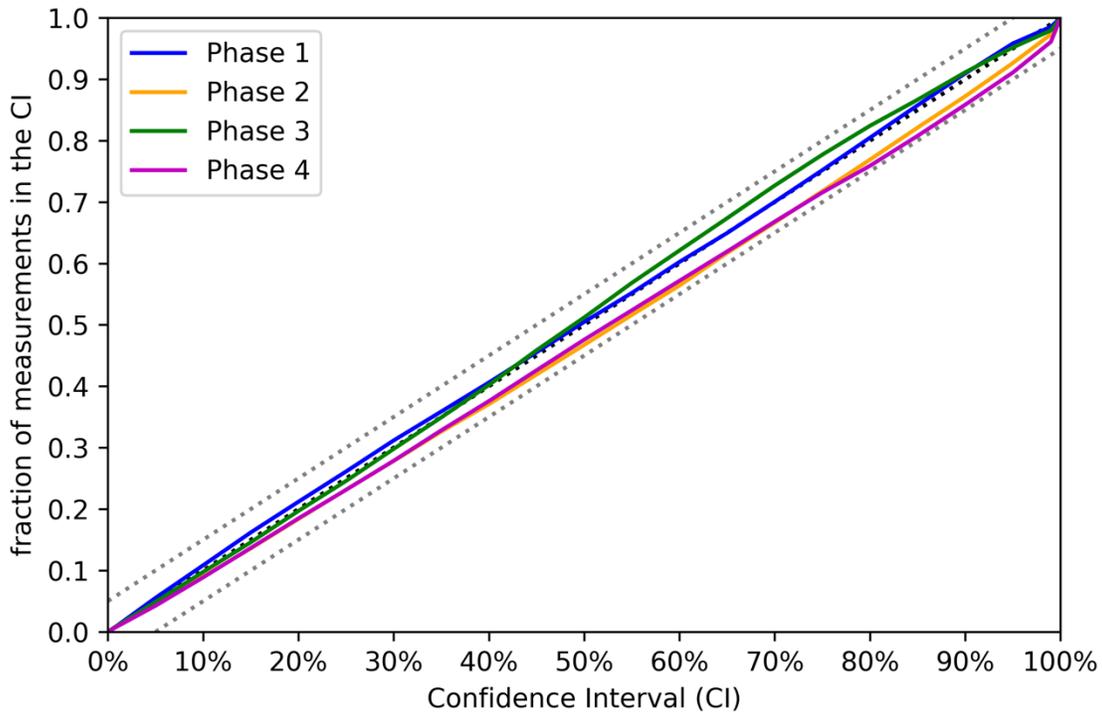
1009 **Figure S6. Empirically determined values for η_1 and η_2 used for New York City in this paper,**
 1010 **as a function of hour of the day (presented in local time).**



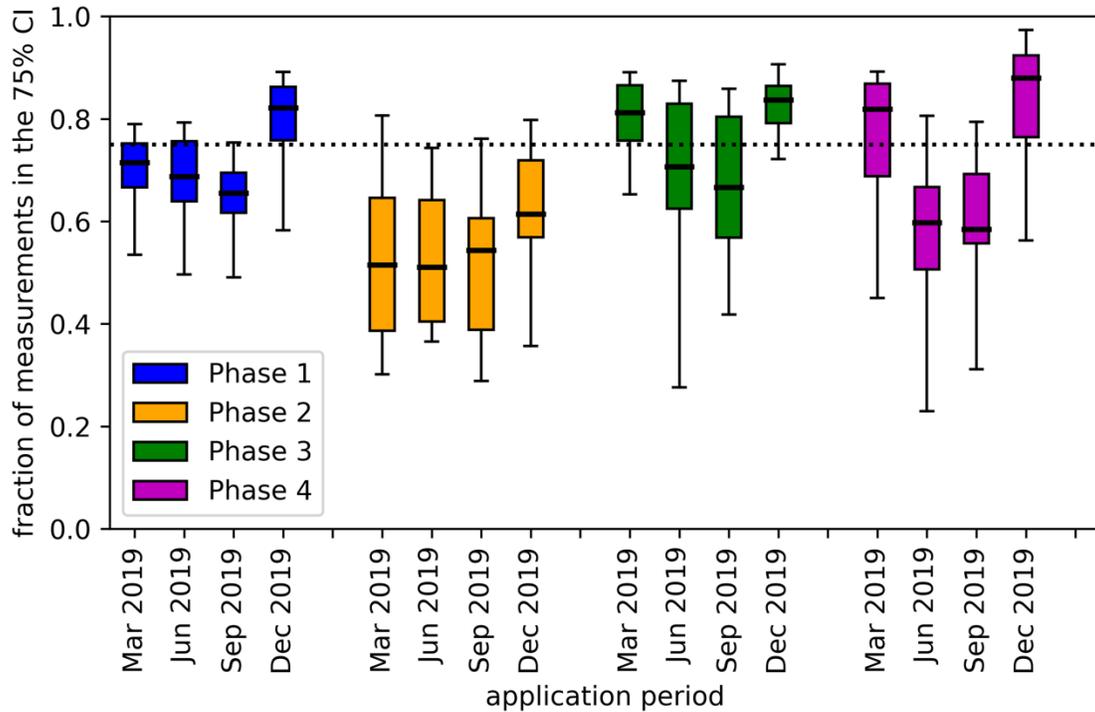
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1012 **Figure S7. Summary performance metrics for the data fusion approach, evaluated for the**
 1013 **San Francisco study region in September 2019 (same results as presented in Figure 2). Plots**
 1014 **depict the Pearson correlation (a) and root mean square error (b) between the estimates of**
 1015 **the various data fusion phases (denoted by colors) as a function of the forecast lead time**
 1016 **on the horizontal axis (note that the horizontal axis is not linearly scaled). The plotted values**

1017 **depict the median value of the performance metrics assessed across the active monitor sites**
 1018 **in the study region.**



1019
 1020 **Figure S8. Assessment of CI coverage for different CI. The horizontal axis reports the**
 1021 **nominal coverage of the CI, and the vertical axis reports the actual fraction of**
 1022 **measurements falling within that CI. The assessment was conducted for zero lead time**
 1023 **estimates in the San Francisco study region for September 2019 (same results as presented**
 1024 **in Figure 2). Coverage is assessed across all data simultaneously, i.e., the fraction of hourly**
 1025 **measurements falling within the CI across all sites and all hours in the month is presented.**
 1026 **Different colored lines represent different phases of the data fusion. The black dotted lines**
 1027 **denote a one-to-one relationship (the ideal result), and grey dotted lines indicate results**
 1028 **within 5 percentage points of this ideal.**



1029

1030 **Figure S9. Fractions of measurements falling within the estimated 75 % CI for different**
 1031 **phases of the data fusion process, with phases represented by different colors, presented**
 1032 **for different application months. Box-and-whisker plots denote ranges of these fractions**
 1033 **across active NO₂ monitor sites in New York City during that month, with the horizontal line**
 1034 **in the box denoting the median, the box denoting the 25th-to-75th-percentile range, and the**
 1035 **whiskers denoting the full range. The horizontal dotted line across the figure indicates the**
 1036 **goal, i.e., 75 % of measurements falling within the 75 % CI.**