Supersaturation Fluctuations from Scalar Transport in Moist Rayleigh-Bénard Convection: One-Dimensional-Turbulence Simulation

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Introduction

A careful characterization of moisture fluxes and supersaturation statistics in atmospheric convection is significant for cloud microphysical processes and dynamics. The small-scale supersaturation fluctuation could be an important mechanism for droplet size-distribution broadening [1]. In an idealized sense, atmospheric boundary-layer convection is equivalent to Rayleigh-Bénard convection [2, 3]. Here, continuous plume eruption from boundaries transports heat and moisture and produces the mixed layer [4]. The presence of multiple driving scalars (water vapor and temperature), with slightly different diffusivities, adds to the degree of complexity. Additionally, in the supersaturation statistics, a nonlinear coupling between water-vapor and temperature fields makes the problem intriguing. Important parameters for this problem are [5]:

$$Ra_{moist} \approx \frac{g\Delta T H^3}{T_0 \nu D_t} + \frac{g\epsilon\Delta q_v H^3}{\nu D_t}, \quad Pr \equiv \frac{\nu}{D_t}, \quad \text{and } Sc \equiv \frac{\nu}{D_v}.$$

Numerical Approach

We approach this problem using an idealized, one-dimensional-turbulence (ODT) model that faithfully represents the processes of advection and diffusion in turbulent convection, including fully-resolved boundary layers [6]. The range of Rayleigh number $2.05 \times 10^8 \leq Ra_{moist} \leq 2.75 \times 10^9$ covered in simulations is relevant to the Π-chamber. In order to understand the relative roles of the two diffusivities, we use the following four combinations.

- Actual D_v and D_t: actual diffusivities for the water vapor and thermal fields at 283 K ($D_v/D_t = 1.16$).
- $D_v = D_t$: D_v = the actual thermal diffusivity ($D_v/D_t = 1$).
- $4 \times D_v$: D_v = four times the actual value, and D_t = same as the actual value ($D_v/D_t = 4.63$).
- $4 \times D_t$: D_v = same as the actual value, and D_t = four times the actual value ($D_v/D_t = 0.29$).

Boundary Fluxes

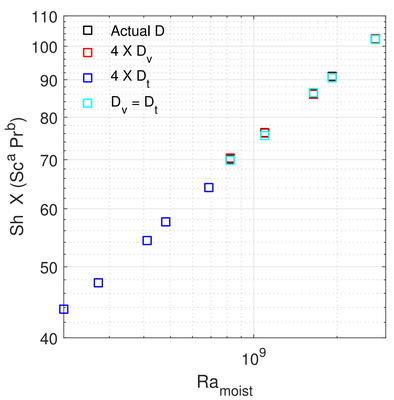

Figure 1: Variation of the scalar fluxes of water vapor (Sh) with moist Rayleigh number (Ra_{moist}). Fitting of the scaled Sh data produce Ra_{moist} exponents around 0.328 ± 0.006.

Supersaturation Fluctuations

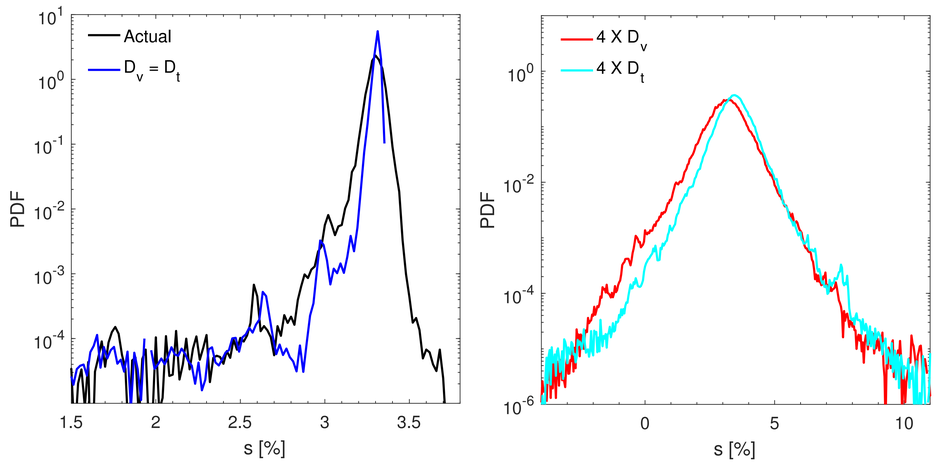

Figure 2: Sample PDFs of supersaturation near the domain center for the different diffusivity cases (8 K applied temperature difference).

Scaling Analysis

Moisture flux:

$$Sh \propto Sc^{1/2} Ra_{moist}^{1/3} \; | Pr \sim 1 \qquad Sh \propto Sc^{1/2} Pr^{-1/3} Ra_{moist}^{1/3} \; | Pr \ll 1$$

Sh : Sherwood Number; Pr : Prandtl Number; Sc : Schmidt Number

Supersaturation Fluctuations:

$$\sigma_s^2 \approx S^2 \left[\left(\frac{\sigma_{e_v}}{\bar{e}_v} \right)^2 + \zeta^2 \left(\frac{\sigma_T}{\bar{T}} \right)^2 - 2\zeta \frac{\overline{T' e_v'}}{\bar{T}\bar{e}_v} \right]$$

$$\left(\frac{\sigma_s}{S} \right)^2 \sim \xi^2 \left[1 - \frac{1}{S} \left(\frac{Pr}{Sc} \right)^{1/2} \right]^2 Pr^{-2} (1 + \sqrt{Pr})^{-1} \; Ra_{moist}^{5/3} \left(\frac{z}{H} \right)^{-1}$$

σ_T: temperature STD; σ_{qv} : water-vapor STD; z: vertical position; H : domain height; $\xi \propto H^{-3}$
ΔT: applied temperature difference; Δq_v: applied water vapor mixing-ratio difference
ν: kinematic viscosity; D_t: thermal diffusivity; D_v: water vapor diffusivity

Supersaturation Fluctuations in Bulk: Contributions from Both Scalars

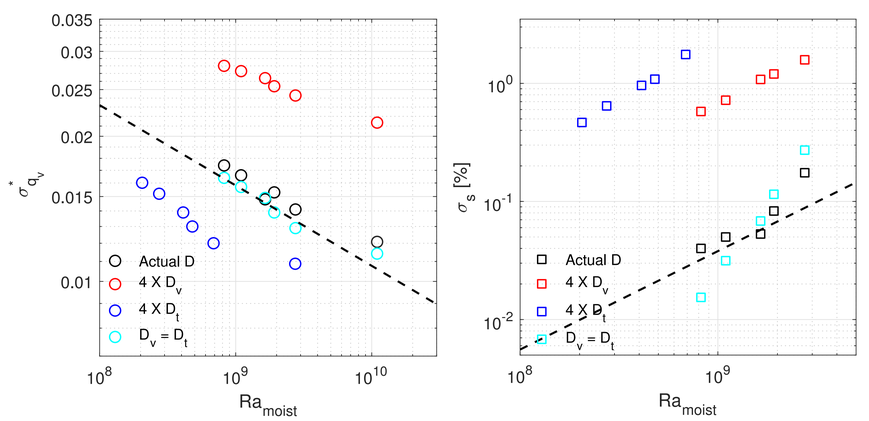

Figure 3: STDs of normalized water vapor mixing-ratio (**left**) and supersaturation fluctuation (**right**) at the domain center versus Ra_{moist}, and their comparison with scaling relations (dash-line) $Ra_{moist}^{-1/6}$ and $Ra_{moist}^{5/6}$.

Mixing Diagram

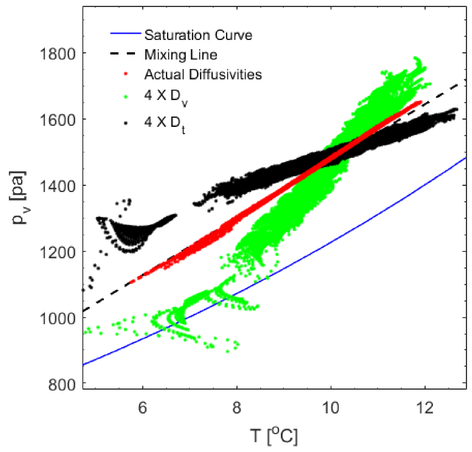

Figure 4: Mixing diagram for different diffusivity cases at 20K applied temperature difference.

Conclusion

- Scaling relations for heat and moisture fluxes are obtained as a function of Ra_{moist}, Pr, and Sc.
- In the bulk fluid, σ_T^* and σ_{qv}^* both follow a $Ra_{moist}^{-1/6}$ scaling relation. Moreover, the magnitude of scalar fluctuations increases with an increase in the respective scalar diffusivity.
- PDFs of supersaturation become broader with an increase in absolute value of diffusivity difference. Also, PDFs are slightly negatively skewed for cases with low diffusivity difference, unlike the T and q_v PDFs.
- Both, scaling and numerical output suggests: $\sigma_s^2 \propto Ra_{moist}^{5/3}/H^6$.
- The analysis of numerical output shows similar order contributions to the supersaturation variance from both scalar variance and covariance.
- Distribution of points in a pressure-temperature mixing diagram deviate from the classical mixing line for isobaric mixing, when $D_v \neq D_t$.

References

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Acknowledgements

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