The Effect of Lorentz Stresses on the Solar Frequency Spectrum: The Forward Problem

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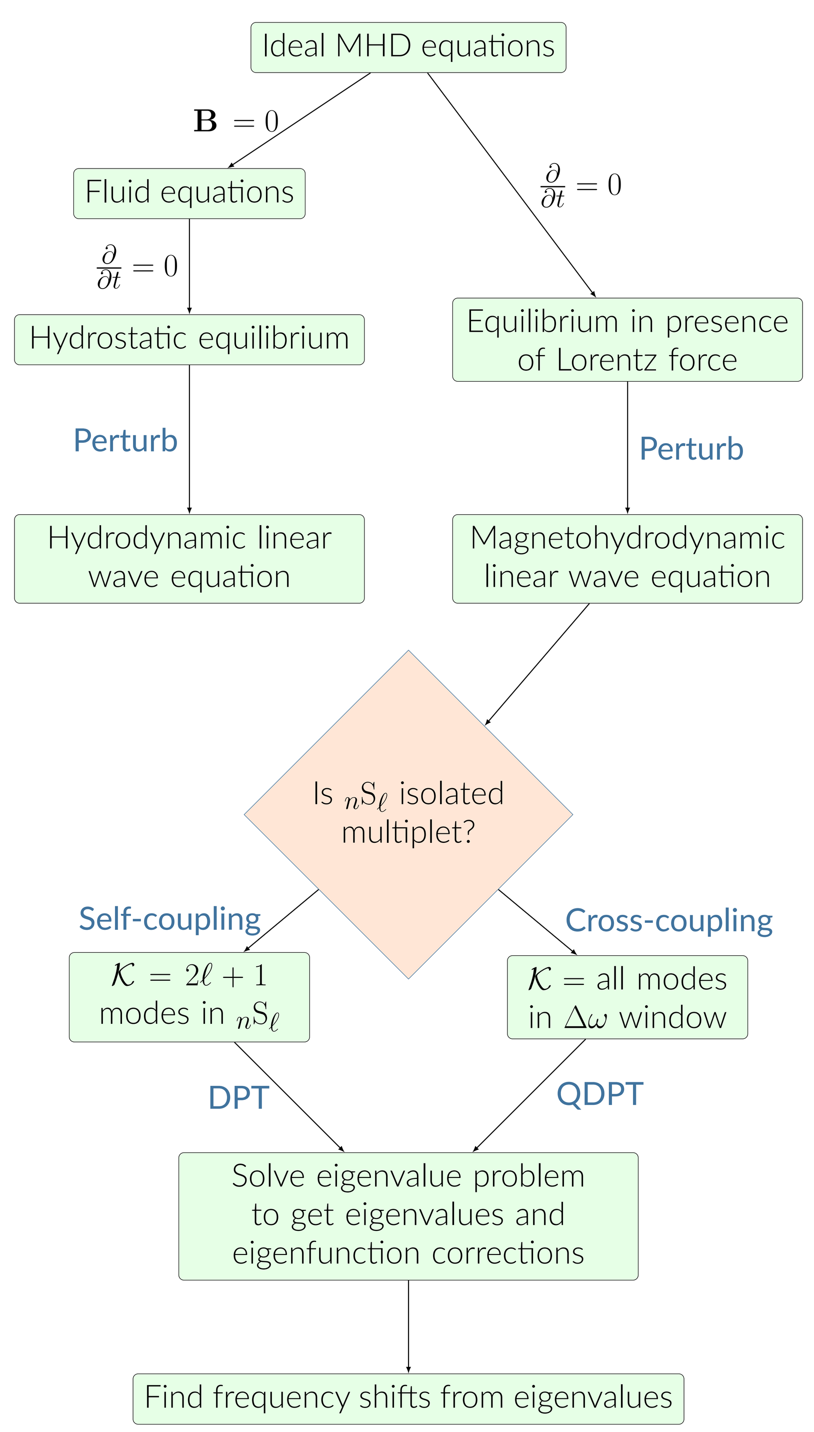
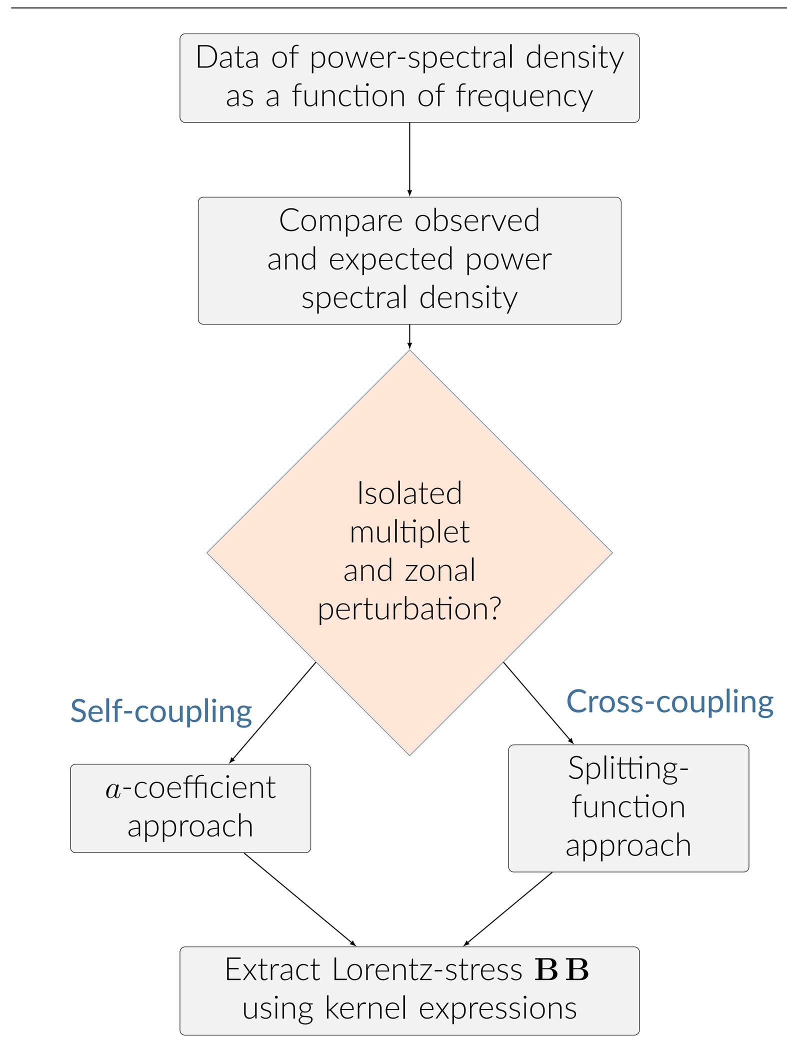
Abstract

Departures from standard spherically symmetric solar models, in the form of perturbations such as global and local-scale flows and structural asphericities, result in the splitting of eigenfrequencies in the observed spectrum of solar oscillations. We find the Lorentz-stress sensitivity kernel for a general magnetic field, and therefore, also propose the sensitivity kernels for frequency splittings (a- coefficients) due to axisymmetric Lorentz stresses in the Sun. These results pave the way to formally pose an inverse problem, and infer solar (stellar) internal magnetic fields.

Forward Problem: At a glance



Inverse Problem: In a blink



Magnetohydrodynamic (MHD) equations

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v}), \tag{1}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \frac{\mathbf{j}\times\mathbf{B}}{c} - \rho\nabla\phi, \tag{2}$$

$$\frac{\partial p}{\partial t} = -\mathbf{v}\cdot\nabla p - \gamma p\nabla\cdot\mathbf{v}, \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}). \tag{4}$$

Definitions

GSH basis : $\hat{e}_- = \frac{1}{\sqrt{2}}(\hat{e}_\theta - i\hat{e}_\phi) \quad \hat{e}_0 = \hat{e}_r \quad \hat{e}_+ = -\frac{1}{\sqrt{2}}(\hat{e}_\theta + i\hat{e}_\phi)$

Expanding the displacement vectors in the basis of normal modes:

$$\boldsymbol{\xi}(\boldsymbol{r},\omega) = \sum_k \boldsymbol{\xi}_k e^{i\omega_k t} = \sum_{st}\sum_{\mu} \xi^\mu_{st}(r) Y^\mu_{st}(\theta,\phi)\hat{e}_\mu$$

No toroidal components in $\boldsymbol{\xi}$. Similarly,

$$\mathbf{B} = \sum_{st}\sum_{\mu} B^\mu_{st}(r) Y^\mu_{st}(\theta,\phi)\hat{e}_\mu,$$

$$\boldsymbol{\mathcal{H}} = \mathbf{B}\,\mathbf{B} = \sum_{st}\sum_{\mu\nu} h^{\mu\nu}_{st}(r) Y^{\mu+\nu}_{st}(\theta,\phi)\hat{e}_\mu\hat{e}_\nu, \quad \mu\leftrightarrow\nu.$$

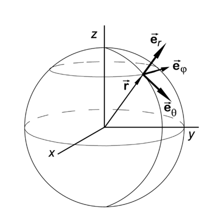


Figure 1:Spherical co-ordinate geometry

Background solar model: SNRNMAIS

We adopt the method of perturbation theory. Therefore, we need a background model of the Sun which shall be perturbed to 'fit' the real Sun. It is standard practise in helioseismology to start with the following simplified model, referred to as **SNRNMAIS**:

- **S**pherically symmetric,
- **N**on-**R**otating,
- **N**on-**M**agnetic,
- **A**diabatic,
- **I**sotropic,
- **S**tatic.

We use Model-S.[1]

Modeling a custom magnetic field B

- Inner toroidal upto tachocline, $r \leq 0.7R_\odot$.
- Outer dipolar beyond $r \geq 0.95R_\odot$.
- Intermediate mixed (toroidal + spheroidal) field.

$$B_{st}(r) = \begin{cases} -i\frac{\alpha(r)}{\gamma_1}\begin{pmatrix}1\\0\\-1\end{pmatrix} - \frac{\beta(r)}{\gamma_1}\begin{pmatrix}b-r\dot{b}\\-2b\\b-r\dot{b}\end{pmatrix}, & \text{for}(s,t)=(1,0)\\ 0, & \text{for}(s,t)\neq(1,0),\end{cases}$$

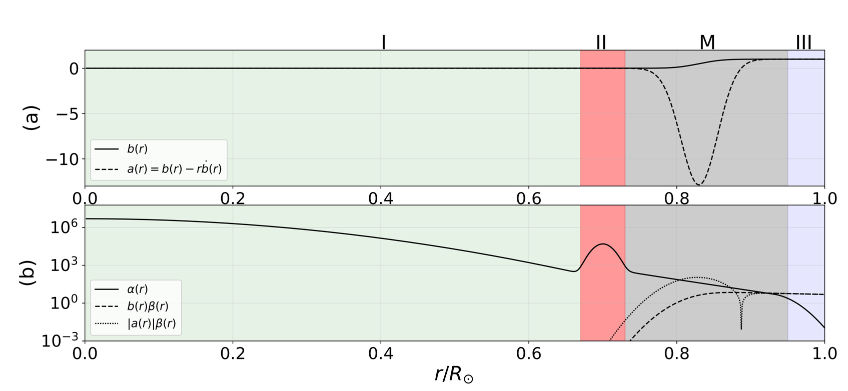
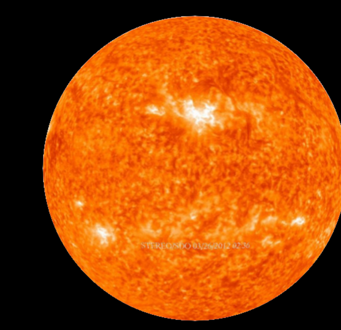


Figure 2:(a) Profiles for tunable parameters $a(r)$ and $b(r)$ chosen in this study. (b) Strength associated to each GSH component of B^μ_{10}.

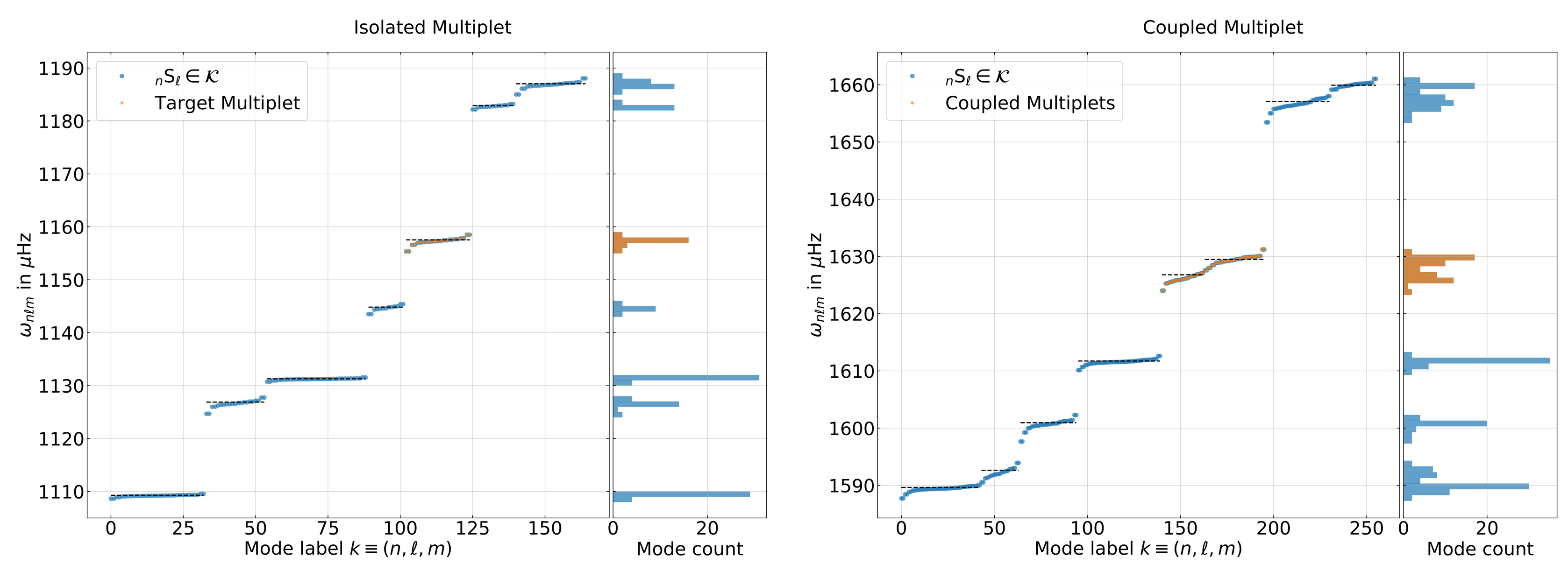
How to find **Sun's internal magnetic fields** by observing the solar **surface oscillation frequency**?

1. Ideal MHD,
2. Normal-mode helioseismology,
3. Quasi-degenerate perturbation theory.



Isolated multiplet vs. coupled multiplet

$$\sum_k\{\Lambda_{k'k} - (\omega^2_{\text{ref}} - \omega^2_k)\delta_{k'k}\}c_k = \sum_k \delta(\omega^2)\delta_{k'k}c_k \quad \text{where,} \quad \Lambda_{k'k} = \sum_{st}\sum_{\mu\nu}\int_0^{R_\odot} dr\, r^2 \mathcal{B}^{\mu\nu}_{st}(r) h^{\mu\nu}_{st}(r). \tag{5}$$



Labelling multiplets: Isolated or coupled?

$$L_2^{QDPT} = \sqrt{\sum_m(\delta\omega^Q_{n\ell m})^2} \quad \text{for cross-coupling,} \qquad L_2^{DPT} = \sqrt{\sum_m(\delta\omega^D_{n\ell m})^2} \quad \text{for self-coupling.} \tag{6}$$

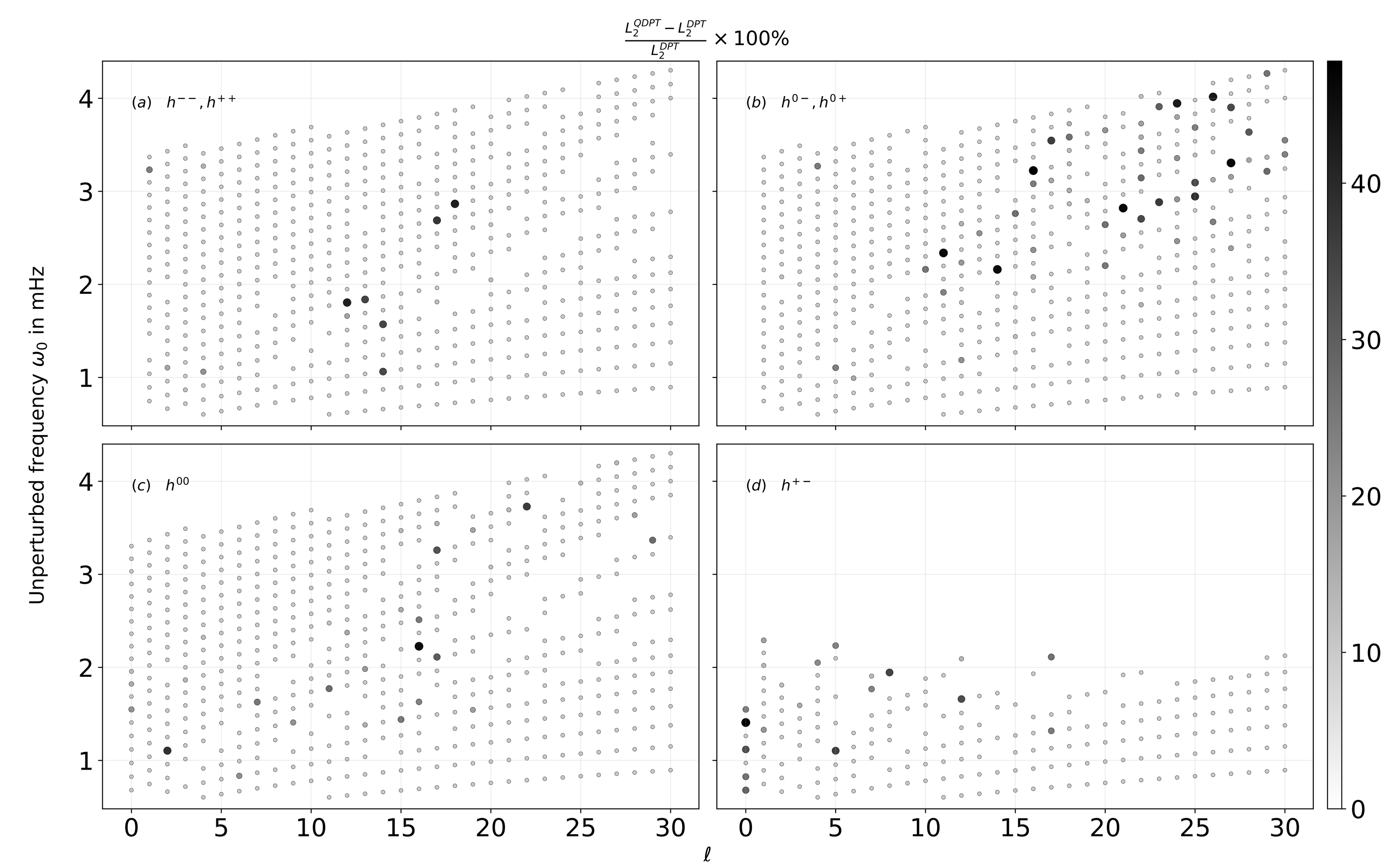


Figure 3:Darker and larger `o' represents multiplets where cross-coupling causes significant relative frequency offset with respect to self-coupling. We have used Lorentz-stress field with $s=\{0,1,2,3,4,5,6\}$ including all $t$ for a certain $s$. The strength of GSH components $h^{\mu\nu}_{st} = h_0(r)$ where $h_0(r)$ is the `total' field strength as shown in Figure 4.

Inversion technique using a-coefficients

- Isolated-multiplet approximation,
- Axisymmetric perturbation $\implies t=0$,
- Extensively used for inversion of differential rotation.[2]

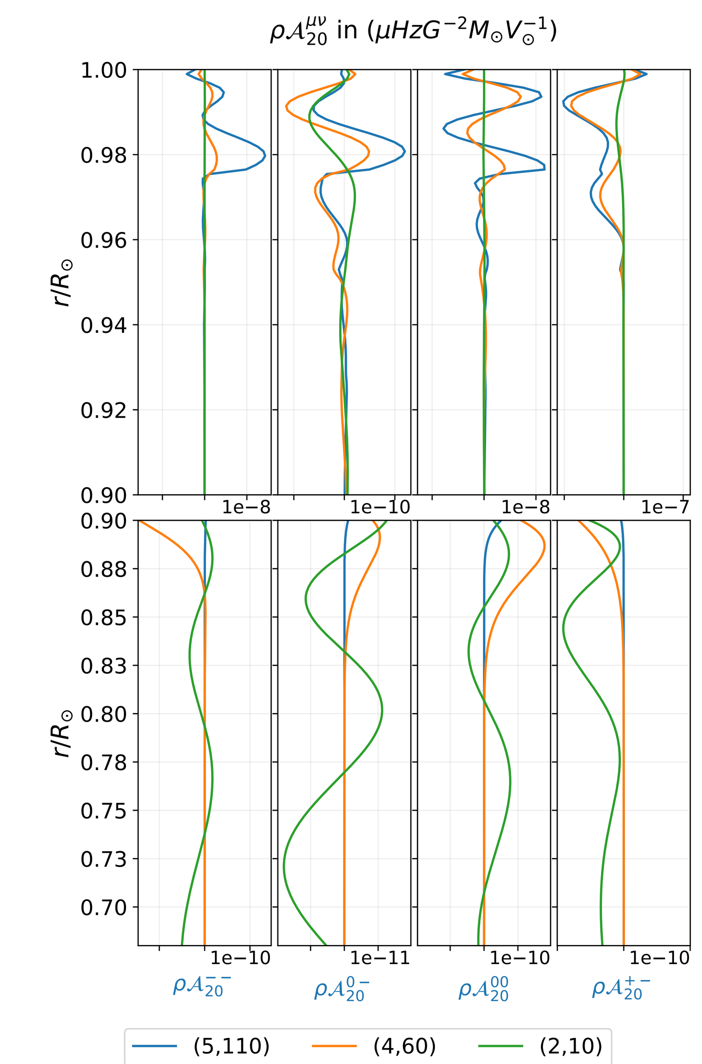
The inverse problem →

$$\delta\omega^m_{n\ell} = \sum_{j=0}^{j_{max}} a^{n\ell}_j \mathcal{P}^{(\ell)}_j(m),$$

$$a^{n\ell}_s = \int_0^{R_\odot} dr\, r^2 \sum_{\mu,\nu}\mathcal{A}^{\mu\nu}_s h^{\mu\nu}_{s0},$$

$$\text{where,} \quad \mathcal{A}^{\mu\nu}_s(r) = \frac{\mathcal{K}^{\mu\nu}_s(r)}{2\omega_{n\ell}} \propto \frac{\mathcal{B}^{\mu\nu}_{s0}(r)}{2\omega_{n\ell}}.$$

Self-coupling kernels for a-coefficients



Varying depth sensitivity of modes

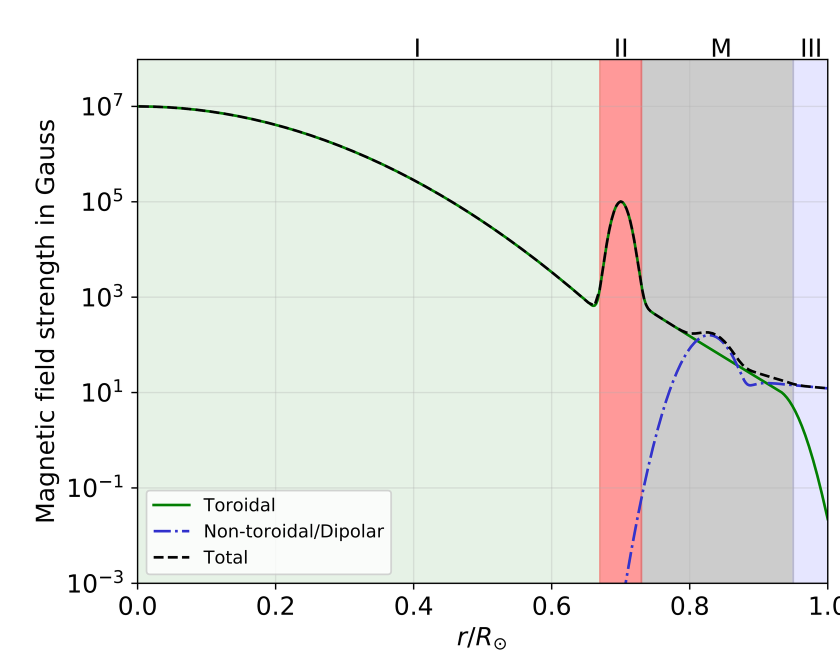


Figure 4:Magnetic field strength (Model M1 in Table 1) associated with three different regions with the following field configurations: (A) Purely toroidal for $r < 0.7R_\odot$, (B) Toroidal + spheroidal extending from $r = 0.7R_\odot \leq r \leq 0.95R_\odot$ and (C) Purely dipolar for $r > 0.95R_\odot$.

	M1	M2	M3	M4	M5	M6	M7	M8
B_I	Low	Low	Low	Low	High	High	High	High
B_{II}	Low	Low	High	High	Low	Low	High	High
B_{III}	Low	High	Low	High	Low	High	Low	High

Table 1:Strength ratio High:Low = 10:1. M1 = Sun-like.

Modes	$_2S_{10}$		$_5S_{110}$	
a-coefficients	a_0	a_2	a_0	a_2
M1	4.138	-0.501	2.399	0.628
M2	4.991	-0.167	239.986	62.863
M3	399.696	-62.360	2.399	0.628
M4	400.549	-62.026	239.986	62.863
M5	14.274	11.824	2.399	0.628
M6	15.127	12.158	239.986	62.863
M7	411.551	-50.349	2.399	0.628
M8	412.405	-50.015	239.986	62.863

Table 2:Mode-response in a-coefficients (nHz).

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$$\mathcal{B} = \frac{1}{2}\left\{\nabla\boldsymbol{\xi}_k\cdot(\nabla\boldsymbol{\xi}^*_{k'})^T + \nabla\boldsymbol{\xi}^*_{k'}\cdot(\nabla\boldsymbol{\xi}_k)^T\right\} + \frac{1}{2}\left\{\nabla\boldsymbol{\xi}^*_{k'}\cdot\nabla\boldsymbol{\xi}_k + \nabla\boldsymbol{\xi}_k\cdot\nabla\boldsymbol{\xi}^*_{k'}\right\} + \frac{1}{2}\left\{\boldsymbol{\xi}^*_{k'}\cdot\nabla\nabla\boldsymbol{\xi}_k + \boldsymbol{\xi}_k\cdot\nabla\nabla\boldsymbol{\xi}^*_{k'}\right\} - \frac{1}{2}\left\{\boldsymbol{\xi}_k\nabla\nabla\cdot\boldsymbol{\xi}^*_{k'} + \boldsymbol{\xi}^*_{k'}\nabla\nabla\cdot\boldsymbol{\xi}_k\right\} - \frac{3}{2}\left\{\nabla\boldsymbol{\xi}_k\nabla\cdot\boldsymbol{\xi}^*_{k'} + \nabla\boldsymbol{\xi}^*_{k'}\nabla\cdot\boldsymbol{\xi}_k\right\} + \boldsymbol{I}\nabla\cdot\boldsymbol{\xi}^*_{k'}\nabla\cdot\boldsymbol{\xi}_k$$