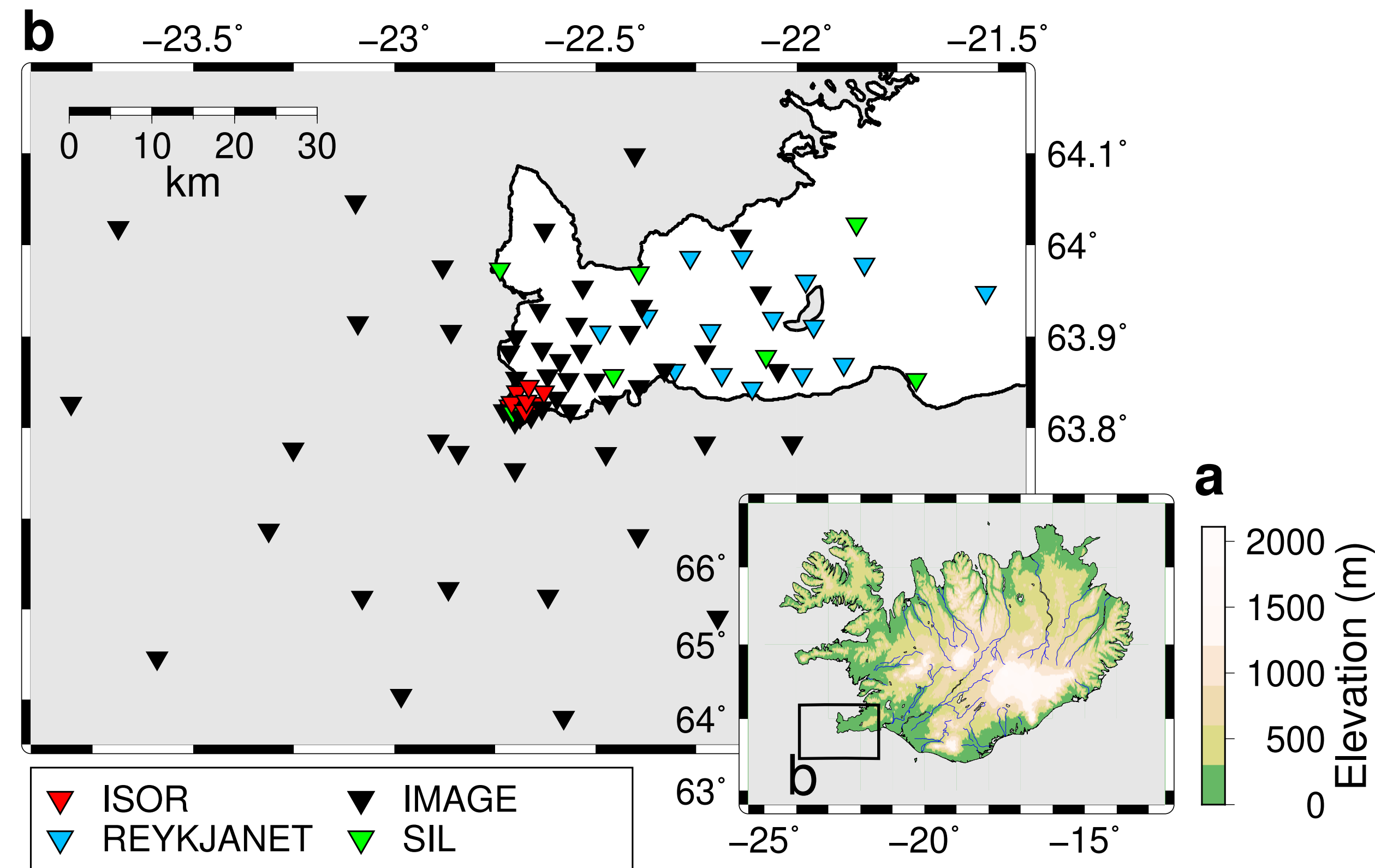


## 1. The Reykjanes seismic array

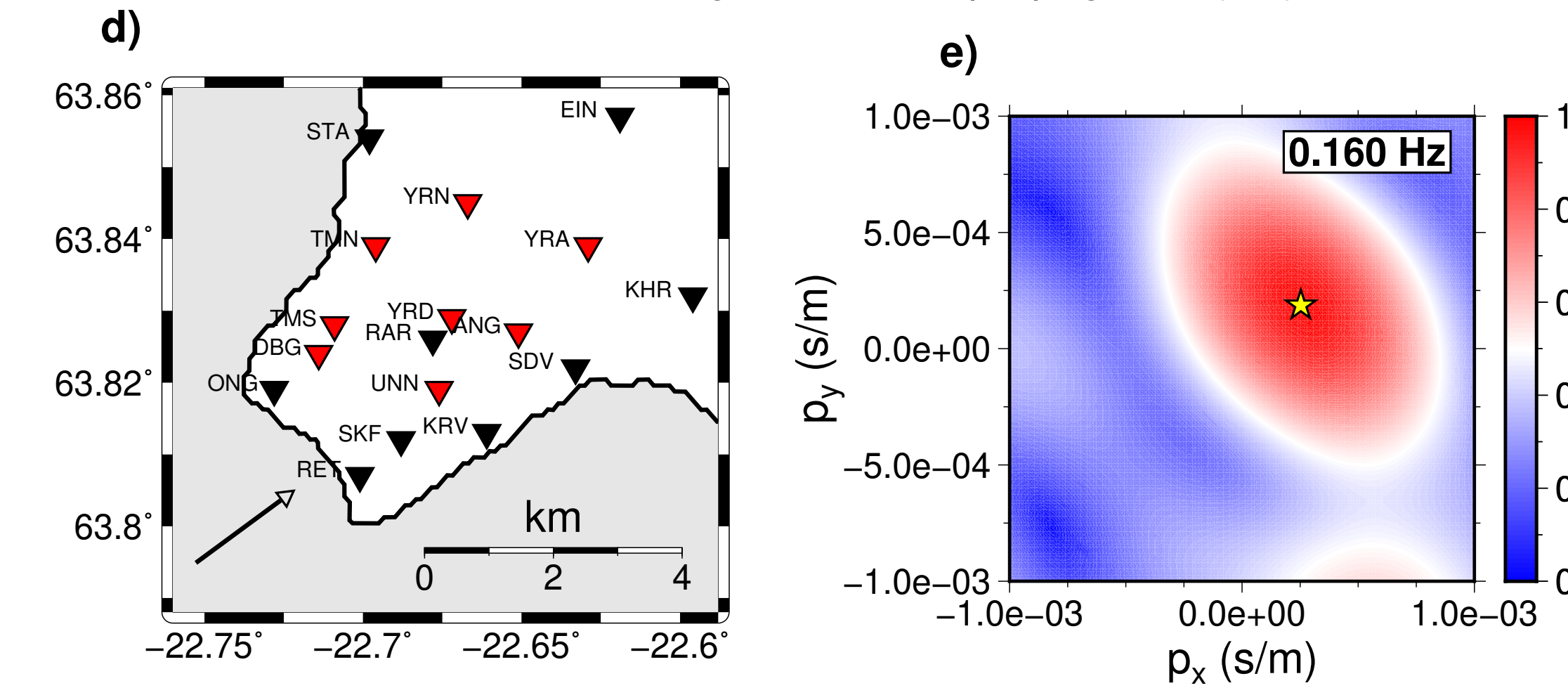
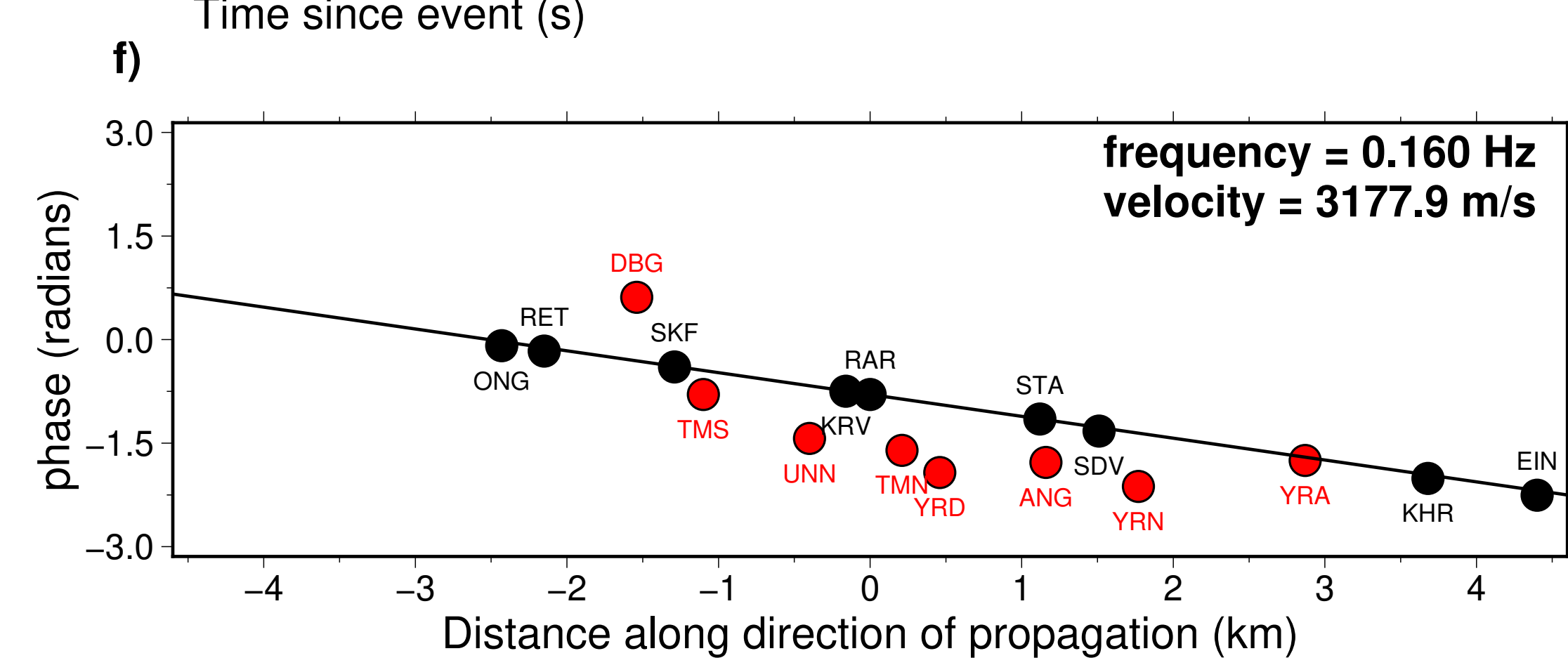
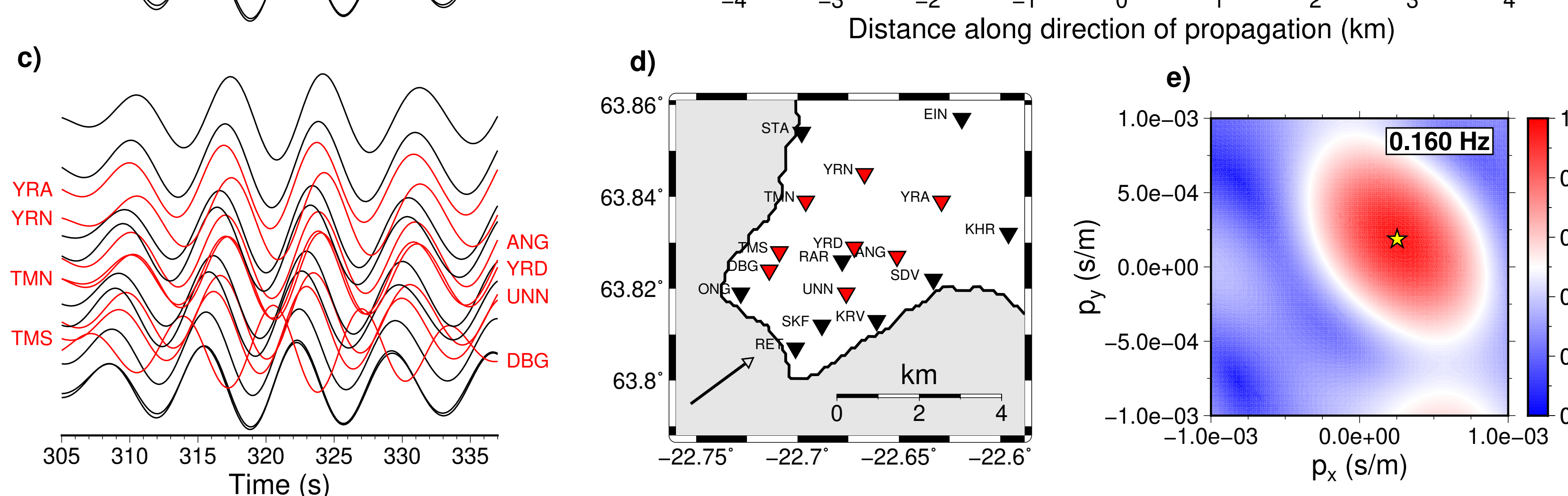
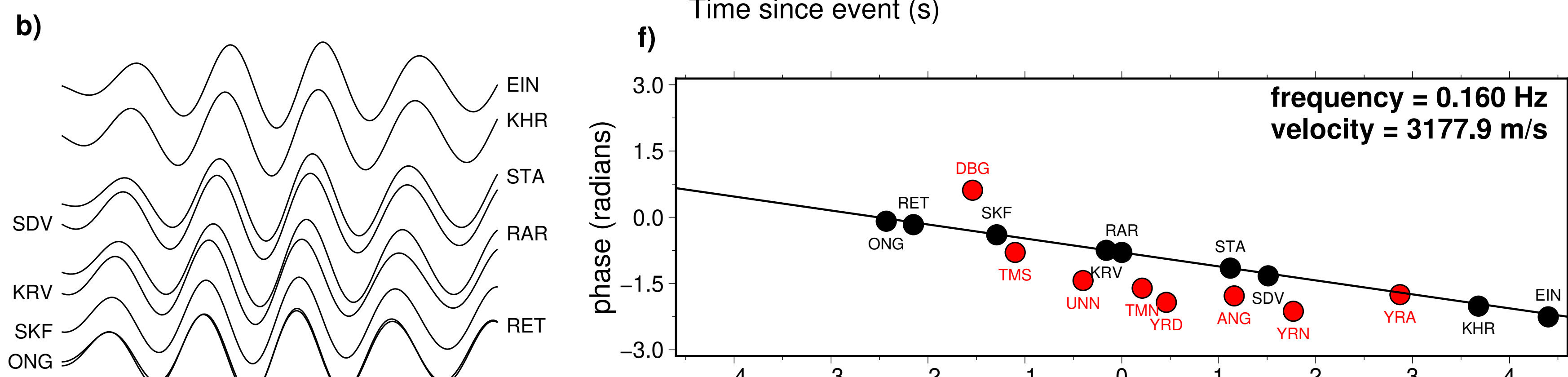
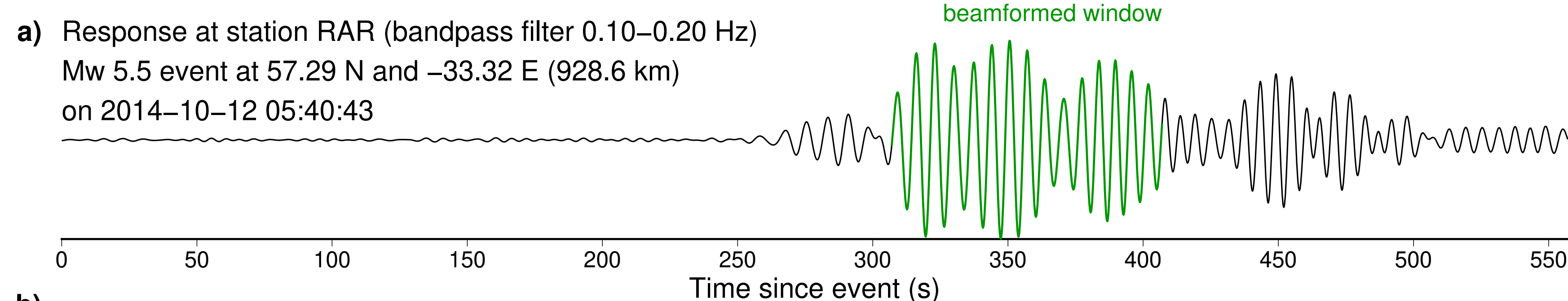
This study is motivated by the recent deployment of 56 stations in the context of IMAGE (Integrated Methods for Advanced Geothermal Exploration). Together with stations from three other networks (REYKJANET, ISOR, and SIL), a composite seismic array can be formed. We refer to this array as the Reykjanes seismic array (RARR). Station locations are depicted in the figure on the right; colors indicate which network a station belongs to. Details regarding instrumentation are listed in the table below. Thus far, we have only used the vertical components.



Network	Instrument	Stations
IMAGE	Trillium compact 120s Mark L-4C-3D Sensor Güralp-40T (OBSs)	BER, GEV, EIN, HAH, HAS, HOS, KEF, KUG, LFE, ONG, PAT, PRE, RAH, RAR, RET, SDV, SKG, SKH, STA, SUH NEW, HOP, KHR, KRV, SKF, STF, STK, VSV, ARN, MER O01, O02, O03, O04, O05, O06, O07, O08, O10, O11, O12, O14, O15, O16, O17, O18, O19, O20, O21, O22, O23, O25, O26
REYKJANET	Güralp-40T	FAF, HRG, KLV, LSF, ELB, LAT, LHL, STH, MOH
ISOR	Lennartz 3D/1s	ASH, SEA, ISS, GEI, LAG, HDV
SIL	Lennartz 3D/5s	UNN, ANG, TMN, TMS, YRA, YRD, YRN, DBG GRV, NYL, RNE, VOG, KRI, VOS, KAS

## 2. Problem statement

After removal of the instrument responses, the recordings by the Lennartz 3D/1s remain subject to a time shift. Communication with the institute of geophysics of the CAS learned us that the Lennartz 3D/1s (a simple commercial 4.5 Hz geophone) has a response that is electronically extended down to 1.0 Hz, but that this instrument response is not guaranteed below 1 Hz (particularly the phase response may differ significantly from the theoretical one). Despite this phase shift, the sensors are sensitive enough to recover vertical particle velocity between 0.1 and 0.5 Hz and hence allow the retrieval of interferometric surface-wave responses (see “3. Seismic interferometry”). Scrutiny of the surface waves generated by a distant earthquake bear witness of the incurred phase shifts:



## Abstract

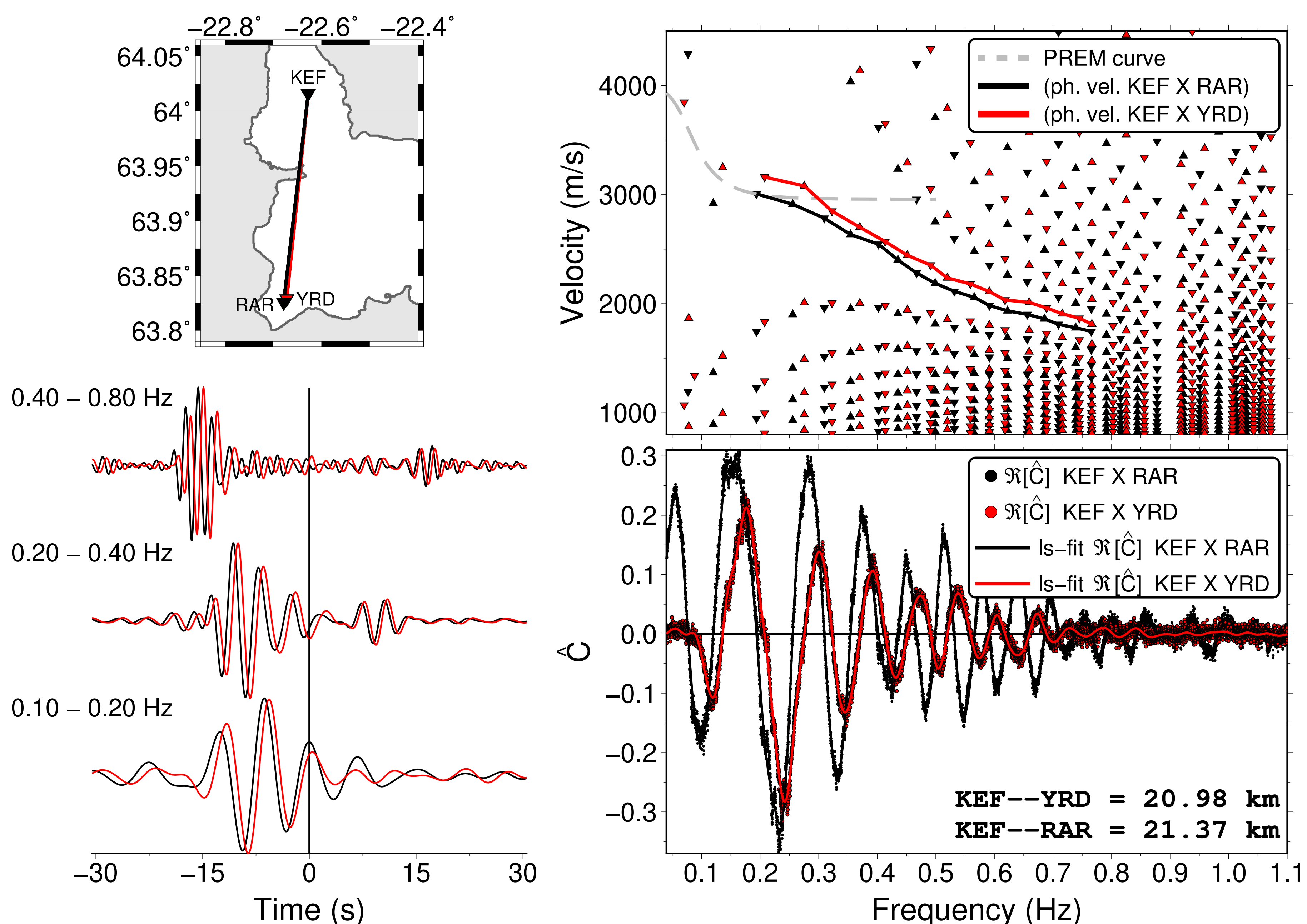
Seismic interferometry (SI) allows one to generate new seismic responses by crosscorrelating observations at different station locations. In application to recordings of ambient seismic noise, SI involves temporal averaging of time-windowed crosscorrelation measurements. The retrieved interferometric responses are typically dominated by surface waves. Time-averaged crosscorrelations therefore often approach the surface-wave part of the medium's Green's function. Assuming a uniform illumination pattern, the Green's function's time-reverse is additionally retrieved, implying time-symmetry of the time-averaged station-station crosscorrelations. By comparing the arrival time of the interferometric surface waves at positive time to the arrival time of the interferometric surface waves at negative time, this time-symmetry can be exploited for the purpose of estimating relative timing errors. This involves the formulation of an inverse problem, which can be solved uniquely in case at least one of the stations is known to be devoid of clock/phase errors. We validate our approach using synthetic data, as well as field data recorded by the Reykjanes seismic array (RARR).

## 3. Seismic interferometry

Under appropriate conditions, the surface-wave part of the Green's function between  $\mathbf{x}_i$  and  $\mathbf{x}_j$ , plus its time-reverse, can be recovered from the time-averaged crosscorrelation  $C_{ij}(t)$  of the noise recorded at those locations (e.g., Halliday and Curtis, 2008):

$$C_{ij}(t) \propto [G(\mathbf{x}_j, \mathbf{x}_i, t) + G(\mathbf{x}_j, \mathbf{x}_i, -t)].$$

Below, the time-averaged noise crosscorrelations associated with the station couples KEF-RAR (black) and KEF-YRD (red) are presented (bottom left). The paths associated with these station couples nearly coincide (top left). The fact that the time-averaged crosscorrelations exhibit a time-shift with respect to each other, however, leads to the conclusion that either station RAR, or station YRD, is subject to a clock/phase error. In this case, we are confident that both KEF and RAR are devoid of such errors. Equating the zeros of the real part of  $\hat{C}_{ij}^T(\omega)$  (bottom right), which represents the temporal Fourier-transform of  $C_{ij}(t)$ , to the zeros of the real part of the surface-wave Green's function associated with a laterally uniform subsurface, phase velocities can be estimated. The repercussions of YRD's clock/phase error when it comes to the extraction of phase velocities are clear (top right).



## Acknowledgements

We are grateful to the institute of geophysics of the Czech academy of sciences (CAS) and the Icelandic meteorological office (MET office) for allowing us to use the data recorded by REYKJANET and SIL, respectively. Instruments for the IMAGE project were provided by the Geophysical Instrument pool Potsdam and the German instrument pool for amphibian seismology. The research leading to these results has received funding from the European Community's Seventh Framework Programme under grant agreement No. 608553 (Project IMAGE).

## 4. Determining clock/phase errors

We denote the arrival times of the medium's direct surface-wave responses at positive time and negative time by  $t_{ij}^{(+)}$  and  $t_{ij}^{(-)}$ , respectively. In case the recordings at  $\mathbf{x}_i$  are subject to clock errors, this will result in an additional time shift  $\delta t_i^{(\text{ins})}$ . Deviation of the illumination pattern from uniformity may result in an additional time shift  $\delta t_{ij}^{(\text{src})}$ . We distinguish between  $\delta t_{ij}^{(+, \text{src})}$  and  $\delta t_{ij}^{(-, \text{src})}$  because the retrieved responses at positive and negative time are associated with opposing stationary-phase regions. Accounting for the introduced time shifts, the observed arrival times of the direct responses at positive and negative time are given by

$$t_{ij}^{(+, \text{obs})} = t_{ij}^{(+)} + \delta t_i^{(\text{ins})} - \delta t_j^{(\text{ins})} + \delta t_{ij}^{(+, \text{src})} \quad \text{and} \quad t_{ij}^{(-, \text{obs})} = t_{ij}^{(-)} + \delta t_i^{(\text{ins})} - \delta t_j^{(\text{ins})} + \delta t_{ij}^{(-, \text{src})}.$$

Summing the left-hand and right-hand sides of these two equations, we find

$$t_{ij}^{(+, \text{obs})} + t_{ij}^{(-, \text{obs})} = 2\delta t_i^{(\text{ins})} - 2\delta t_j^{(\text{ins})} + \delta t_{ij}^{(+, \text{src})} + \delta t_{ij}^{(-, \text{src})}.$$

Extending this formulation to a total of  $N$  simultaneous recording receivers, we obtain the matrix equation

$$\mathbf{T}^{(\text{obs})} = \mathbf{A}\mathbf{T}^{(\text{ins})} + \mathbf{T}^{(\text{src})},$$

where the rows and columns of  $\mathbf{A}$  relate to different station couples and stations, respectively. The column vector  $\mathbf{T}^{(\text{ins})}$  holds the  $N$  instrumental time-shifts we aim to estimate, and the column vectors  $\mathbf{T}^{(\text{src})}$  and  $\mathbf{T}^{(\text{obs})}$  hold the  $t_{ij}^{(+, \text{obs})} + t_{ij}^{(-, \text{obs})}$  and  $\delta t_{ij}^{(+, \text{src})} + \delta t_{ij}^{(-, \text{src})}$ , respectively. If  $t_{ij}^{(+, \text{obs})} + t_{ij}^{(-, \text{obs})}$  can be determined for all combinations of  $i$  and  $j$ , the matrix  $\mathbf{A}$  will contain  $N(N-1)/2$  rows. In the ideal case that the surface-wave illumination of the RARR is uniform (from all angles),  $\mathbf{T}^{(\text{src})} = \mathbf{0}$ . In practice, however, this assumption is often not valid (e.g. Weemstra et al., 2017).

A stationary-phase approximation of  $\delta t_{ij}^{(+, \text{src})} + \delta t_{ij}^{(-, \text{src})}$  is derived by Weaver et al. (2009). Assuming a sufficiently smooth illumination pattern, these authors find:

$$\delta t_{ij}^{(+, \text{src})} + \delta t_{ij}^{(-, \text{src})} \sim \frac{B''(\theta_{ij})}{2t_{ij}\omega_0^2 B(\theta_{ij})} - \frac{B''(\theta_{ij} - 180)}{2t_{ij}\omega_0^2 B(\theta_{ij} - 180)}$$

where  $B(\theta_{ij})$  denotes the power of the (time-averaged) noise flux as a function of azimuth ( $\theta_{ij}$  points from  $\mathbf{x}_i$  to  $\mathbf{x}_j$  and is measured counterclockwise from North),  $B''(\theta_{ij})$  its second derivative, and  $\omega_0$  the central angular frequency of the correlation waveform. Using this approximation, the least-squares estimator of  $\mathbf{T}^{(\text{ins})}$ , which we denote by  $\tilde{\mathbf{T}}^{(\text{ins})}$ , is given by,

$$\tilde{\mathbf{T}}^{(\text{ins})} = (\mathbf{A}^T \mathbf{W}_d \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W}_d \mathbf{T}^{(\text{obs})}.$$

The weight matrix  $\mathbf{W}_d$  is a diagonal matrix whose dimension coincides with the number of observations. For each station couple  $i, j$ , the weight factors (diagonal elements) are given by the inverse of the approximation above. In case  $B(\theta_{ij})$  and  $t_{ij}$  are not known, the weighting can be simplified by approximating the inverse by the station-station distance  $|\mathbf{x}_j - \mathbf{x}_i|$ . Because the system of equations is under-determined, an infinite number of combinations of the time shifts  $\tilde{\delta t}_i^{(\text{ins})}$  ( $i = 1, \dots, N$ ) exist that minimize the least-squares error. The appropriate least-squares solution, however, can be obtained by imposing the additional constraint:

$$\min_{\tilde{\delta t}_i^{(\text{ins})}} \sum_{i=1}^N \left( \tilde{\delta t}_i^{(\text{ins})} - \delta t_i^{(\text{ins})} \right)^2.$$

## 6. Application to synthetic and field data

Single-mode surface waves propagating along the surface of a laterally homogeneous Earth were modeled to create two synthetic data sets. The first synthetic data set is the result of a uniform illumination by uncorrelated plane waves; the second data is associated with an arbitrary (but smooth) non-uniform illumination pattern. In both cases, the OBS recordings were given an arbitrary (constant) clock error between -2 and 2 seconds. The residual errors, i.e., after inversion, are shown on the right for various inversion approaches. In application to field data (below), we obtained the instrument-related time shifts for 83 receivers of the RARR, based on noise crosscorrelations of 650 station couples. In all cases (synthetic and field data) the (non OBS) IMAGE stations were assumed to be devoid of instrument-related time shifts.

