Supporting Information for "Io's Long Wavelength Topography as a Probe for a Subsurface Magma Ocean"

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Introduction This supplement expands on points raised in the main paper, but that were not necessarily the focus of that paper. Section S1 details a calculation of the minimum thickness of Io's lithosphere needed to support its mountains. Section S2 shows the mismatch in Io's observed global shape, and that which one expects from a satellite in hydrostatic equilibrium. Table S1 accompanies Section S2.

Text S1. Minimum Thickness of Io's Lithosphere

Assuming Io's heat flow was dominated entirely by thermal conduction, a minimum heat flow of $F = 2$ W m^{-2} (Veeder et al., 1994; Simonelli et al., 2001; McEwen et al., 2004; Rathbun et al., 2004; de Kleer et al., 2019) would imply a lithosphere only 2.25 km thick (assuming constants in Table 1 of the main text). Yet Io's landscape includes mountains ~ 10 km high (Carr et al., 1979, 1998; Schenk et al., 2001). Assuming a floating elastic lithopshere 5 km thick, O'Reilly and Davies (1981) calculated a mountain 10 km high and 10 km wide would generate a maximum bending stress of 6 kbar (60 MPa), while the strength of Earth's lithosphere at low pressure was estimated to have a maximum of 1-2 kbar (10-20 MPa). This led to the conclusion that most of Io's heat was advected to to the surface via heat-pipe volcanism (O'Reilly & Davies, 1981). Repeating from the main text, O'Reilly and Davies (1981) describe the combined conductive and advective heatl flux through Io's lithosphere as,

$$F = v\rho \left[\Delta H_f + C_p \left(T_m - T_s\right)\right] + \frac{v\rho C_p \left(T_m - T_s\right)}{e^{vd/\kappa} - 1}, \tag{1}$$

where v is the resurfacing rate, ρ is the magma density, ΔH_f is the latent heat of fusion, C_p is the specific heat, T_m is the melting temperature, T_s is the surface temperature, κ is the thermal diffusivity, and d the lithospheric thickness. Under Equation 1, the lithosphere could have an arbitrarily high thickness when the volcanic emplacement rate is high.

In order to qualify our predictions for long-wavelength topography as a result of tidal heat flux variations (Section 3 of the main text), it would help to have a minimum lithosphere thickness as a point of comparison. Carr et al. (1998) find the lower limit of 30 km set forth by Nash, Yoder, Carr, Gradie, and Hunten (1986) to be reasonable, even if "the origin of this 30-km number was obscure." By modeling the magmatic differentiation of Io, Keszthelyi and McEwen (1997) estimate a lithosphere thickness of 50 km. Then Jaeger et al. (2003) estimate that the minimum lithosphere thickness to support the volume of every mountain on Io is 12 km.

We revisit the method used by O'Reilly and Davies (1981) to formulate our own estimate of minimum lithosphere thickness. O'Reilly and Davies (1981) cite McNutt (1980), but the same approach is covered in Walcott (1976); Banks, Parker, and Huestis (1977); Turcotte and Schubert (2014). Imagine an elastic lithosphere of thickness d. In response to some line-load P at $x = 0$ (where x is a horizontal coordinate along the surface of the lithosphere), there will be a deflection $w(x)$ (where w is positive downward, beneath the undeflected surface) such that

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = 0, \tag{2}$$

where $\Delta\rho$ is the density contrast between the crustal (lithospheric, in our approximation) density ρ_c and mantle density ρ_m, g is gravitational acceleration, and D is the flexural rigidity of the lithosphere, defined

$$D = \frac{Ed^3}{12\left(1-\nu^2\right)}, \tag{3}$$

for the Young's Modulus E and Poisson's Ratio ν of the lithosphere (e.g., Walcott, 1976; Banks et al., 1977; Turcotte & Schubert, 2014). The maximum bending stress experienced by the lithosphere is

$$\sigma_{max} = -Ez\frac{d^2x}{dx^2}, \tag{4}$$

where z is depth below the midway point of the lithosphere (i.e., σ_{max} at the base of the lithosphere is at $z = d/2$; Walcott, 1976). In response to a line load P, one can find the maximum curvature of the lithosphere

$$\left.\frac{d^2w}{dx^2}\right|_{x=0} = -\frac{2w_0}{\alpha^2}, \tag{5}$$

where w_0 is w at $x = 0$ and the flexural parameter α is defined

$$\alpha^4 = \frac{4D}{\Delta\rho g}, \tag{6}$$

(Walcott, 1976). The maximum deflection w_0 in response to a line-load P is

$$w_0 = \frac{P\alpha^3}{8D}, \tag{7}$$

following Turcotte and Schubert (2014).

Combining the preceding equations, we find the maximum bending stress experienced at the base of a floating, elastic lithosphere under a line-load $P = \rho_c g h \lambda$ (where h is the height and λ is the half-width of the infinitely long line-load) is

$$\sigma_{max} = \frac{1}{8}\rho_c h \lambda \left\{ \frac{4E\left[12g\left(1-\nu^2\right)\right]^3}{\Delta\rho d^5} \right\}^{1/4}, \tag{8}$$

where one can see that the larger the lithosphere thickness d is, the lower the maximum bending stress at the base of the lithosphere is.

As O'Reilly and Davies (1981) did not provide the exact equations they used in their estimation, we double check our formulae against their result ($\sigma_{max} = 6$ kbar) to be sure that we are solving for the right value. Following O'Reilly and Davies (1981), for a 10 km high mountain that is 10 km wide ($\lambda = 5$ km) under Io gravity $g = 1.8$ m s^{-2} on a floating, elastic lithosphere with thickness $d = 5$ km, Young's Modulus $E = 80$ GPa, Poisson's ratio $\nu = 0.25$, $\rho_c = 3000$ kg m^{-3}, and density contrast with the mantle $\Delta\rho = 500$ kg m^{-3}; we find a maximum bending stress of 6.75 kbar. This is marginally larger than O'Reilly and Davies (1981)'s estimate, but that may have resulted from a difference in the assumed Young's Modulus or the assumed geometry of the surface load.

Satisfied that we are on the same track as O'Reilly and Davies (1981), we can now solve for the minimum lithosphere thickness that can support the observed topography on Io, where $\sigma_{max} < 2$ kbar. All else held constant for the assumed physical parameters of Io's lithosphere, Equation 8 reduces to

$$\sigma_{max} = 6.75\text{kbar} \times \left(\frac{5\text{km}}{d}\right)^{5/4}. \tag{9}$$

We then invert the equation to solve for d given $\sigma_{max} < 2$ kbar, and find a minimum lithosphere thickness $d > 23$ km.

Text S2. The Global Shape of Io

The shape $H(\theta, \lambda)$ of nearly-spherical bodies such as satellites can be described as function of the distance between the satellite's surface from its center of mass as a function of colatitude θ ($\frac{\pi}{2}$ subtracted by the latitude, where Northern latitudes are positive) and longitude λ (where East is positive). As a surface defined in spherical coordinates, one may then describe the shape using spherical harmonics. Here, some function $f(\theta, \lambda)$ is the sum of spherical harmonics with coefficients $C_{l,m}$ and $S_{l,m}$ for each degree l and order m,

$$f(\theta, \lambda) = \sum_{l=0}^{\infty} \sum_{m=0}^{l} \left(C_{l,m} \cos m\lambda + S_{l,m} \sin m\lambda\right) P_{l,m}(\cos\theta), \tag{10}$$

where $P_{l,m}(\cos\theta)$ is an associated Legendre function (e.g. Blakely, 1995). The spherical harmonic degree l indicates the length-scale (or wavelength) over which some value oscillates across a sphere. This wavelength is (approximately) the sphere's circumference divided by the degree l.

As tidal heating varies spatially in even orders of spherical harmonic degrees 2 and 4 (e.g. Beuthe, 2013), we use only those spherical harmonic coefficients of shape to isolate for the topography that could have arisen from variations in tidal heating. In this paper, we will refer to the spherical harmonic coefficients of shape as $H_{l,m}$. Spherical harmonic coefficients $H_{2,0}$ and $H_{2,2}$ may be calculated from the total triaxial shape of Io, which is

$$H^{tri}(\theta, \lambda) = \frac{1}{2} H_{2,0} \left(3\cos^2\theta - 1\right) + 3H_{2,2} \cos 2\lambda \sin^2\theta. \tag{11}$$

For a massive enough satellite, its self-gravity should ensure that the satellite adopts a practically spherical shape in hydrostatic equilibrium. Spinning bodies will become oblate due to rotational flattening. Further, because a satellite orbits a planet, the planet will raise a tidal bulge upon the satellite. When a satellite is in a synchronous orbit, there is an average, "permanent," bulge along the axis that points from the satellite to its host planet. Approximated as a triaxial ellipsoid, the length of each of a satellite's orthogonal axes can be denoted a, b, and c, where $a > b > c$ and a is the ellipsoid's largest possible axis. With the axes defined as such, a must then point from the satellite towards the planet ($\theta = \pi/2$, $\lambda = 0$), while c is the satellite's spin pole ($\theta = 0$), leaving b to point along the path of the satellite's orbit ($\theta = \pi/2$, $\lambda = \pi/2$). Thus, using these axes with Equation 11, one can calculate these spherical harmonic coefficients as $H_{2,0} = c - R_0$ and $H_{2,2} = (a - b)/6$. For Io, these axes a, b, and c are 1829.7, 1819.2, and 1815.8 km, respectively; with an error of 0.3 km (Thomas et al., 1998). With this measurement, we may then calculate the degree $l = 2$ terms of even order for Io's shape (Table S1).

Then, we calculate spherical harmonic coefficients of shape $H_{l,m}$ for degrees $l \geq 3$ and orders m for Io from limb profiles (Thomas et al., 1998; Nimmo & Thomas, 2013; White et al., 2014) (Table S1). We list only the terms for even orders of spherical harmonic degrees 2 and 4, as only those matter for inferring the tidal heating pattern. These spherical harmonic coefficients have not been normalized in any fashion (cf., Nimmo et al., 2011). The errors in degree $l = 2$ and $H_{4,0}$ topography are about an order of magnitude less than the coefficient, while errors in $H_{4,2}$ and $H_{4,4}$ are the same order as the coefficient.

To analyze any relationship between Io's topography and its spatial variations in tidal heating, we must first remove the contribution to its topography of this rotational flattening and tidal bulge. Due to the axial symmetry of both rotational flattening and the tidal bulge, we need only the cosine terms of Equation 10 in even orders of degree-2. The second-order approximation of a satellite's hydrostatic shape from the theory of figures that accounts for rapid rotation (i.e., a spin period of less than a few days, as derived by Beuthe et al., 2016) are defined as a function of the fluid Love number h_2^F (of order unity), such that

$$H_{2,0}^{hyd} = -\frac{5}{6} h_2^F R_0 q \left(1 + \frac{76}{105} h_2^F q\right), \tag{12}$$

$$H_{2,2}^{hyd} = \frac{1}{4} h_2^F R_0 q \left(1 + \frac{44}{21} h_2^F q\right), \tag{13}$$

where q is the ratio of rotational and gravitational forces, $q = \frac{\omega^2 R_0^3}{GM}$ (cf., Zharkov & Gudkova, 2010; Tricarico, 2014). By dropping the higher order term within the parentheses, the ratio $-H_{2,0}^{hyd}/H_{2,2}^{hyd}$ can readily be calculated as its first order approximation, 10/3. Because the term $H_{2,2}^{hyd}$ has a greater second-order increase compared respectively to the second-order increase of $H_{2,0}^{hyd}$, the actual ratio $-H_{2,0}^{hyd}/H_{2,2}^{hyd}$ will shrink from 10/3. We include the higher order terms for completeness but find they are insignificant for Io, as $q = 0.0017$.

For a hydrostatic body, the fluid Love number h_2^F is related to the body's mean moment of inertia C (a measure of mass distribution) by the Darwin-Radau relation (e.g. Munk

& MacDonald, 1960),

$$h_2^F = \frac{5}{1 + \left[\frac{5}{2}\left(1 - \frac{3}{2}\frac{C}{MR_0^2}\right)\right]^2}, \tag{14}$$

where the moment of inertia has been normalized by the satellite's mass M and mean radius R_0 squared. The normalized moment of inertia for a sphere of uniform density is $0.4\ MR_0^2$, and lower if more mass is concentrated in the core. For Io, we know its moment of inertia to be $0.3782MR_0^2$ from gravity measurements assuming it is in hydrostatic equilibrium (Schubert et al., 2004), thus finding $h_2^F = 2.3$ using Equation 14. This allows us to calculate Io's hydrostatic shape as $H_{2,0}^{hyd} = -5.95$ km and $H_{2,2}^{hyd} = 1.80$ km. Unfortunately, this means that when we eliminate the hydrostatic contribution to Io's shape, the remaining topography relative to the hydrostatic shape (and thus the topography we would assume is due to isostatic variations) is only $H_{2,0}^{rem} = 0.15$ km and $H_{2,2}^{rem} = 0.05$ km, which is less than the error in degree-2 topography (Table S1). Thus, it is unlikely we could make any conclusion on Io's tidal heating pattern from its global shape.

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Table S1. Spherical harmonic coefficients of Io's shape[a]

l	m	$H_{l,m}$ (km)		$\sigma_{Hl,m}$ (km)
2	0	-5.8	$\pm$	0.4
2	2	1.7	$\pm$	0.1
4	0	-0.06	$\pm$	0.02
4	2	-0.0016	$\pm$	0.0016
4	4	-0.00016	$\pm$	0.00016

[a] $l = 2$ terms were calculated with Equation 11, while $l = 4$ terms were calculated with the method of White et al. (2014) using smoothing parameter $r = 3 \times 10^7$. These terms are not normalized.