Modelling economic decision-making under uncertainty: a comparison of approaches

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Key Points:

- Climate change is a grey-swan problem comprising both probabilistic and non-probabilistic (uncertain) outcomes
- A common approach for modelling uncertainty in economics is via expected values but critical learning, innovation and adaptation responses are not well represented in such models
- In a context where adaptation outcomes from climate change will be critical for policy design, assessment and modification effective modelling approaches will be important
- We compare state contingent analysis as a modelling alternative for representing innovation and learning to illustrate if usefulness in the climate and water policy space

Abstract

Modelling economic decision-making under uncertainty typically involves approaches that assume perfect foresight, or that future information is knowable and awaiting discovery (white-swan problems). By contrast, grey-swan problems where some information may be known probabilistically, and other information remains uncertain (no probability measures), is less commonly modelled in the literature. Climate change is a classic grey-swan problem that will shape complex economic, social and environmental decisions for generations to come. Within the global debate on water scarcity and inequality strategies for (re)allocating water resources in response to climate change impacts are of particular concern. Analysts need to be able to inform decision makers on how to best adapt given uncertain futures. This paper contrasts the outputs from (re)allocating water resources via expected value and stochastic state contingent modelling approaches—where both are specified to represent white- and grey-swan problems. The models represent how water resources (re)allocate under current climate settings and two alternative future climate change scenarios: i) a reduction in total water availability/use, and ii) the increased occurrence of bad (e.g., drought) events. Expected value models mask innovation and adaptation reactions by decision-makers in response to external stimuli (e.g., increased droughts), and under-represent water (re)allocation outcomes. Conversely, state contingent models more clearly represent and report decision-maker reactions that can be more readily interpreted and linked to stimuli including policy interventions.

1 Introduction

According to Raffaelli (2003) Marshall characterised the human mind as an evolving self-organisation with a unique asset of routines that, when current routines fail, will grow with experience as chance variation and foresight motivate new pathways. If true, complex systems thus react to external stimuli, especially when innovation is supported rather than the routine (Dow, 2020, Chavas and Nauges, 2020). Within the discipline of economics, neoclassical approaches constrain innovation under an assumption that all present and future information is known—or knowable awaiting discovery. By contrast, ecological economic theory treats economic, social and physical environments as interrelated parts of a system where expected outcomes are fundamentally uncertain and thus not wholly amenable to quantitative probabilities (Kapp, 1970). Further, if reversibility is presumed, then attempts to mathematically analyse interrelated systems exhibiting structural irreversibility will be invalid (Foster, 2011). For these reasons economic modelling of uncertainty remains at the frontier of theoretical and applied research.

Climate change and its impacts on economic and social environments is uncertain. In a complex interrelated system there is scope for unintended consequences to public and/or private decision-making (Dow, 2020). Pindyck (2011) and Quiggin (2019) describe the relevance of correctly identifying, and describing the impacts of, thin- and/or fat-tailed probability distributions resulting in very different management options/selections. Climate change is also a good example of future uncertainty coupled to irreversibility that necessitates care when choosing quantitative analysis or modelling techniques. When modelling the new pathways that may stem from individual decision-maker routines or innovation in response to climate change, it is necessary that the role of institutions, policy-makers and any firm/household decision-makers be included (*ibid*.). Further, if radical innovation of goals, institutions and/or behaviour is required to address climate change then consensus between all parties will be needed for effective implementation (Raffaelli, 1993). However, any economic theoretical framework that reverts to maximising growth through perfect foresight pricing mechanisms will create problems for uncertain climate adaptation modelling (Dow, 2020).

1.1 Black-, white- and grey-swan events

Future outcomes and contingent decision choices can be defined via three levels of increasing uncertainty: black-swan, white-swan and grey-swan (Taleb, 2007) problems. At one extreme, black-swan problems are characterised by complete uncertainty where both future outcomes and decision choices are fully undescribed (i.e., public/private decision-makers are totally unaware of these events and their likelihood). Such outcomes often arise from bounded awareness heuristics (Chugh and Bazerman, 2007) and are unrepresentable in typical modelling approaches (Chichilnisky, 2010). However, once realised, black-swan events will shake the foundations of decision-making and cause non-linear responses in knowledge and choice selection.

By contrast, white-swan problems align with neoclassical economic assumptions of perfect knowledge where possible outcomes and contingencies are completely described. Discrete probability parameters make such problems relatively easy to specify and model, if we accept the assumptions discussed above. However, climate change and its impacts on decision-making does not fit neatly into either problem definition. While we know that future climate outcomes (e.g., rainfall, temperature, etc.) will be different, we have an incomplete set of decision contingencies (innovation/adaptation choices) because these events have not occurred and/or rarely occur. This then prevents the complete contingency set and their outcomes from being described. This problem is defined as a grey-swan event where decision-makers, either through inductive reasoning or experiencing the failure of existing routines, may realise a need to innovate in response to uncertainty (Grant and Quiggin, 2012). Importantly, different innovation/learning approaches are why some decision-makers correctly anticipate uncertain outcomes and adopt appropriate contingencies in response via incomplete/bounded choice sets (state and appropriate response matrices) that mimic both imperfect recall and inexperience (*ibid*.). A challenge remains, though, in how to represent decision-making under uncertainty and its flow-on effects through economic modelling.

In order to deal with grey-swan problems including climate change, modelling approaches must be capable of representing goal, institution or behavioural change in the form of innovation; such that public/private investment decisions can be pivoted to effectively target, mitigate, prevent or adapt to climate change shocks (Mazzucato, 2013). Where open-system thinking remains routinised, along with mainstream economic thought, both an evolution of thought as well as an understanding of that evolution across public/private decision-makers requires direct attention (Dow, 2020). The purpose of this paper is to contrast a common (i.e., expected value or EV) and alternative (i.e., state contingent analysis or SCA) approach to modelling uncertainty in complex economic, social and physical environments and evaluate the insights these alternative approaches provide for innovation and learning outcomes. This process allows us to answer three questions. First, is it possible to adapt the perfect foresight (i.e., white-swan) SCA model used by Adamson et al. (2009) through an incorporation of stochastic bounds to represent grey-swan problems such as climate change adaptation and decision-making? Second, if this is possible, what can we learn from comparing stochastic bounded SCA model results to perfect foresight (i.e., white-swan) EV models of how decision-makers' may innovate/learn to (re)allocate water resources in response to uncertain climate change? Third, what do these results suggest for future modelling of decision-making under uncertainty in complex interrelated system settings?

Decision-making and innovation/learning insights from contrasting these modelling approaches will be based on agricultural production and land/water use (re)allocation choices in Australia's Murray–Darling Basin (MDB). Climate change in the MDB illustrates a need to shift from routine to innovative choices where future outcomes may be known (e.g., decreased rainfall), but the full set of choices (both allowing and motivating innovative adaptation) remains incomplete. We begin with a description of the MDB and its production/water systems as a basis for the model data before shifting to an explanation of the two analytical approaches.

2 Study context: Australia's MDB

2.1 Water resources

In Australia's Murray-Darling Basin (MDB) water resources are over-allocated. There are approximately 19,300GL (gigalitres or one billion litres) of water right entitlements on issue, while annual conjunctive and allocable water resources average around 11,000GL per annum (BoM, 2020). This significant over-allocation creates conflict between economic, social and environmental users and climate change is expected to reduce the volume of water available for all users in future, exacerbating conflict. Any misallocation of water resources will increase the fragility of the economic, social and environmental systems and lower their capacity to respond to future unknowns.

The MDB dominates eastern and southern Australia and its river networks drain over one million square kilometres (km^2) (MacDonald and Young, 2001) or 14 per cent of the Australian continent. The two major rivers are the Darling River running north to south and the Murray River, including its significant tributary the Murrumbidgee River, which both run east to west. The total river network extends over 440,000km suppling water to consumptive users (e.g., towns and irrigators) and contributing to the inundation of up to 30,000 wetlands (Murray-Darling Basin Authority (MDBA), 2010). Ten per cent of Australia's population resides within the Basin (Australian Bureau of Statistics, 2008b) but its water resources also supplement the City of Adelaide's potable water supply which adds a further five per cent of Australia's population into the water use mix (Australian Bureau of Statistics, 2010). Over 80 per cent of the MDB is dedicated to agriculture but only two per cent is irrigated. On a national perspective the MDB comprises over 40 per cent of all irrigators, over 65 per cent of total irrigated land, and accounts for 68 per cent of all water diverted for irrigation (Australian Bureau of Statistics, 2008c). Subject to climatic variability, the MDB produces 35 to 40 per cent of Australia's total gross value of agricultural production and one third of this value is derived from irrigation activities (Australian Bureau of Statistics, 2008a, Australian Bureau of Statistics, 2009). It is this gross value return from irrigation that has driven over 100 years of domestic policy within the MDB attempting to deal with water (re)allocations (Cummins and Watson, 2012) arguably at the expense of other irrigators and dry-land producers (Davidson, 1969), the environment (Young and McColl, 2009), rural communities (Buikstra et al., 2010), traditional land owners (Loch et al., 2020b) and overuse that directly and negatively impacts other water users through externalities (Keating et al., 2002, Murray-Darling Basin Authority, 2011, Quiggin, 1988).

On average the MDB has 25,467GL of conjunctive water resources (runoff rather than river or storage inflows) that contributes to the annual allocable water resources shared between users. This volume is primarily derived from rainfall (23,925GL) and groundwater aquifers (1,424GL), while 1,118GL are transferred into the MDB from the Snowy Basin (Adamson et al., 2011). However, averages are misleading when dealing with water resources. The MDB has the second most variable water flows in the world (McMahon and Finlayson, 1991) and the Darling River is its most variable river system (Khan, 2008). Apart from inherent variability, modelling complexity in the MDB is increased by errors in the hydrological data sets including time series, measurement errors, conveyance losses, landscape modification, environmental use, and storages rules (Young and McColl, 2009).

2.2 Climate change

Climate change is expected to increase the frequency of MDB extreme events as well as the variability and inaccuracy described above. Climate change alters both the spatial and temporal patterns of rainfall. The general consensus for Australia is that droughts will become more frequent and more severe (IPCC, 2018). As 88 per cent of irrigation supplies in the Basin are derived from surface flows (Adamson et al., 2011) understanding future rainfall and runoff patterns is critical. However, outcome predictions are bounded by the complexities involved in upscaling and downscaling climate models (Berrocal et al., 2012). This scaling issue creates conflicting information. If we used proportional downscaling models based on an assumption that 4 per cent of MDB rainfall becomes surface runoff (Australian Bureau of Statistics, 2008b) then we could assume a linear relationship between rainfall to runoff. However, Austin et al. (2010) found that this relationship is not linear. Due to the spatial and temporal nature of rainfall and the landscape rainfall-runoff attributes incur an elasticity of two to three-times (i.e., a 10% decline in rainfall equates up to a 20 to 30% reduction in runoff), making predictions difficult.

New bounds shape the bio-physical characteristics of soil moisture that decision-makers must adapt to. If a decision-maker fails to consider grey-swan impacts to production inputs (e.g., water) and outputs (i.e., changing comparative advantage) then capital will also be exposed (Kingwell and Farre, 2009). Jodha (1991) argues that to prevent capital loss, decision-makers need to incorporate flexibility into their management systems and rapidly innovate based on past and new information when signals are 'ecologically rational' (Goldstein and Gigerenzer, 2002). In this paper 'ecologically rational' refers to a state of nature (drought, normal or wet). As Mallawaarachchi and Foster (2009) and Loch et al. (2012) discuss, any adaptation to climate signals is dependent on both the flexibility of institutional frameworks (e.g., capacity to trade water), the decision-maker's cognitive capacity (e.g., attitude to learning) and existing production system constraints. Importantly, by explicitly representing what we understand about future states, alternative public/private capacity to recognise those states, and the incomplete response set applicable to these settings we can identify when/how institutional or decision-maker solutions fail—and identify alternative (innovative) management solutions to increase adaptive flexibility. This flexibility should theoretically lead to robust outcomes which not only adapt to the forecast settings but also provide greater capacity to respond to future grey-swan events.

3 Modelling uncertainty and resource allocation

With this context in mind and, given our focus on white- and grey-swan problems, we next detail the EV and SCA approaches used in our comparisons. Each approach involves a discrete (certainty/white-swan) and stochastic (uncertainty/grey-swan) model specification. We begin with a general discussion of economic uncertainty modelling.

3.1 Traditional production economics approaches to modelling uncertainty

Uncertainty is typically modelled via sensitivity analysis to explore the mean and variance of a probability distribution whose variables positively/negatively impact costs/benefits (Merrifield, 1997). However, in such analyses the decision-maker remains passive to external stimuli and incapable of innovation/learning or operating within nonlinear parameters. Downside risk has also motivated researchers to expand beyond mean-variance approaches (e.g., by including skewness) to explore innovation/technology adoption impacts on our thin- or fat-tailed payoff distributions (Chavas and Nauges, 2020). For example, in production economic applications, modelling resource (re)allocation often involves a passive decision-making response based upon the mean and an error term. This formed the basis of Just & Pope's (1978) critical review of stochastic production functions where the general form of their model is:

$$\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{X}, \boldsymbol{\varepsilon}). \tag{1}$$

The error term (ε) provides a stochastic description of final output based on set combinations of inputs (X). However, the error term is frequently based on past data where known mean and variance parameterise a probability distribution function. Monte-Carlo simulations allow for a series of outcome-runs to determine the likelihood of investment decisions making a return given expected pricing outcomes. Taking these issues into account, Just and Pope concluded that while the generalised function is appropriate for empirical work, it remains unsatisfactory for dealing with future uncertainty. Prior to this, Rothschild & Stiglitz (1970, 1971) also noted the limitations of relying on mean-variance by illustrating the results of choosing between variables with the same expected value, but different mean distributions. A critical limitation, commonly known as *Mean Preserving Spread*, identifies how alternative weights in the distribution of tails can result in investors choosing riskier rather than safer investments by assuming the decision-maker remains passive to signals provided by the source(s) of uncertainty. As discussed, Pindyck (2011) argues climate change events can have thin- or fat-tailed outcomes and may transition between these two outcomes; possibly within short time frames due to the complex systems involved. Thus, modelling that fails to account for these possible outcomes may represent a decision-maker (e.g., farmer) as one who refuses to adapt in the face of required change no matter the uncertainty signal; which as discussed is unrealistic given different approaches to innovation/learning. Despite this conclusion, the use of stochastic production functions is common within the literature when dealing with production decisions under uncertainty. We detail a common modelling approach below.

3.2 Expected value modelling approaches

In expected value (EV) models future economic return $E[Y]$ is calculated by multiplying each possible outcome by the likelihood it will occur and then summing those values to provide a long-run average or mean. In a simple application/equation, discrete parameters are used. For example, in agricultural production decision-makers will maximise returns with respect to available area A of commodity options δ. Final returns are subject to yield Q, market prices P and costs of production C (Equation 2):

$$\boldsymbol{E}[\boldsymbol{Y}] = \sum_{\delta}\left[\boldsymbol{A}^{\boldsymbol{\delta}} \times \boldsymbol{Q}^{\boldsymbol{\delta}} \times (\boldsymbol{P}-\boldsymbol{C})^{\boldsymbol{\delta}}\right] \tag{2}$$

This then provides a single point estimate of an outcome (Figure 1a). In such a world we are modelling certainty. However, we can relax the discrete (white-swan) equation above and adapt it to climate change (grey-swan) problems by incorporating stochastic representations via the addition of an error term ε (as in Figure 1b) to describe final outputs based on random combinations of inputs, and represent uncertainty (Equation 3):

$$\boldsymbol{E}[\boldsymbol{Y\varepsilon}] = \sum_{\delta}\left[\boldsymbol{A}^{\boldsymbol{\delta}} \times \boldsymbol{Q}^{\boldsymbol{\delta}} \times (\boldsymbol{P}-\boldsymbol{C})^{\boldsymbol{\delta}}\right] \tag{3}$$

Recall that stochastic representations acknowledge the inherent natural variability within a system but also assume decision-makers do not innovate/learn in response to external stimuli, thereby potentially misallocating resources (Chambers and Quiggin, 2000, Chavas et al., 2010). However, this version of the model is more comparable to alternative state contingent analysis (SCA) approaches for determining how decision-makers (re)allocate resources in response to a changing climate.

3.3 State contingent analysis modelling approaches

SCA allows for active decision-making responses to revealed or anticipated state of nature outcomes (e.g., climate change impacts such as reduced future water availability or increased frequency of drought events). It therefore reduces problems of misinformation and resource misallocation that could be driven by applications of a stochastic EV model (Just and Pope, 1978). Early SCA studies used the term ‘states of nature’ when discussing the assessment of production choices under exogenous uncertainty.

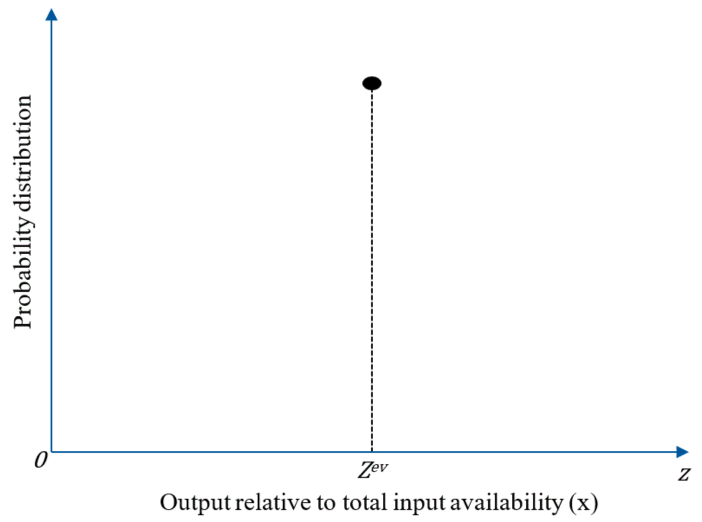

Figure 1a: Discrete EV state outcome representation.

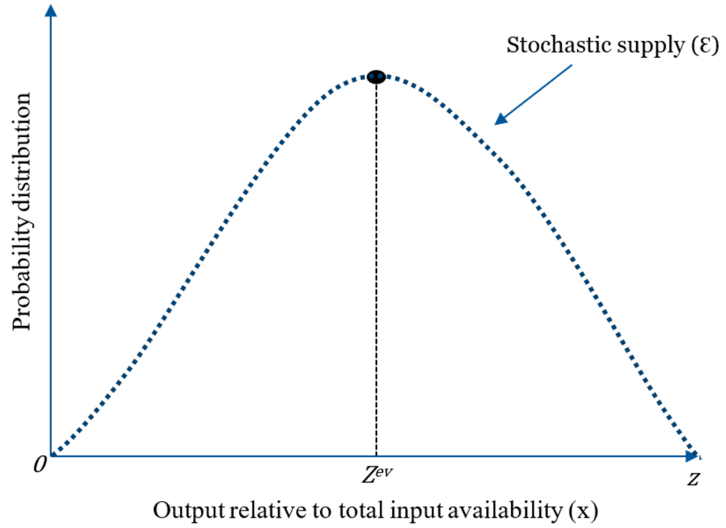

Figure 1b: Discrete EV state outcome representation with error terms.

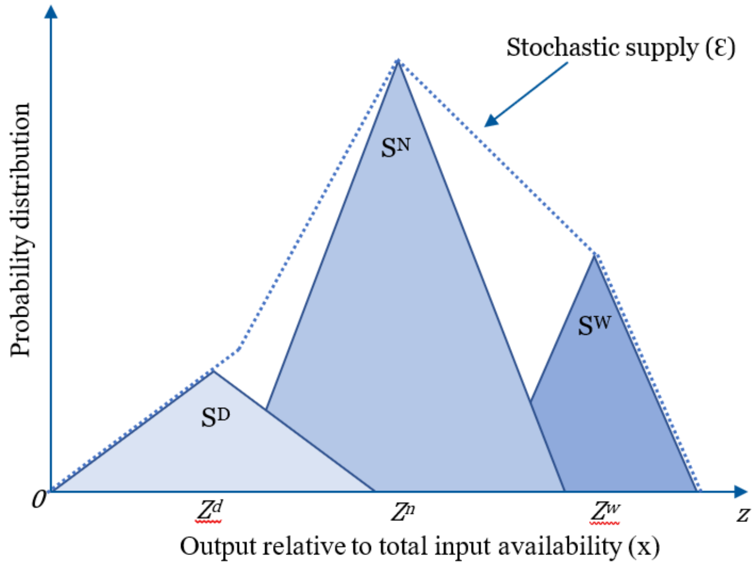

Figure 1c: SCA discrete state-described outcomes (D = dry, N = normal, W = wet) within the stochastic frontier.

The earliest work was undertaken by Arrow (1953) and Debreu (1959), providing capacity to represent how decision-makers respond to realised alternative states (e.g., drought/flood events). Graham (1981) used this approach to explore farmers' willingness to pay for a public dam project that provided water supply in dry states of nature, and flood mitigation in wet states. However, it was Hirshleifer's (1965, 1966)[1] work that articulated clear differences between dominant mean-variance approaches and state of nature representations of risk or uncertainty. According to Hirshleifer (1965), state of nature approaches remove the "vagueness" (pg. 534) associated with other uncertainty methodologies as it allows the decision-maker to precisely identify both the natural endowments provided in a given state, and the factors of production required to obtain an output in that state. This finding has been reiterated in more recent studies (e.g. Hildebrandt and Knoke, 2011, Adamson and Loch, 2021).

Chambers and Quiggin (2000) subsequently extended the state of nature approach by merging it with dual optimisation to illustrate how resource allocations represent adjusting input use in all states by time, place and type2 (Rasmussen, 2003). Following this work, the state of nature approach became the SCA approach. In the SCA approach, nature $(\boldsymbol{\Omega})$ defines the state space that can be divided into a series of states of nature (s) to define real and mutually-exclusive sets (S) describing uncertainty $(\boldsymbol{\Omega} = \{1, 2, \dots, s, \dots, S\})$. Importantly the decision-maker has no ability to influence which s occurs; s is determined exogenously (Figure 1c where triangles represent distribution by dry, normal and wet states). Further, the decision-maker's subjective belief about the frequency $(\boldsymbol{\pi})$ of each s occurring is a probability vector described by $(\boldsymbol{\pi} = \pi_1, \dots, \pi_s)$. However, for each s the decision-maker has a set of management options giving rise to alternative production possibilities (technology set). This can be represented (Equation 4) by a "continuous input correspondence, $\boldsymbol{X}: \boldsymbol{\Re}_{+}^{S} \rightarrow \boldsymbol{\Re}_{+}^{N}$, which maps state-contingent inputs into output sets that are capable of producing that state-contingent output vector" (Chambers and Quiggin, 2002, pg. 514):

$$\boldsymbol{X}(\mathbf{z}) = \{\boldsymbol{x} \in \boldsymbol{\Re}_{+}^{N}: \mathbf{x\ can\ produce\ z}\}. \quad (4)$$

The basic form of the state contingent approach to model production risk and uncertainty is:

$$\mathbf{y_S} = \mathbf{f_s}(\mathbf{x}) \quad \mathbf{s} \in \boldsymbol{\Omega} = \{\mathbf{1}, \dots, \mathbf{S}\} \quad (5)$$

where output (y) is described from a specific crop (x) produced within a single state of nature (s). Rasmussen (2006) argues that outcomes (i.e., yields and prices) arise from states of nature, implying the use of stochastic functions. Chavas (2008) used this concept to illustrate the output from a decision when inputs had to be allocated before the state was fully realised. By highlighting the reliability of state conditions or what we expect within a state (e.g., quantity of rainfall in a drought) we can illustrate production heterogeneity within the model (i.e., variable yields in a given state of nature), the amount of input required by state (i.e., water requirements by commodity by state of nature) and importantly the management response to extant conditions (i.e., variation with a state decreases through time via learning). Now climate change can be also be represented by altering the states' variance. This can be written as:

$$\mathbf{y_S} = \mathbf{f_s}(\mathbf{x}, \boldsymbol{\varepsilon}) \quad \mathbf{s} \in \boldsymbol{\Omega} = \{\mathbf{1}, \dots, \mathbf{S}\}. \quad (6)$$

This sets climate variability and resource (re)allocation by placing boundaries on incomplete awareness of future states and illustrating a decision-makers' ability to learn and apply appropriate contingency measures. When the re-introduction of stochastic errors are applied independently to either the state of nature, the state specific inputs requirements, or the

[1] Note Hirshleifer (1965) uses the term 'state-preference' rather than Arrow's (1953) states of nature.

[2] Refers to three input types: i) ***non-state-specific (or state-general)*** inputs that must be allocated *ex-ante* to the s being realised, and which influence z in all s; ii) ***state-specific inputs*** that are applied *ex-post* to the realisation of s, and which influence z in only that s; and iii) ***state allocable (flexible) inputs*** that are applied *ex-ante* to s being realised, but where benefits accrue once s is realised.

state described outputs then the environmental signal and the response to that signal can be separated. This separation minimises multiplicative and additive uncertainty found in approaches with multiple stochastic functions. Applications of this approach can either follow Chavas' (2008) two stage decision model (i.e., fixed inputs allocated before the state was revealed) or mimic capacity to (re)allocate resources once the state is revealed. The concept of a two stage decision is analogous to describing the error term from Equation 5 to simulate the solution with a Monte-Carlo method (Liddle and Monahan, 1988). For our paper this is referred to as simulating or the *ex-ante* solution (i.e., before the state is revealed). The second approach occurs when decision-makers can (re)allocate resources as the state reveals itself. This is analogous to having perfect awareness (certainty) and is modelled through Monte-Carlo optimisation using Equation 6 where new data is drawn from the discrete distribution. In this paper, we label this the *ex-post* solution. A triangular distribution has been used to set hard bounds (Equation 7). As the distance between bounds increase the fuzziness of the grey-swan proportionally increases.

$$y_s = \begin{cases} \frac{2(y_s - a_s)}{(c_s - a_s)(b_s - a_s)} \text{ if } a_s \leq y_s \leq c_s \\ \frac{2(b_s - y_s)}{(b_s - c_s)(b_s - a_s)} \text{ if } c_s \leq y_s \leq b_s \end{cases} \tag{7}$$

Note that state of nature is a special case in this formula. River flow can only be $a_s \geq 0$ but the minimum and maximum water supply is confined by other state of nature mean flows and must not exceed c of the proceeding and subsequent states of nature. For example, if state 1 = drought, state 2 = normal, and state 3 = wet then $c_1 \leq b_2 \leq c_2$. This prevents obscuring the signal.

In our SCA models, a complete set of states explicitly internalises uncertainty and allows for discrete approaches using certainty to be estimated. This specification means the decision-maker has perfect awareness about all future states of nature (i.e., a drought state is always identical through time), any commodity always requires the same volume of irrigation water (and other inputs) in each state of nature, and outputs through time are constant within each state. This equation can be expanded to mimic the stochastic EV model specification above, where the probability of state occurrence π, production areas, commodity selections, yields, market prices and costs are all state specific (Equation 8):

$$\boldsymbol{E}[\boldsymbol{Y}] = \sum_{\delta} \sum_{\boldsymbol{S}} \boldsymbol{\pi_S} \left[\boldsymbol{A}^{\delta} \times \boldsymbol{Q}_{\boldsymbol{S}}^{\delta} \times \ (\boldsymbol{P} - \boldsymbol{C})_{\boldsymbol{S}}^{\delta}\right] \tag{8}$$

This version of the model is aligned to a perfect knowledge (white-swan) outcome, where all probabilities are known in advance. Chambers and Quiggin (2000) argue that representing white-swan (i.e., perfect foresight) problems within a state contingent approach allows for solutions to be generated using standard optimisation techniques applied to problems not involving uncertainty. For example, using a perfect foresight model, Adamson et al. (2009) illustrated how SCA can encapsulate economic, social and environmental objectives while predicting how irrigators respond to climatic variability and change. By explicitly representing water variability by state (i.e., drought, normal and wet) and decision responses to that state (change in inputs or production system) they determined that changes in the frequency of drought states was a greater factor in decision-maker allocation of capital than the reduction in water supply by state outcome. However, they concluded that when optimising with complete discrete awareness, while the theoretical optimum may be obtained, the solution will be inflexible as any natural variability in the description of the state and the inputs required in each state is ignored. Further, in that case, as the solution was derived from a directed river flow/network model any inherent uncertainty would violate the optimisation constraints and ultimately result not only in misallocated resources but in solutions that could compound negative externalities.

To counter these outcomes, a stochastic determination and incorporation of probability outcomes in the model may be achieved via a stochastic SCA version. To model a grey-swan agricultural production problem the model must incorporate a stochastic representation of either the state of nature outcome (e.g., drought) and/or those inputs required by each state of nature (e.g., additional water resources). Further, by examining each option separately it prevents the environmental signal and the response to that signal from being misinterpreted. Importantly, relaxation of the discrete values permits decision-makers to innovate/learn in response to new signals within defined bounds. Further, a stochastic representation of outputs by state of nature is not needed as the ultimate constraint is the total future volume of water to share between all users. State described output is a function of the state and the inputs available as below:

$$\boldsymbol{E}[\boldsymbol{Y_\varepsilon}] = \sum_{\boldsymbol{\delta}} \sum_{\boldsymbol{S}} \boldsymbol{\pi_S} \left[\boldsymbol{A^\delta} \times \boldsymbol{Q_S^\delta} \times (\boldsymbol{P}-\boldsymbol{C})_{\boldsymbol{S}}^{\boldsymbol{\delta}}\right]. \tag{9}$$

As π_s is the probability of the state occurring, $\sum \pi_s = 1$ (i.e., every state is identified), where $0 < \pi \leq 1$ (i.e., the states must have a chance of occurring). Here π 1 to 3 = (0.5, 0.3, 0.2). The three states of nature (S) modelled represent alternative MDB inflows. These states are normal (the expected long term average inflows derived from (MDBC, 2006)), drought (0.6 x Normal Inflows), and wet (1.2 x Normal Inflows) and decision-makers are expected to respond to these state outcomes by (re)allocating output.

3.4 Additional model specification requirements

Climate change compounds the externalities associated with over-allocated and ill-defined water property rights in the MDB (Young and McColl, 2009). These negative externalities include loss of natural capital (Kingsford, 2000), high salinity (Keating et al., 2002, Yaron and Bresler, 1970), urban water quality reduction (Adamson et al., 2008) and the inability of the irrigation network to deliver water to all users when required (Robertson and Wang, 2004). Consequently, to deal with inequitable resource (re)allocations, future shares of water in the MDB are constrained by social and environmental objectives to internalise these externalities (Commonwealth of Australia, 2008). We account for these issues with extensions to the model specifications as below. The model aims to maximise economic return (Equations 2, 3, 8 and 9) from using water for irrigation in a spatially explicit representation of regional comparative advantage in production by state of nature (S = 3: drought, normal and wet). Commodity options δ equal 16 in the EV models and 21 in the SCA models. This difference is due to the SCA model's capacity to transition in and out of production return-generating activity Y by state of nature. The river system is modelled as an undeveloped network with natural inflows and salt loads by state of nature. As water is extracted for irrigation the return flows transport salt back into the river network, thereby highlighting the opportunity cost of water use by location. The models are optimised from a national good perspective (i.e., a benevolent individual controlling all resources to maximise returns across catchments subject to environmental needs and social requirements for salinity levels in water). This then mimics the complex interrelated system described at the beginning of the paper.

For each model specification the relevant equations are summarised in Table 1. Maximised economic return is always subject to maintaining the City of Adelaide's water quality at less than 800 EC in each state of nature. The measurement of salinity in milligrams per litre (σ) is converted into electrical conductivity (EC) by dividing it by 0.64 (Equations 10-13). The volume of water used in the basin must also always be less than the CAP[3] on average (i.e., as long as the average CAP is not violated you may use up to the CAP in a given state of nature) as specified in Equations 14-17. In the model extractions described for the urban and dryland use under the CAP, all catchments apart from Adelaide are removed from inflow before

[3] CAP as in the limit on long term diversions. Here the term CAP is interchangeable with CDL or SDL depending on which scenario is run.

the model is optimised to ensure that they received their allocations. The CAP has been transformed simply into diversions for irrigation purposes. Equations 18-21 ensure that water use in a catchment must be less than or equal to the flow in that catchment, while Equations 22-25 state that the area dedicated to horticulture in any catchment must be less than equal to the horticultural constraint in that area. Equations 26-29 ensure that the total area dedicated to irrigation in any region must be less than the total area available in that region. Finally, Equations 30-33 ensure that there is sufficient operator labour to undertake the irrigation activity mix in a region (r).

Table 1: Summary of the full model specification, by approach and discrete/stochastic setting

EV	Eq.	EV, Stochastic	Eq.	SCA	Eq.	SCA, Stochastic	Eq.
$E[Y] = \sum_{\delta} [A^{\delta} \times Q^{\delta} \times (P-C)^{\delta}]$	(2)	$E[Y_{\varepsilon}] = \sum_{\delta} [A^{\delta} \times Q^{\delta} \times (P-C)^{\delta}]$	(3)	$E[Y] = \sum_{\delta} \sum_{S} \pi_S [A^{\delta} \times Q_S^{\delta} \times (P-C)_S^{\delta}]$	(8)	$E[Y_{\varepsilon}] = \sum_{\delta} \sum_{S} \pi_S [A^{\delta} \times Q_S^{\delta} \times (P-C)_S^{\delta}]$	(9)
Subject to:							
$\sigma^{20}/0.64 \leq 800\ \mathrm{EC}$	(10)	$\mathrm{VaR}_{0.95}\left(\sigma^{20}/0.64 \leq 800\ \mathrm{EC}\right)$	(11)	$\sigma_s^{20}/0.64 \leq 800\ \mathrm{EC}$	(12)	$\mathrm{VaR}_{0.95}\,(\sigma_s^{20}/0.64 \leq 800\ \mathrm{EC})$	(10)
$\sum K \leq \mathrm{CAP}$	(14)	$\sum K \leq \mathrm{CAP}$	(15)	$\sum K_s \pi_s \leq \mathrm{CAP}$	(16)	$\sum K_s \pi_s \leq \mathrm{CAP}$	(11)
$wk \leq fk$	(18)	$Va\delta_{0.95}\,(wk_{\varepsilon} \leq fk_{\varepsilon})$	(19)	$wk_S \leq fk_s$	(20)	$Va\delta_{0.95}\,(wk_{S\varepsilon} \leq fk_{s\epsilon})$	(21)
$A_k \delta_{1..5} \leq \mathrm{AHort}_k$	(22)	$A_k \delta_{1..5} \leq \mathrm{AHort}_k$	(23)	$A_k \delta_{1..5} \leq \mathrm{AHort}_k$	(24)	$A_k \delta_{1..5} \leq \mathrm{AHort}_k$	(25)
$A_k \delta_{1..16} \leq \mathrm{Atotal}_k$	(26)	$A_k \delta_{1..16} \leq \mathrm{Atotal}_k$	(27)	$A_k \delta_{1..22} \leq \mathrm{Atotal}_k$	(28)	$A_k \delta_{1..22} \leq \mathrm{Atotal}_k$	(12)
$\sum L_{rk} \leq L_k$	(30)	$\sum L_{rk} \leq L_k$	(31)	$\sum L_{rk} \leq L_k$	(32)	$\sum L_{rk} \leq L_k$	(33)
$fk^{21} \geq 1{,}000\ \mathrm{GL}$	(34)	$Va\delta_{0.95}\,(fk^{21} \geq 1{,}000\ \mathrm{GL})$	(35)	$fk_s^{21} \geq 1{,}000\ \mathrm{GL}$	(36)	$Va\delta_{0.95}\,(fk_{s\varepsilon}^{21} \geq 1{,}000\ \mathrm{GL})$	(37)

405 **Table 2:** Production System by EV and SCA models

Production System	**State Contingent Crop**			**Normal (EV)**
	Drought	**Normal**	**Wet**	
Citrus-H	Citrus-H	Citrus-H	Citrus-H	Citrus-H
Citrus-L	Citrus-L	Citrus-L	Citrus-L	Citrus-L
Grapes	Grapes	Grapes	Grapes	Grapes
Stone Fruit-H	Stone Fruit-H	Stone Fruit-H	Stone Fruit-H	Stone Fruit-H
Stone Fruit-L	Stone Fruit-L	Stone Fruit-L	Stone Fruit-L	Stone Fruit-L
Pome Fruit	Pome Fruit	Pome Fruit	Pome Fruit	Pome Fruit
Vegetables	Melons	Vegetables	Fresh Tomatoes	Vegetables
Cotton Flex	Dryland Cotton	Cotton Flex	Cotton	
Cotton Fixed	Cotton Fixed	Cotton Fixed	Cotton Fixed	Cotton Fixed
Cotton/Chickpea	Chickpea	Cotton Flex	Cotton	
Cotton Wet	Dryland Cotton	Dryland Cotton	Cotton	
Rice PSN	Rice PSD	Rice PSN	Rice PSW	Rice PSN
Rice Flex	Dryland Wheat	Rice PSN	Rice PSW	
Rice Wet	Dryland Wheat	Dryland Wheat	Rice PSW	
Wheat	Wheat	Wheat	Wheat	Wheat
Wheat Legume	Wheat Legume Dry	Wheat Legume	Wheat Legume Wet	Wheat Legume
Sorghum	Sorghum	Sorghum	Sorghum	Sorghum
Oilseeds	Oilseeds	Oilseeds	Oilseeds	Oilseeds
Sheep Wheat	Sheep Wheat Dry	Sheep Wheat	Sheep Wheat Wet	Sheep Wheat
Dairy-H	Dairy-H	Dairy-H	Dairy-H	Dairy-H
Dairy-L	Dairy-L	Dairy-L	Dairy-L	Dairy-L

Notes: The EV model has less production systems available as the ability to alter production systems by state of nature is not considered.
H= intensive irrigation capital (i.e., drip lines)
L = low irrigation capital (i.e., furrows)

406 The production costs for producing one hectare of commodity for each K in each S can
407 be written as the sum of capital costs (i.e., capital costs do not change by state of nature and are
408 modelled as an annual cost) plus operator labour costs LC (i.e., hours L is multiplied by a
409 constant price LP plus variable costs VC (Equation 38). Finally, Equation 39 details the
410 variable production costs which are derived from the sum of casual labour CL (i.e., hours
411 multiplied by a constant price), contractor costs Con, machinery costs Ma, chemical costs Ch,
412 plus water use W multiplied by water price Wp and the sum of any other costs Ot.

$$\mathrm{R_{ks}} = \sum(\mathrm{CC_k} + \mathrm{LC_{ks}} + \mathrm{VC_{ks}}) \tag{38}$$

$$\mathrm{VC_{ks}} = \sum(\mathrm{CL_{ks}} + \mathrm{Con_{ks}} + \mathrm{Ma_{ks}} + \mathrm{Ch_{ks}} + (\mathrm{W_{ks}} \times \mathrm{Wp_{ks}}) + \mathrm{Ot_{ks}}) \tag{39}$$

Once again, Equations 33-33 in Table 1 deal with the amount of operator labour L required to produce ∑δ in K. Here we ensure that the amount of labour in a region derived from ABS (2004) data and based on number of farms multiplied by two people by 2,500 hours/person is adequate to meet the needs the chosen production systems.

Table 3: Water Use by State (Condamine catchment example only)

Production System	State Contingent Water Use			Normal (EV)
	Drought	Normal	Wet	
Citrus-H	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
Citrus-L	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
Grapes	(2.5,5.0,7.5)	(2.5,5,7.5)	(3,6,9)	(2.5,5,7.5)
Stone Fruit-H	(1.7,3.33, 5.0)	(1.7,3.3,5)	(2,4,6)	(1.7,3.3,5)
Stone Fruit-L	(3.2,6.4,9.7)	(3.2,6.4,9.7)	(3.9,7.7,11.6)	(3.2,6.4,9.7)
Pome Fruit	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
Vegetables	(2.0,4.0,6.0)	(0,0,0)	(3,6,9)	(0,0,0)
Cotton Flex	(2.5,5.0,7.5)	(0,0,0)	(2.5,5,7.5)	
Cotton Fixed	(2.5,5.0,7.5)	(2.5,5.0,7.5)	(2.5,5,7.5)	(2.5,5.0,7.5)
Cotton/Chickpea	(2.5,5.0,7.5)	(1.9,3.8,5.6)	(2.5,5,7.5)	
Dryland Cotton	(0,0,0)	(0,0,0)	(3,6,9)	
Rice PSN	(0,0,0)	(0,0,0)	(0,0,0)	(0,0,0)
Rice Flex	(0,0,0)	(0,0,0)	(0,0,0)	
Dryland Wheat	(0,0,0)	(0,0,0)	(0,0,0)	
Wheat	(0.8,1.5,2.3)	(0.8,1.5,2.3)	(0.9,1.8,2.7)	(0.8,1.5,2.3)
Wheat Legume	(1.3,2.6,3.9)	(0.8,1.7,2.5)	(1.6,3.2,4.7)	(0.8,1.7,2.5)
Sorghum	(2.0,4.0,6.0)	(2.0,4.0,6.0)	(2.4,4.8,7.2)	(2.0,4.0,6.0)
Oilseeds	(2.0,4.0,6.0)	(2.0,4.0,6.0)	(2,4,6)	(2.0,4.0,6.0)
Sheep Wheat	(2.4,4.8,7.1)	(3.7,7.4,11).0	(1.7,3.5,5.2)	(3.7,7.4,11).0
Dairy-H	(4.5,9.0,13.5)	(3.2,6.3,9.5)	(5.4,10.8,16.2)	(3.2,6.3,9.5)
Dairy-L	(5.0,10.0,15.0)	(3.0,6.0,9.0)	(6,12,18)	(3.0,6.0,9.0)

Notes: For both the State-contingent and expected value the discrete data is 'c' of the middle number. If all values 0 then the commodity is not grown in that region. If the values are 0 in states and >0 in others it means that the production system has transitioned from dryland to irrigated. If the cell is blank for the EV that production system is not possible in the EV model

3.5 Models and data

The models were developed in Microsoft Excel using the Risk Solver Platform from Frontline systems v.12. The platform uses a Monte-Carlo approach to simulate the stochastic descriptions of inflows and uses 1,000 trials for a single simulation. The optimisation algorithm used was the Large Scale SQP Engine to deal with the non-linearity of river flow. Each of the models uses a conjunctive approach to water resources as described above. Consequently, total water inflows are dependent upon surface supplies, ground water supplies and inter-basin transfers. The model uses a directed flow network where the Basin is divided into 21 catchments (K) which consists of the 19 irrigation areas plus the City of Adelaide and the Coorong (default flow to sea). Production area by catchment A is a matrix of production systems $(\mathrm{K} \times \delta) \times S$ (Table 2). There are 23 production systems (δ) consisting of 21 irrigation activities, the City of

Adelaide's water supply, and a dryland production system. Catchments (e.g., the Condamine as described in Table 3 and further detailed in Appendix 1) are based on disaggregated Catchment Management Regions to help model the directed flow network (water and salt). Water flows (*fks*) out of a given catchment are equal to inflows (net of evaporation and seepage) less extractions (net of return flows). Extractions are determined endogenously by land use decisions as described below, subject to limits imposed by the availability of both surface and ground water (Equations 34-37, Table 1). This structure allows for the determination of total irrigation use, the flow to the Coorong, and water quality arriving at the City of Adelaide.

The second critical factor in describing A is the matrix δ where the state contingent production systems are defined. Each state of nature δ outcome will derive an independent representation of yields Q, prices P, costs of production C and input requirements N such that each matrix has a form of (21 x 23). This data is based on a series of regional gross margin budgets that provide the data for the five inputs modelled (N = water, land, labour, capital and cash input). The production systems are derived from $(\mathrm{K} \times \mathrm{M}) \times \mathrm{S}$, where in this case M represents commodities. A commodity is a single enterprise under a given state in a given catchment. This version of the model has 15 distinct commodities (M) plus urban water for the City of Adelaide and water for the Coorong. Consequently, there are (M+2) x S distinct state-contingent commodities. Yield Q has a dimension of $(\mathrm{K} \times \delta) \times S$ which represents the output derived for that state of nature. Net return per hectare is described in the model as (P-C). Price P paid for output has a matrix of $(\mathrm{M} \times \mathrm{S})$. For simplicity it has been assumed that the price paid in all regions for each commodity is uniform by state of nature.

Because the model is solved on an annual basis, the process of capital investment is modelled as an annuity representing the amortised value of the capital costs over the lifespan of the development activity. This allows us to permit the modelling of a range of pricing rules for capital, and to represent the imposition of appropriate constraints on adjustment to derive both short-run and long-run solutions. Finally, the state contingent approach allows for discontinuous environmental and production functions to be classified as alternative functions within each state of nature. This specification of environmental, urban or private requirement by state of nature helps determine the type and number of water property rights needed to meet that demand. For all scenarios examined in this paper, Equations 34-37 apply only to those scenarios that specify a minimum flow of 1,000 GL reaching the Coorong. Alternative studies could incorporate environmental targets along the river system to stipulate river flow constraints along the system as either flow targets by each state of nature (i.e., drought, normal or wet) or on average over the states of nature.

4 Results

Recall that our first research question queried if it is possible to represent grey-swan problems via stochastic bounds across the model types. In simple terms this is possible, and our results suggest that discrete (certain) EV models align well to stochastic (uncertain) SCA model specifications. This can be illustrated via a consideration of how land and water resources are (re)allocated across the MDB in response to different event outcomes (Table 4). EV model economic returns during dry and wet events, even when modelled by input or state, are non-existent. By contrast, the discrete, input and state SCA models return more nuanced results across all states for both current and future climate conditions. Note though that both salinity and Coorong flow results are differentiated between the discrete and input/state EV models, where wet and dry condition outcomes drive some reallocation above zero values. This suggests that the ex-post inputs model and both the ex-ante and ex-post state EV models have some capacity to respond to stochastic events with respect to those model constraints—but not others. Thus, the discrete EV models appear to ignore (not represent) environmental signals.

Table 4: Results Comparison

	Model Run	Water Use (GL)				Coorong Flow (GL)				Salinity (EC)				Economic Return ($'m)			
		Normal	Dry	Wet	Avg	Normal	Dry	Wet	Avg	Normal	Dry	Wet	Avg	Normal	Dry	Wet	Avg
Current climate	**EV, Discrete**	9,162	0	0	9,162	6,221	0	11,753	6,221	228	0	222	228	$2,591	$0	$0	$2,591
	EV, Inputs Ex-ante	9,412	0	0	9,412	6,191	0	11,746	6,191	231	0	223	231	$2,644	$0	$0	$2,644
	EV, Inputs Ex-post	7,065	0	0	7,065	7,833	1,256	13,475	7,833	158	380	166	158	$2,270	$0	$0	$2,270
	EV, State Ex-ante	9,368	0	0	9,368	760	3,858	11,755	760	1,045	203	222	1,045	$2,591	$0	$0	$2,591
	EV, State Ex-post	6,792	0	0	6,792	2,572	6,056	13,606	2,572	407	94	161	407	$2,013	$0	$0	$2,013
	SCA, discrete	8,930	5,849	11,757	9,162	6,383	1,287	10,482	6,594	235	288	285	260	$2,644	$1,085	$3,872	$2,701
	SCA, Input Ex-ante	9,116	6,092	11,994	9,374	6,398	1,254	10,461	6,588	234	295	286	262	$2,644	$1,085	$3,871	$2,701
	SCA Input Ex-post	8,013	5,517	10,763	8,339	7,170	1,772	11,323	7,336	190	214	243	211	$2,396	$1,026	$3,590	$2,480
	SCA, State Ex-ante	9,136	6,055	11,963	9,368	880	5,771	10,478	4,738	1,061	91	285	634	$2,644	$1,085	$3,872	$2,701
	SCA, State Ex-post	6,829	5,516	14,146	8,762	2,465	6,212	8,956	5,161	409	70	342	321	$2,084	$901	$3,663	$2,321
Climate change 550 Avg.*	**EV, Discrete**	9,000	0	0	9,000	3,756	0	8,117	3,756	350	0	311	350	$2,502	$0	$0	$2,502
	EV, Inputs Ex-ante	9,228	0	0	9,228	3,740	0	8,109	3,740	350	0	312	350	$2,348	$0	$0	$2,348
	EV, Inputs Ex-post	7,853	0	0	7,853	4,703	0	9,119	4,703	262	0	253	262	$2,292	$0	$0	$2,292
	EV, State Ex-ante	9,206	0	0	9,206	0	1,627	8,136	0	1,305	411	310	1,305	$2,502	$0	$0	$2,502
	EV, State Ex-post	5,311	0	0	5,311	2,079	4,962	10,980	2,079	656	133	210	656	$1,852	$0	$0	$1,852
	SCA, discrete	9,179	4,632	12,154	9,162	3,630	1,000	6,430	3,944	374	266	445	374	$2,578	$1,075	$3,752	$2,629
	SCA, Input Ex-ante	9,392	4,842	12,369	9,375	3,625	997	6,424	3,939	375	268	445	374	$2,348	$959	$3,447	$2,400
	SCA Input Ex-post	8,636	4,396	10,752	8,422	4,155	1,398	7,556	4,624	315	183	347	298	$2,116	$884	$3,242	$2,208
	SCA, State Ex-ante	9,385	4,838	12,360	9,368	0	4,583	6,449	2,851	0	86	444	150	$2,578	$1,075	$3,752	$2,629
	SCA, State Ex-post	5,293	3,687	15,032	7,894	2,097	5,641	4,570	3,548	440	51	664	429	$1,853	$869	$3,472	$2,142
Frequency	**SCA, discrete**	10,535	3,492	14,234	9,162	5,260	3,409	8,748	5,402	297	87	367	248	$2,809	$1,279	$3,786	$2,546
	SCA, Input Ex-ante	10,752	3,682	14,447	9,370	5,253	3,423	8,743	5,402	296	86	367	247	$2,546	$1,192	$3,431	$2,317
	SCA Input Ex-post	9,599	3,194	12,919	8,342	6,059	3,863	9,813	6,151	247	62	308	204	$2,355	$1,048	$3,281	$2,148
	SCA, State Ex-ante	10,741	3,698	14,440	9,368	0	7,866	8,745	4,109	1,538	42	368	855	$2,809	$1,279	$3,786	$2,546
	SCA, State Ex-post	7,074	4,640	17,575	8,444	2,396	7,079	6,558	4,634	436	55	528	340	$2,126	$1,009	$3,694	$2,104

Notes: Abbreviations: Dry is used for Drought, Irr is abbreviated for irrigation, Avg used for Average. Shaded areas are results when compared the EV estimated water use is compared to the SCA model's estimation of flow for that state. Average in the EV are the Normal Values to make the table easier to read.
0 for salinity means that as there was no water flowing there was no salinity.

* The 550 Average scenario denotes a 550 parts-per-million CO^2 representative conceptration pathway, or a relatively high level of future climate change impact.

Note also the water use results in Table 4. In both current and future climate conditions water use in dry and wet conditions falls to zero across the full set of EV models, with only the mean (normal) values being reported. While all models have the same bounds, the capacity of the SCA models to capture and report differences in the frequency of state events allows for kinks in the available water supply in gigalitres (Figure 2). Akin to thinking by Guttman et al. (2006), differences between rational experience, informed decision-making and discontinuities arising from innovation or learning may appear as kinks. Their work finds no evidence of smoothing in financial markets; why then would we expect to find it in complex natural systems? In our models, changes in event frequency (e.g., more drought) causes decision kinks to move inversely. In the EV models these might be interpreted as noise, rather than adaptation decisions in response to altered state outcomes.

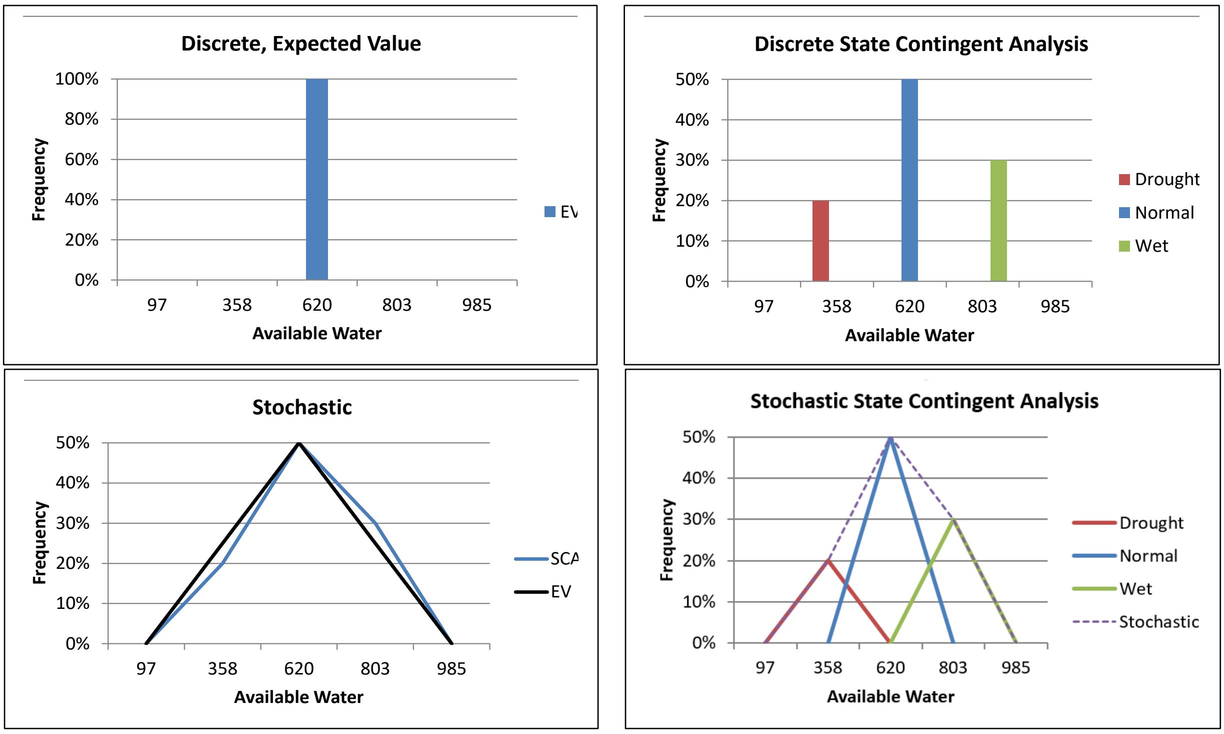

Figure 2: Charts of water use derived from the alternative modelling approaches

Production (re)allocation decision results are also interesting across the model runs, especially for horticulture crops between the EV and SCA models (Table 5). Note what happens in the area by state of nature. Under increasing climate change uncertainty the EV model is unable to respond to the extreme tails in the distribution, sticking instead to mean values. This shrinks the total area dedicated to irrigation. By contrast, the SCA model reallocates land under production during good seasons to offset lower returns in the drought (e.g., wheat/cotton). In this situation we see the SCA model reallocates water away from those production systems that always use water towards those that stop irrigating in drought years (Flex cotton and Flex Rice). Opportunistic irrigation will only occur in wet years (Dryland Cotton and Dryland Wheat[4]), when the frequency of drought states increases. This represents decision-maker adaptation to the uncertainty signals provided and modelling capacity to represent innovation/learning over adherence to familiar decision pathways based on experience (i.e., routines).

[4] Remember Table 3 where water use by SCA production system is produced. Rice is not grown in the Condamine so appears as 0 ML/Ha in all states.

The requirement for models to take event tails into consideration—and prompt innovation or learning (note the change in area between SCA State, ex-ante and SCA State ex-post between Rice Flex and Wheat Dryland)—is also borne out in predicted water flows by state which enables us to look more closely at the importance of variability. Under the discrete EV and SCA models demand will quickly outstrip supply in drought threatening river system shutdown. But, equally, the models will fail to report decision-makers' willingness to use abundant water resources during wet periods. Model solutions that fail to account properly, or at all, for system bounds and the full extent of outcome variance ultimately fail. Figure 3 puts this into perspective across the model runs. The EV model data range for both discrete and stochastic representations of inputs falls within the estimated bounds of the discrete SCA model. This suggests that the EV model cannot progress past the range of the discrete SCA, is constrained by the tails, and that decision-makers cannot learn to adapt or innovate to change. This is unrealistic as we have discussed above. Further, differences between the ex-ante (uncertain) and ex-post (certain) models also illustrate the positive benefits of innovation/learning by decision-makers.

As both Quiggin (2019) and (Pindyck, 2011) have noted, tails in the distribution have serious implications for understanding climate change and rare events often have far greater consequence than we give them credit for. Thus, any approach that cannot represent both the impact and the possible solution to rare events may lead to a serious underinvestment in climate responses leading to long term social losses. Considering the time required to optimise between a discrete model and a stochastic model and the outcomes from the model, a discrete SCA model provides significant insights for minimal time.

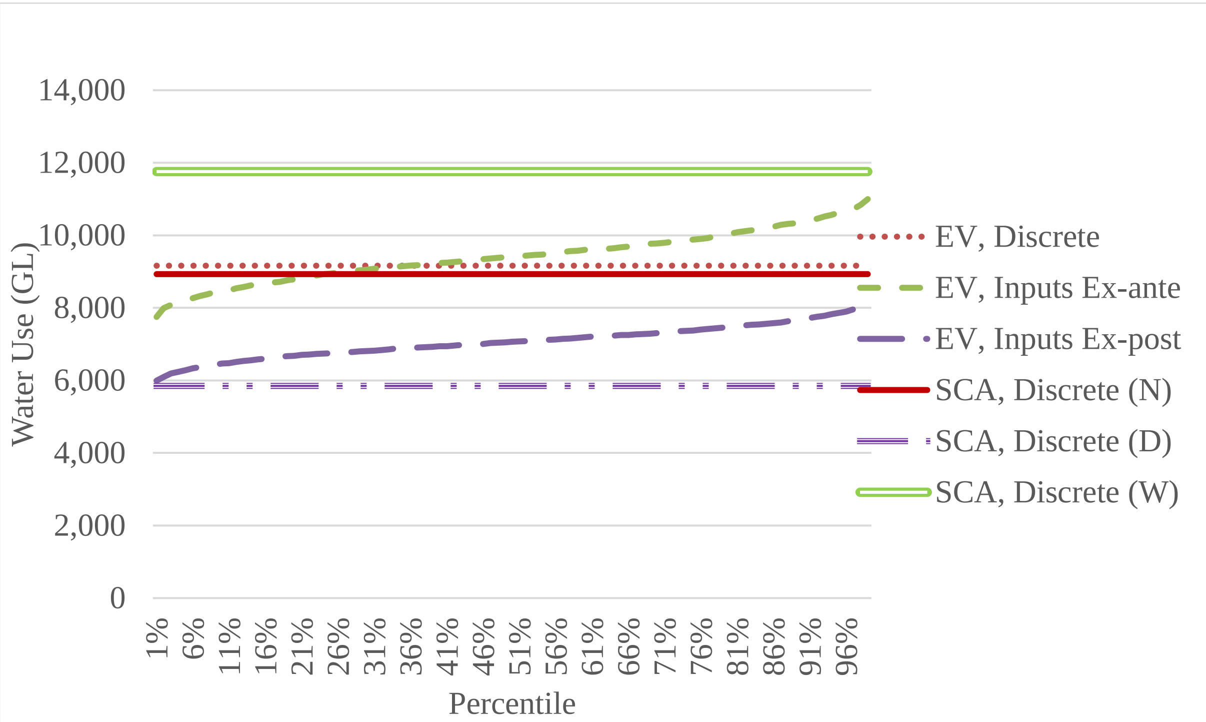

Figure 3: Water use (GL) results for the EV and SCA model comparisons

Table 5: Area irrigated by commodity (‘000 hectares)

		Citrus-H	Citrus-L	Grapes	Stone Fruit-L	Pome Fruit	Veg	Cotton Flex	Cotton Fixed	Dryland Cotton	Rice PSN	Rice Flex	Dryland Wheat	Wheat	Dairy-H	Dairy-L
Current climate	**EV, Discrete**	9	127	14	4	7	57	0	390	0	403	0	0	229	0	195
	EV, Inputs Ex-ante	9	127	14	4	7	57	0	390	0	403	0	0	229	0	195
	EV, Inputs Ex-post	9	127	14	4	7	57	0	253	0	240	0	0	301	0	144
	EV, State Ex-ante	9	127	14	4	7	57	0	390	0	403	0	0	229	0	195
	EV, State Ex-post	9	127	14	4	7	57	0	132	0	238	0	0	275	0	191
	SCA, discrete	0	136	72	4	7	0	438	0	243	392	0	0	33	24	186
	SCA, Input Ex-ante	0	136	72	4	7	0	438	0	243	392	0	0	33	24	186
	SCA Input Ex-post	0	136	72	4	7	0	304	41	240	314	39	0	67	24	139
	SCA, State Ex-ante	0	136	72	4	7	0	438	0	243	392	0	0	33	24	186
	SCA, State Ex-post	0	136	72	4	7	0	167	0	378	389	0	435	150	24	57
Climate change 550 Avg.*	**EV, Discrete**	9	127	14	4	7	57	0	335	0	456	0	0	280	0	155
	EV, Inputs Ex-ante	9	127	14	4	7	57	0	335	0	456	0	0	280	0	155
	EV, Inputs Ex-post	9	127	14	4	7	57	0	269	0	364	0	0	313	0	115
	EV, State Ex-ante	9	127	14	4	7	57	0	335	0	456	0	0	280	0	155
	EV, State Ex-post	9	127	14	4	7	57	0	104	0	158	0	0	204	0	122
	SCA, discrete	0	135	72	4	7	0	423	0	244	253	213	0	33	24	147
	SCA, Input Ex-ante	0	135	72	4	7	0	423	0	244	253	213	0	33	24	147
	SCA Input Ex-post	0	136	72	4	7	0	315	0	129	188	271	0	164	24	108
	SCA, State Ex-ante	0	135	72	4	7	0	423	0	244	253	213	0	33	24	147
	SCA, State Ex-post	0	136	72	4	7	0	117	0	309	0	0	755	116	24	234
Frequency	**SCA, discrete**	0	136	72	4	7	0	443	57	230	0	528	0	122	24	186
	SCA, Input Ex-ante	0	136	72	4	7	0	443	57	230	0	528	0	122	24	186
	SCA Input Ex-post	0	136	72	4	7	0	414	0	205	0	488	0	157	24	141
	SCA, State Ex-ante	0	136	72	4	7	0	443	57	230	0	528	0	122	24	186
	SCA, State Ex-post	0	136	72	4	7	0	124	43	422	29	61	694	154	36	300

* The 550 Average scenario denotes a 550 parts-per-million CO^2 representative conceptration pathway, or a relatively high level of future climate change impact.

These findings provide insights for our second research question about why models of decision-making must be capable of representing innovation or learning in response to future uncertain events (e.g., climate change). Logically, as inputs change and the comparative advantage of production systems alter decisions should alter as well. For example, as certainty about the significance of a drought state increases (ex-ante) the need to transition away from production systems that require water in all states (i.e., horticulture) will increase. As Adamson and Loch (2021) show, insufficient water for horticulture crops across all state outcomes may result in irreversible capital losses (i.e., root stock loss). Models that do not take irreversible outcomes into account are invalid in agricultural contexts comprising horticulture and may skew the interpretation or full set of public/private decision options. As grey-swan events impose input shocks public/private goals, institutions or behaviour decision sets must innovate and adapt in order to achieve long-run resilience; and models must appropriately capture and reflect such innovation.

Figure 4 illustrates this requirement using Coorong flow results as a test of model robustness. The SCA model provides a stable flow of water that is consistent with policy and system constraints (i.e., realistic). The EV model runs transfer an increasing volume of water to Coorong flows as water is transitioned away from production under mean (normal condition) use assumptions that, in reality, are an unsustainable outcome. Importantly, in Figure 4 we see that EV models cannot represent a change in the frequency of alternative states of nature (or bad events occurring) as the current climate and frequency runs are close to identical. In other words, it appears as noise in the outcomes.

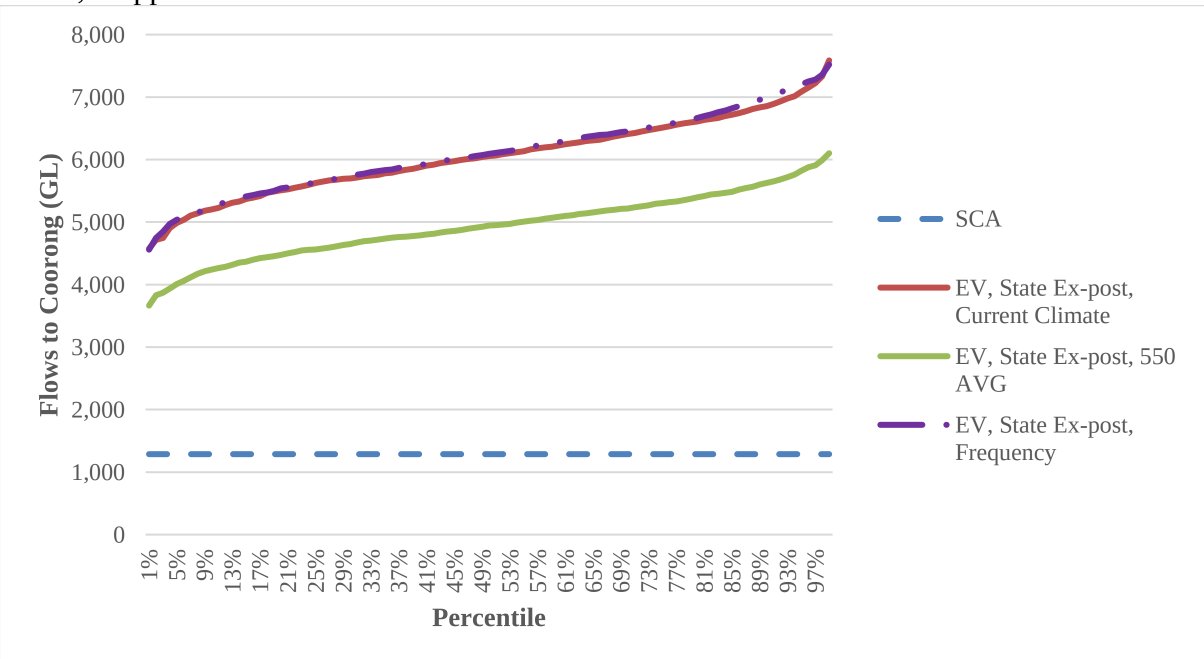

Figure 4: Coorong flow results under drought conditions by EV/SCA model (ex-post)

This finding is contrary to that of Adamson et al. (2009) where the frequency of bad states had greater impacts on reallocating investments than a proportional mean reduction in average inflows. In those earlier models production area decreased but the investment decisions remained constant. Thus, the EV model is incapable of seeing the difference in state frequencies and generates the same results for current and future climate; even once the climate has altered (i.e., EV state, ex-post frequency). If the model is representing a mean-reduction in water availability (i.e., the 550 ex-post run), then a difference in the EV results will appear. This is a problem for drought policy. If we model future climate adaptation as an increase in the frequency of bad events, an EV model will not represent change. This also links back to our Figure 1 representations of the model differences, and supports the expected model outcomes.

Finally, we compared all model representations of drought water flows to the Coorong Wetlands, as shown in Figure 5. It is clear that the discrete EV and ex-ante EV results lead to situations where no flow to the Coorong occurs, as those models fail to understand the risk to the conjunctive resource base. In essence the EV model then places bounds around the analysis and would create a black-swan outcome. Any failure to consider water demand upstream could lead to the collapse of the Coorong; a key management flow target with irreversible loss implications. In reality, basin managers might allocate less water to irrigation which could in turn lead to a black-swan event creating irreversible private capital losses.

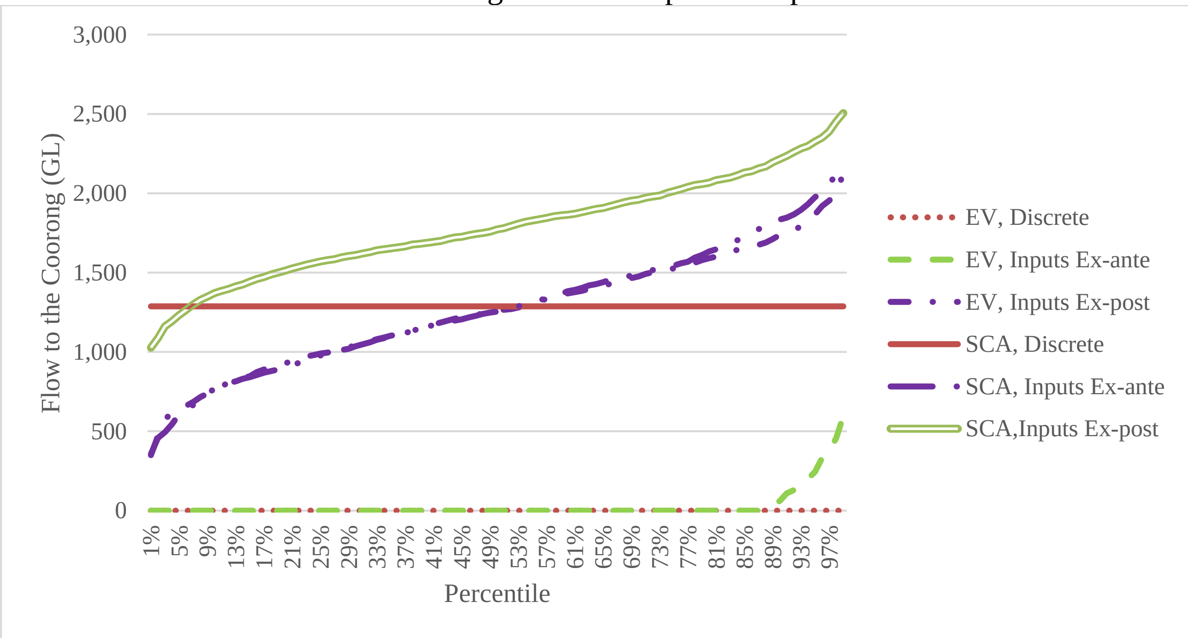

Figure 5: Comparison of EV and SCA results on environmental drought flows to Coorong

By acknowledging that alternative states of nature exist, the SCA model constraints are achieved in the discrete and the ex-post input evaluation. The discrete model (i.e., SCA, Inputs ex-ante results) would be expected to fail in 50% of years and not 5% of years. This suggests that production systems must transition away from always requiring water in each state of nature (i.e., reduction in perennial crops), as detailed by Loch et al. (2020a). Critically the difference between the discrete and ex-post SCA model occurs as the ex-post model actively reallocates resources towards opportunistically irrigating with substantial shifts towards “dryland cotton” and “dryland wheat” as illustrated in Table 2.

5 Concluding Comments

We have shown that climate change problems are well characterised as grey-swan events with some uncertainty over the complete set of outcomes and the full set of appropriate decisions in response. Our model comparisons suggest that SCA modelling of land and water (re)allocation more robustly represents and evaluates public policy and private investment decisions than discrete/stochastic EV models—the more commonly adopted approach for economic modelling of uncertainty. This is based on our findings that applications of state contingent analysis using discrete data allows for an improved representation and understanding of grey-swan events (i.e., adverse and positive states of nature) together with model recognition that decision-makers innovate/learn—which is consistent with Marshallian views. Stochastic SCA models also facilitate assessment of policy or investment goals so that they can be tested for fragility, unrealistic conclusions, and/or irreversible loss outcomes.

Applications of the stochastic SCA model description and its bounds allows for an exploration of the level of risk associated with state-described input use, thus overcoming issues associated with fat-tailed event distributions and non-linear climate change events (Rosser, 2011). This is not possible using EV model approaches. By comparing ex-ante and ex-post results we are able to identify the value of being prepared for future adverse events, and selecting adaptation/investment choices in response to preserve capital (i.e., natural, economic, social, cultural etc.) Further, discrete SCA models provide better outcomes than stochastic EV models due to a capacity to represent management responses to thin- or fat-tailed outcomes. This conclusion is supported by the SCA model's capacity to clearly separate the signal from the decision response, and thus inform how distribution tails contribute to (re)allocation choices. This separation helps identify the importance of tail events and helps identify where existing knowledge, technology and known management responses fail. We could further apply such analysis and undertake sensitivity testing to determine when systems may fail and/or enter the active set of possibilities, and this would provide lessons for on-going management adaptation at both private and public levels. Future research paths and questions will be informed by an increased awareness of the full set of contingencies that may/may not be applicable under future climate change. Such research will enhance future innovation and adaptation. However, in practice, the success of those choices will still be constrained by decision-maker bounds to awareness, and any black-swan events that may arise.

Acknowledgments, Samples, and Data

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