Conlusion on the fully forgetting model with different porbability of buying and selling

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January 14, 2014

Abstract

One considere agents which fully forget. First we derive the low temperature limit and then observe the finite temperature limit.

1 Quantities of the model

In the $N \to \infty$ limit, the trading prices are fixed to $tp_i = \theta_i \langle b \rangle + (1-\theta_i)\langle a \rangle$ At a market labeled i the probability of (bidding/asking) a valid offer will be

$$A_i = \int_{-\infty}^{tp^{(i)}} \frac{\exp(\frac{-(x-\langle a \rangle)^2}{2})}{\sqrt{2\pi}dx} \tag{1}$$

$$B_i = \int_{-\infty}^{tp^{(i)}} \frac{\exp(\frac{-(x-\langle b \rangle)^2}{2})}{\sqrt{2\pi}dx} \tag{2}$$

One can also get the average transition probability of an agent who succesfully (asked/bid) at market i

$$S_i^A = \int_{-\infty}^{tp_i} \frac{\exp(\frac{-(x-\langle a \rangle)^2}{2})}{\sqrt{2\pi}A_i} \exp(tp^{(i)} - x)/(3 + \exp(tp^{(i)} - x))dx \tag{3}$$

$$S_i^B = \int_{tp_i}^{+\infty} \frac{\exp(\frac{-(x-\langle b \rangle)^2}{2})}{\sqrt{2\pi}A_i} \exp(x - tp^{(i)})/(3 + \exp(x - tp^{(i)}))dx \tag{4}$$

2 Update equation for the trading probabilities

let's for example look at the probability of asking at market 1 at time $t+1$. Three events can lead you to ask at market 1.

- You succesfully selled at market 1, hence you hade a probability S_i^A do the same thing at $t+1$
- You succesfully traded at one of the market, hence you hade a probability $\frac{1}{3}(1 - S_{choice})$ to sale at market 1
- You didn't succeed to trade at any of the markets, hence you ask at market 1 with probability $\frac{1}{4}$

Hence by using the notation ... one derive the following equation for the probability of selling

$$P_1^A(t+1) = P_1^F(t)F_1S_1^A + \tag{5}$$

$$1/4 - 1/4P_1^F(t)(S_1^B + S_1^S) - 1/4P_j^F(t)(S_2^B + S_2^S) + \tag{6}$$

$$\frac{1}{3}(P_1^F F_1(1 - S_1^A)) + \frac{1}{3}(P_2^F F_2(2 - S_2^A - S_2^B)) \tag{7}$$

This equation can be simplified and generalized to any other probability :

$$P_i^U(t+1) = P_i^F(t)F_iS_i^U + 1/4 - 1/4P_i^F(t)(S_i^B + S_i^A) - 1/4P_j^F(t)(S_j^B + S_j^S) + \frac{1}{3}(P_i^F(t)F_i(1 - S_i^{-U})) + \frac{1}{3}(P_j^F(t)F_j(2 - S_j^A - S_j^B)) \tag{8}$$

2.1 Trading probabilities difference

Using the notations $\Delta S_i = S_i^B - S_i^A$ and $\Delta P_i(t) = P_i^B(t) - P_i^A(t)$ one get the following identity for the probability difference :

$$\Delta P_i(t+1) = \frac{1}{3}(4F_i P_i^F(t)\Delta S_i - F_{-i}P_{-i}^F(t)\Delta S_{-i}) \quad (9)$$

In the low temperature limit, equatil (9) become $\Delta P_i(t+1) = 0$

2.2 $\theta \rightarrow 1-\theta$ symmetry

when the $\theta_i \rightarrow 1-\theta_i$ hence the only modification of which arise in the equation are $B_i \leftrightarrow A_i$ and $S_i^A \leftrightarrow S_i^b$. In the low temperature limit the behaviour of the equation is the same because the population of buyer in a market is the same as the population of sellers. In the non zero temperature phase I cannot find some answer yet. As a first intuition, assuming that the changing $\theta \rightarrow 1-\theta$ would lead to a changing of F then the equation (8) would remain the same which mean that P_i^U would remain unchanged.

3 Comparaison between numerics and analytics

3.1 In the case of a variation of θ at $T = 0.11$

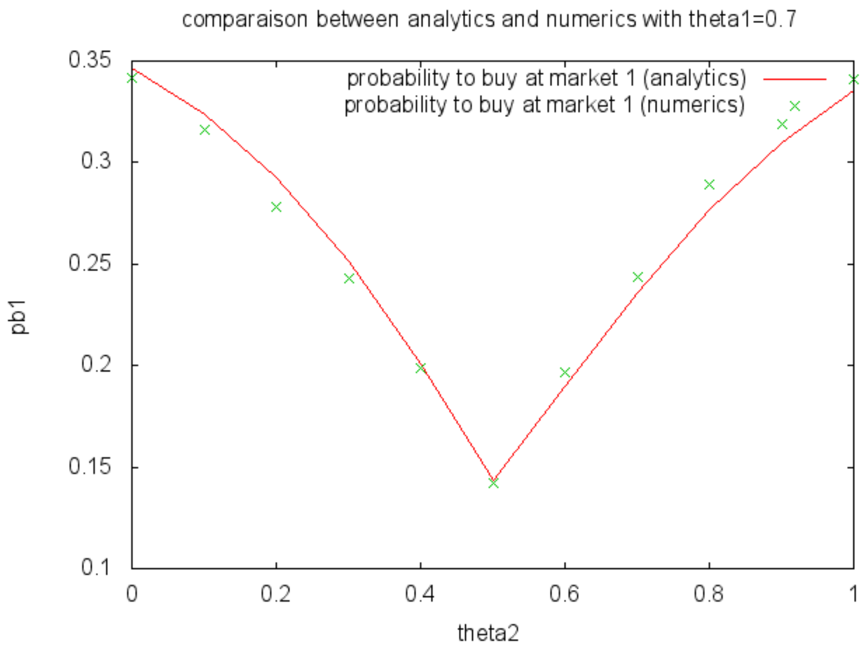

Figure 1: Here T remain fixed at 0.11 and θ_1 at 0.7

3.2 In the case of symmetric markets and varying temperature

In the case of symmetric market, the analytics fit well the numerics. Bothe of the limits when $T \rightarrow 0$ and $T \rightarrow \infty$ are coherent.

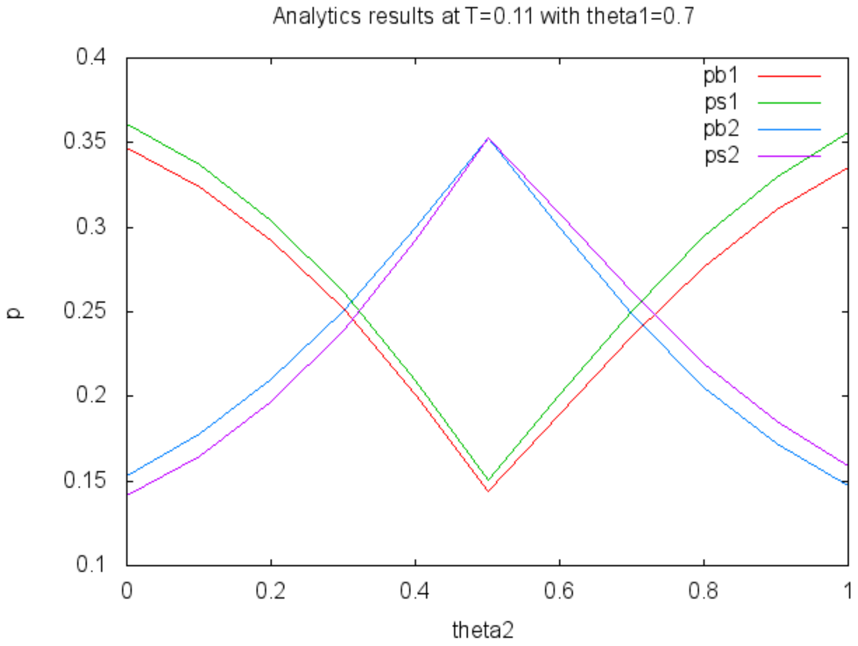

Figure 2: Here T remain fixed at 0.11 and θ_1 at 0.7 (analytic results)

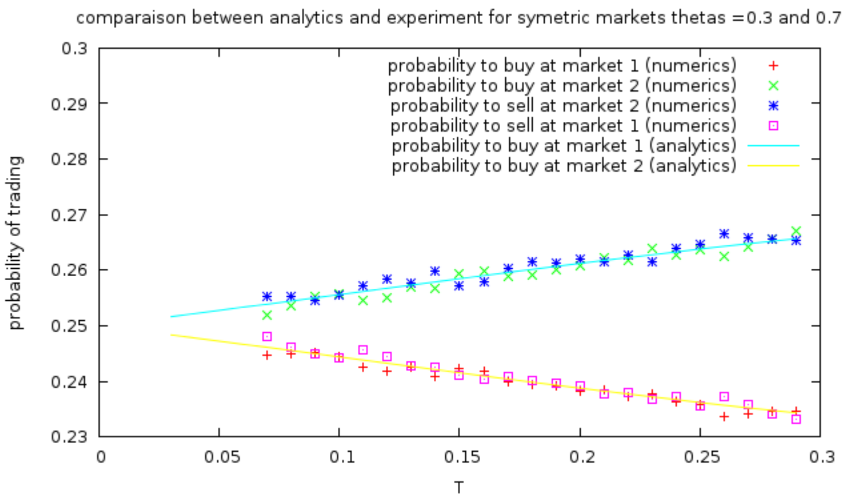

Figure 3: Fixed thetas and symmetric markets and T varying

3.3 Not symmetric markets

4 favorability of the markets

The equation which rules the steady state depend wether $\frac{P_i^B B_i[\theta_i]}{P_i^A A_i[\theta_i]} \geq 1$. The singularity in the equation correspond to the value of theta for which $\frac{P_i^B B_i[\theta_i]}{P_i^A A_i[\theta_i]} = 1$. A densityplot done with mathematica let us presume that the threshold of favorability of the market is 0.5 but I couldn't get analytical results

4.1 Low temperature limit

In the case of low temperature, the probability of buying and asking at a same market are equals. Hence the ratio $\frac{P_i^B B_i[\theta_i]}{P_i^A A_i[\theta_i]}$ become $\frac{B_i}{A_i}$ which is plotted in figure 6 One see that the for every θ lower than 0.5 the ration will be greater than 1 and for all theta greater than 0.5 the ratio will be lower than 1. In the limit of low temperature the guess done according to figure 4 is true.

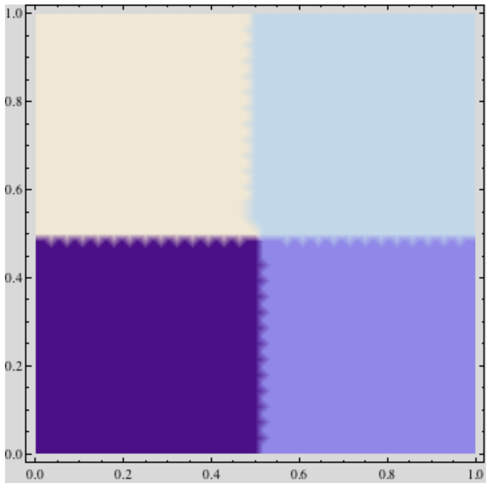

Figure 4: Plot of the favorability of the market varying with theta

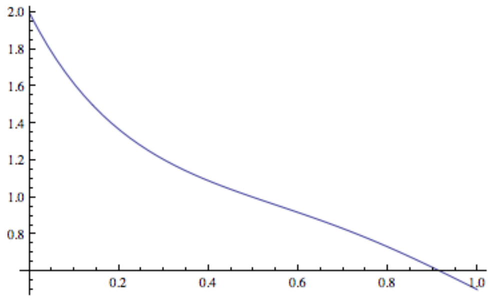

Figure 5: ratio $\frac{B_i}{A_i}$ as a function of θ

5 Analytic solution at $T = 0$

At zero temperature, all the scores are equal to 1. In other words, if you had a succesfull strategy, then you will use the same next turn with probability 1. Hence, equation (5) is highly simplified into

$$P_i^U(t+1) = 1/4 + \frac{1}{2}(P_i^F F_i - P_{-i}^F F_{-i}) \tag{10}$$

Which enable us to get an analytical solution for the trading probabilities. The left hand side of equation (10) doesn't depend of U which mean that $P_i^A = P_i^B$. Hence, if we get the on of the four probabilities we will be able to deduce all the other one.

favorability of the market The value of F_i only depend of the ratio B_i/A_i because the probabilities to ask and bid at a same market ar equal. According to the graph 6 the ratio is higher than 1 is $\theta \leq 0.5$ and vice versa. Hence, an analytical expression for the probability of trading at market 1 is

$$P_1 = \frac{-1 + F_1}{-2 + F_1 + F_2} \tag{11}$$

The probabilities also verify a balance relation :

$$P_1(1 - F_2) = P_2(1 - F_1) \tag{12}$$

When $\theta 1 < 0.5$ then, the chances trading succesfully in market 1 are fixed by A_i, hence the only wether we prefer market 1 or market 2 will depend if A_1 id greater or lower than A_2. One also know that $A(\theta)$ is increasing which explain why p1 is also increasing when $\theta_1 < 0.5$. At $\theta_1 = 0.5$ the chances of buying at market 1 are non longer ruled by A_1 but by B_1 which is a decreasing function of theta. It explain the brutal change of slope at $\theta = 0.5$ which correspond of the change of slope when F_1 goes from A_1 to B_1.

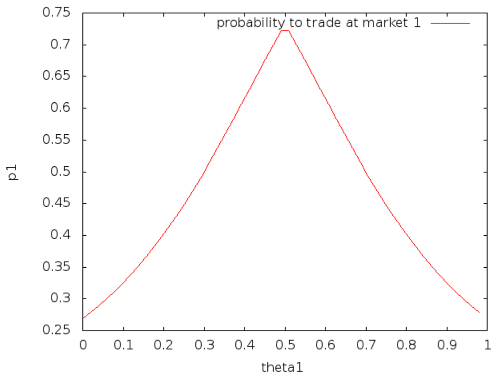

Figure 6: Plot of the probability to buy at market as a function of θ_1 with θ_2 fixed to 0.7

5.1 What happen in the normal temperature case

In the normal temperature case. The same events happens but I didn't succeed to get the equation which describe them due to the fact that the 4 probabilities of trading are different. The difference of probabilities to buy and sell in a market i also depend on the probabilities to trade at market -i. that coupling which do not appear in the equations make the study of the favorability of the market much harder.