Complex permittivity formulation for induced polarization effect in transient electromagnetics

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Key Points:

- Dielectric function is a dispersive total permittivity related to susceptibility and implicitly bears the medium's conductivity information
- Permittivity at low frequency with induced polarization effect may provide different information beyond that given by the conductivity only

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Abstract

We have studied the relevance of permittivity as an indicator of the IP effect in transient electromagnetics. The approach is based on a mathematical formulation of a dielectric model, where we assume that the free electrons of the induced current are weakly bound electrons included in a complex frequency-dependent permittivity. We combine this dielectric function to Maxwell's equations in order to obtain a modified Helmholtz equation whose new term contains the polarization parameters of the medium. The resultant convolution in Ohm's Law was solved by the concept of memory variables transforming the convolution into a set of time-dependent ODE, where the storage of the full-time series of the electric field is no longer needed. The final model allows any Cole-Cole Model (CCM) in the electromagnetic fields through the Debye Function Expansion (DFE) and makes it possible to incorporate additional information into the canonical property of conductivity through the dispersive permittivity.

Plain Language Summary

In geophysical exploration, the observation of a natural phenomenon is based on measuring physical properties such as electrical conductivity. This property brings essential information about the solid and fluid phases of earth and materials. Its measure is carried out by low-frequency electromagnetic devices such as coils and antennas widely used in environmental studies worldwide. Those systems are not only sensitive to conductivity but other electromagnetic properties such as electrical permittivity. Is in this area where our main contribution lie, we propose a novel mathematical model to quantify the contribution of both properties, and whit it, identify the natural phenomenon called induced polarization, which is an electrochemical process due to electrical conduction and the chemical reaction that takes place in some rocks and minerals. The combined contribution of these properties in the transient electromagnetic methods has been barely studied before. The approach introduced here provides novel information for understanding more complex phenomena on the earth.

1 Introduction

In recent years, the Induced Polarization (IP) technique, also known as complex resistivity, has been widely used in mineral exploration (J. C. Wynn, 1988; Flores & Peralta-Ortega, 2009; Kwan et al., 2015), reservoir studies (Davydycheva et al., 2006; Burtman & Zhdanov, 2015), environmental applications (Slater & Lesmes, 2002; Kemna et al., 2004) and to detect alteration zones in geothermal studies (Lévy et al., 2019). Remarkable, all these scenarios comprise electrolytes immersed in electronic conducting media.

Assimilation of IP phenomenon in electromagnetic modeling is most commonly implemented in frequency-domain using galvanic sources and applying a well-known Cole-Cole relaxation Model (CCM) (Cole & Cole, 1941), which was originally introduced in geophysics by Pelton et al. (1978) as a frequency-dependent complex impedance:

$$Z(\omega) = R_0 \left[1 - m \left(1 - \frac{1}{1 + [i\omega\tau]^c} \right) \right], \tag{1}$$

which depends on four parameters: the chargeability (m), time relaxation (τ), the frequency exponent (c) and the resistivity in direct current (R_0). Despite the fact that there are more complete relaxation models, CCM explains well enough the dispersive behavior of the electric resistivity with a relatively simple mathematical expression.

Regarding inductive sources, and independently of the acquisition domain, most of the electromagnetic data are transformed into the frequency domain to explain the IP effect mainly because the relaxation model in equation (1), is posed in that domain, despite the computational burden brought by the Fourier transform (Marchant et al., 2018). By contrast, there are very few developments of IP modeling directly in the time-domain systems due to the mathematical complexity that arises from the temporal variation in conductivity, which results in a convolution and fractional derivatives in Ohm's Law. To overcome this difficulty, there have been some developments not only in geophysics, but also in the physics of dielectrics and antennas.

In geophysics, Smith et al. (1988) decomposed the total electromagnetic response into fundamental and polarization responses to show that the polarization response can be written approximately as a convolution. Differently, Kelley and Luebbers (1996) proposed a piecewise linear recursive convolution based on the trapezoidal rule. Their efficient algorithm is implemented in the high frequency area for the Debye dispersion model, but it could be implemented in geophysical applications. Siushansian and Lovetri (1997) posed a quasi-trapezoidal-based algorithm and a Prony's method to evaluate the convolution integral in a time dependent susceptibility term. More recently, Zaslavsky and Druskin (2010) suggested a new algorithm for the solution of time domain Maxwell equations in dispersive media based on the generalization of the rational Krylov subspace approach for the solution of non-dispersive Maxwell's systems. Years letter, Marchant et al. (2014) rewrote Ohm's law in terms of Cole-Cole parameters and a expanded Debye model using Padè approximation avoiding the computation of the convolution integral and fractional derivatives in an implicit FDTD scheme. Commer et al. (2017) implemented the Debye expansion to fit a Cole-Cole Model and used memory variables to avoid integral convolution in an explicit FDTD Dufort-Frankel scheme of a system of coupled Maxwell's equations. In the same year Cai et al. (2017) used a truncated Padè approximation with adaptive selection of the central frequency to expand the Cole-Cole model in frequency domain and solve the vector Helmholtz equation for the total electric field. Recently, Ji et al. (2018) used a frequency-domain error minimization and least-squares optimization to approximate the fractional-order system and a recursive method to compute the convolution using an implicit FDTD scheme for a system of coupled Maxwell's equations. All of these works are based conventionally on the use of conductivity.

In this work, we propose a theoretical EM model which incorporates IP effect directly in the time-domain electromagnetic signal by a modified vector Helmholtz equation. The model is based on the simplest theory which explains the IP phenomenon, as, the charging and discharging processes of a capacitor. We have introduced a dielectric function, which is essentially a frequency-dependent total permittivity, in terms of a Cole-Cole model approximation using a Debye function. We introduced this approximation into the Maxwell's Equations and derive a new electromagnetic mode, which comprises not only diffusive but also reactive processes.

Finally, we conclude that with this model, it is possible to quantify polarization mechanisms due to current conduction governed by the conductivity and displacement currents characterized by the permittivity as it occurs in capacitors.

2 Derivation of the dielectric model

In addition to the widely accepted polarization mechanisms (membrane and grain polarization), Fuller and Ward (1970) define an interfacial or space charge polarization, that can provide important information about oil and gas reservoirs. Also, the existence of the orientational or dipolar polarization is now accepted and associated to a permanent dipole moments in the material such as water molecules. Both mechanisms give strong responses in permittivity and have important applications. For example, in petrophysics. Table 1 gives a short summary of the different and common types of polarization mechanisms.

Table 1. Polarization mechanisms in materials

Polarization	Based on σ	Based on ε
Electronic	✓	
Membrane	✓	
Orientational		✓
Interfacial		✓

In general, the IP effect thrives on the fact that the Earth behaves not only as a conductor but also as a poor dielectric; in which a significant amount of energy of the electromagnetic field is lost through different layers of the subsurface that in low-frequency process, the displacement current is small enough to be neglected. However, our hypothesis is that this small contribution of energy in the presence of the IP effect may have relevant information since it can contribute to identifying grain/membrane (usually determined through conductivity) from interfacial/orientational polarization where permittivity becomes more relevant. To explain this, we propose a function that relates both properties and, when it is introduced into Maxwell's equations, can add the IP effect in the electromagnetic signal. Hence, let us start with the dispersive form of Maxwell's equations in the frequency domain with a source term $\mathbf{J}_S$:

$$\begin{aligned} \nabla \times \left[\frac{1}{\mu}\mathbf{B}(\mathbf{r},\omega) - \mathbf{M}(\mathbf{r},\omega)\right] &= \mathbf{J}(\mathbf{r},\omega) + i\omega\mathbf{D}(\mathbf{r},\omega) + \mathbf{J}_S(\mathbf{r},\omega), & (2) \\ \nabla \times \mathbf{E}(\mathbf{r},\omega) &= -i\omega\mathbf{B}(\mathbf{r},\omega). & (3) \end{aligned}$$

where $\mathbf{J}$ and $\mathbf{D}$, are the conduction and displacement current density respectively. In the general case, these currents in inhomogeneous media can be related to the electric field through (Ward & Hohmann, 1988):

$$\mathbf{J}(\mathbf{r},\omega) = \left[\sigma'(\mathbf{r},\omega) + i\sigma''(\mathbf{r},\omega)\right]\mathbf{E}(\mathbf{r},\omega) = \sigma(\mathbf{r},\omega)\mathbf{E}. \tag{4}$$

and

$$\mathbf{D}(\mathbf{r},\omega) = \left[\varepsilon'(\mathbf{r},\omega) - i\varepsilon''(\mathbf{r},\omega)\right]\mathbf{E}(\mathbf{r},\omega) = \varepsilon(\mathbf{r},\omega)\mathbf{E}. \tag{5}$$

where the conductivity (σ) and permittivity (ε) in equations (4) and (5) are the frequency-dependent conductivity and permittivity respectively. Remarkably, σ can be defined, for instance, by the Cole-Cole model approximated through a Debye Function Expansion ($\tilde{\sigma}_D$) (Kelley et al., 2007):

$$\sigma(\mathbf{r},\omega) = \sigma_{cc}(\mathbf{r},\omega) \approx \tilde{\sigma}_D(\mathbf{r},\omega) = \sigma_\infty(\mathbf{r}) + (\sigma_0(\mathbf{r}) - \sigma_\infty(\mathbf{r}))\sum_{p=1}^{N_D}\frac{\gamma_p(\mathbf{r})}{1 + i\omega\tau_p(\mathbf{r})}. \tag{6}$$

where the parameter σ_0, σ_∞, τ_p and γ_p are the polarization parameter of the medium which are spatial dependent and are respectively defined as follows: low-frequency conductivity (DC), high-frequency conductivity, weighted relaxation time and dimensionless weighted term equivalent to c exponent in the original CCM (Tarasov & Titov, 2007). From these parameters, the chargeability is defined as m $= \frac{\sigma_0 - \sigma_\infty}{\sigma_\infty}$. The p$th$ weighted parameter goes from 1 to total number of Debye function (N_D) in the sum. From now on, we write $\eta = (\sigma_0 - \sigma_\infty)$ in equation (6) for simplicity. We have chosen this Debye model due to its corresponding exponential function in time domain, given by:

$$\sigma_D(\mathbf{r}, t) = \sigma_\infty(\mathbf{r})\delta(t) + (\sigma_0(\mathbf{r}) - \sigma_\infty(\mathbf{r})) \sum_{p=1}^{N_D} \frac{\gamma_p(\mathbf{r})}{\tau_p(\mathbf{r})} \; \mathrm{e}^{-\left(\frac{t}{\tau_p(\mathbf{r})}\right)} H(t) \tag{7}$$

where $\delta(t)$ is the Dirac delta function, and $H(t)$ the Heaviside function. Equation (7) is the key term in most of the time domain electromagnetic dispersive formulations.

Regarding the frequency-dependent total permittivity, (ε) can be posed in terms of the electric susceptibility (χ) of the material, which in a dispersive medium (Orfanidis, 2016), is defined as:

$$\varepsilon(\omega) = \varepsilon_0 \kappa(\mathbf{r}, \omega) = \varepsilon_0(1 + \chi(\mathbf{r}, \omega)), \tag{8}$$

where κ is the relative permittivity. Therefore, we can write (5) as:

$$D(\omega) = \varepsilon_0 \kappa(\mathbf{r}, \omega)\mathbf{E} = \varepsilon_0 \mathbf{E}(\omega) + \mathbf{P}(\omega), \tag{9}$$

where $\mathbf{P}$ represents the dielectric polarization of the material. Therefore, equation (9) shows that the electric susceptibility, electric field and polarization are closely related.

The frequency-dependent total permittivity (also known as dielectric function) can be incorporated into Maxwell's equations to obtain a kind of vector Helmholtz equation which, provides better coupling of electromagnetic fields in the air (Newman & Alumbaugh, 1995).

In order to include the dielectric function, we take the curl in both sides of (3)

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, \omega) = -i\omega \nabla \times \mathbf{B}(\mathbf{r}, \omega), \tag{10}$$

and substitute (2), (4) and (5) in (3), the equation (10) becomes:

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, \omega) = -i\omega\mu \Big(\sigma(\mathbf{r}, \omega) + i\omega\varepsilon_0\kappa(\mathbf{r}, \omega)\Big)\mathbf{E}(\mathbf{r}, \omega) - i\omega\mu \left[\mathbf{J}_S(\mathbf{r}, \omega) + \mathbf{J}_M(\mathbf{r}, \omega)\right], \tag{11}$$

where we introduce $\varepsilon_0\kappa$ and the magnetic current $\mathbf{J}_M = \nabla \times \mathbf{M}$. We then rewrite the second term on the right-hand side of equation (11) as:

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, \omega) = -i\omega\mu\varepsilon_0 \left(\frac{\sigma(\mathbf{r}, \omega)}{i\omega\varepsilon_0} + \kappa(\mathbf{r}, \omega)\right) i\omega \mathbf{E}(\mathbf{r}, \omega) - i\omega\mu \mathbf{J}(\mathbf{r}, \omega), \tag{12}$$

being $\mathbf{J}_{ms} = \mathbf{J}_m + \mathbf{J}_s$ a magnetic source. From equation (12), we then define the dielectric function, similarly to Ward and Hohmann (1988), Kuzmany (1998), Lesmes and Friedman (2005), Kaiser (2006), Maier (2007), Orfanidis (2016), as:

$$\hat{\varepsilon}(\mathbf{r}, \omega) = \varepsilon_0 \left(\kappa(\mathbf{r}, \omega) + \frac{\sigma(\mathbf{r}, \omega)}{i\omega\varepsilon_0}\right). \tag{13}$$

where $\sigma(\mathbf{r}, \omega)$ is the spatial and frequency-dependent electrical conductivity defined by (6). The separation of bound and free charges can bee explain through equation (13). At low frequencies, ε usually describes the response of bound charges to a driving field, leading to an electric polarization and a consequent generation of displacement currents, as some authors has been studied (Lesmes & Frye, 2001), (Davydycheva et al., 2006), (J. Wynn & Fleming, 2012). On the other hand, σ describes the contribution of free charges to the current flow as typically done in DC current methods.

Substituting equation (13) in equation (12) and simplifying we have:

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r},\omega) = -i\omega\mu\hat{\varepsilon}(\mathbf{r},\omega)i\omega\mathbf{E}(\mathbf{r},\omega) - i\omega\mu\mathbf{J}(\mathbf{r},\omega). \tag{14}$$

With the inverse Fourier transform, we define the compact form of the vector Helmholtz equation for the electric field with IP effect in a heterogeneous media:

$$\nabla \times \nabla \times \mathbf{e}(\mathbf{r},t) + \mu\left(\frac{\partial\hat{\varepsilon}(\mathbf{r},t)}{\partial t} * \frac{\partial\mathbf{e}(\mathbf{r},t)}{\partial t}\right) = -\mu\frac{\partial\mathbf{j}(\mathbf{r},t)}{\partial t}, \tag{15}$$

where the dielectric function in time domain is defined in terms of Debye functions as follows:

$$\begin{aligned}\hat{\varepsilon}(\mathbf{r},t) &= \varepsilon_0\mathscr{F}^{-1}\left[\kappa(\mathbf{r},\omega) + \frac{\sigma(\mathbf{r},\omega)}{i\omega\varepsilon_0}\right] \\ &= \varepsilon_0\left(\kappa(\mathbf{r},t)\delta(t) + \left(\sigma_\infty + \eta\sum_{p=1}^{D_f}\gamma_p\left(1 - \mathrm{e}^{-\frac{t}{\tau_p}}\right)\right)\frac{H(t)}{\varepsilon_0}\right)\end{aligned} \tag{16}$$

whose response requires the computation of the weighted-parameter γ_p and τ_p for any given relaxation model (e.g. CCM). Using this expression we can compute the time derivative of (16), as:

$$\frac{\partial\hat{\varepsilon}(\mathbf{r},t)}{\partial t} = \varepsilon(\mathbf{r},t)\dot{\delta}(t) + \sigma_\infty\delta(t) + \eta\sum_{p=1}^{D_f}\gamma_p\left[\delta(t)\left(1 - \mathrm{e}^{-\left(\frac{t}{\tau_p}\right)}\right) - \frac{H(t)}{\tau_p}\ \mathrm{e}^{-\left(\frac{t}{\tau_p}\right)}\right]. \tag{17}$$

Equation (17) is the polarization rate whose gives account of how the polarization effect changes in a lapse of time. Notably, this expression is fully analogue to the expression for the strain rate in continuum mechanics.

3 IP effect contribution through memory variables

Equation (15) is an integro-differential equation with no trivial solution, for which, we have used the concept of memory variables as widely employed in seismic attenuation problems to transform the convolution into a set of first-order ordinary differential equations in time. Hence using the definition of convolution between two causal signals $(f * g)$, where one of them is reversed and shifted a value ψ, resulting in the following convolution integral:

$$\frac{\partial\hat{\varepsilon}(\mathbf{r},t)}{\partial t} * \frac{\partial\mathbf{e}(\mathbf{r},t)}{\partial t} = \int_{-\infty}^{t}\left[\left(\frac{\partial\hat{\varepsilon}(\mathbf{r},\psi)}{\partial\psi}\right)\left(\frac{\partial\mathbf{e}(\mathbf{r},t-\psi)}{\partial t}\right)\right]d\psi, \tag{18}$$

substituting equation (17) into the above integral and solving respect to shifted variable ψ we obtain the general solution of the convolution between polarization and electric field rates:

$$\frac{\partial\hat{\varepsilon}(\mathbf{r},t)}{\partial t} * \frac{\partial\mathbf{e}(\mathbf{r},t)}{\partial t} = \varepsilon(\mathbf{r},t)\frac{\partial^2\mathbf{e}(\mathbf{r},t)}{\partial t^2} + \sigma_\infty\frac{\partial\mathbf{e}(\mathbf{r},t)}{\partial t} - \eta\sum_{p=1}^{D_f}\frac{\gamma_p}{\tau_p}\left(\mathbf{e}(\mathbf{r},t) - \frac{\mathbf{M}_{\mathrm{p}}(\mathbf{r},t)}{\tau_{\mathrm{p}}}\right), \tag{19}$$

From the right-hand side of equation (19), we follow Carcione et al. (1988b) and define the memory variable function ($\mathbf{M}_p$) that stores the cumulative effect of the full-time series of the electric field as:

$$\mathbf{M}_\mathrm{p}(\mathbf{r},t) = \int_{-\infty}^{t} \mathbf{e}(\mathbf{r},\psi)\ \mathrm{e}^{-\left(\frac{t-\psi}{\tau_p}\right)} d\psi. \tag{20}$$

Similar to Carcione et al. (1988a), Carcione et al. (1988b), Moczo et al. (2007), we transform the integral convolution into a differential equation, by taking the time derivative of the memory variable in equation (20):

$$\frac{\partial \mathbf{M}_p(\mathbf{r},t)}{dt} = \frac{d}{dt}\left[\int_{-\infty}^{t} \mathbf{e}(\mathbf{r},\psi)\ \mathrm{e}^{-\left(\frac{t-\psi}{\tau_p}\right)} d\psi\right]. \tag{21}$$

We then evaluate the right-hand side of the differential equation using the Leibniz integration rule, to obtain:

$$\frac{\partial \mathbf{M}_p(\mathbf{r},t)}{\partial t} = \mathbf{e}(\mathbf{r},t) - \frac{1}{\tau_p(\mathbf{r})}\mathbf{M}_p(\mathbf{r},t). \tag{22}$$

Finally, dropping the explicit dependence of space and time, we substitute the solution of the convolution (equation (19)) in equation (15) and obtain:

$$\nabla\times\nabla\times\mathbf{e}\ +\mu\left[\varepsilon\frac{\partial^2\mathbf{e}}{\partial t^2} + \sigma_\infty\frac{\partial \mathbf{e}}{\partial t} - \eta\sum_{p=1}^{D_f}\frac{\gamma_\mathrm{p}}{\tau_\mathrm{p}}\left(\mathbf{e} - \frac{\mathbf{M}_\mathrm{p}}{\tau_\mathrm{p}}\right)\right] = -\mu\frac{\partial \mathbf{j}}{\partial t}. \tag{23}$$

We note that our procedure recovers the wave equation for the electric field in time domain (Ward & Hohmann, 1988, eq. 1.29) with an extra term to quantify the IP effect. The minus sign refers to the fact that the IP effect opposes the original EM signal.

Taking the first-order diffusion-reaction terms in equation (23) and with the ODE of the memory variable (22) we define the following system of coupled equations:

$$\mu\sigma_\infty\frac{\partial \mathbf{e}}{\partial t} + \nabla\times\nabla\times\mathbf{e} - \mu\eta\sum_{p=1}^{D_f}\frac{\gamma_\mathrm{p}}{\tau_\mathrm{p}}\frac{\partial \mathbf{M}_p}{\partial t} = -\mu\frac{\partial \mathbf{j}}{\partial t}. \tag{24}$$

Equations (20), (22) and (24) fully describe the polarization response of the medium (cf. Carcione et al., 1988a) and are the objective to be solved in order to simulate the IP effect in the transient EM signal.

4 Validation test on a Cole-Cole model approximation

Being that our methodology is based on the use of the Debye model is not intrinsically dependent on any particular IP relaxation model; however, to validate our formulation we apply the methodology described in Kelley and Luebbers (2003) or Kelley et al. (2007) to approximate the well-known Cole-Cole model trough a weighted sum of linear functions. First, we set the Cole-Cole model (left-hand side of equation 25) through the real and imaginary part of the corresponding Debye function expansion (right-hand side of equation 25), writing it as follows:

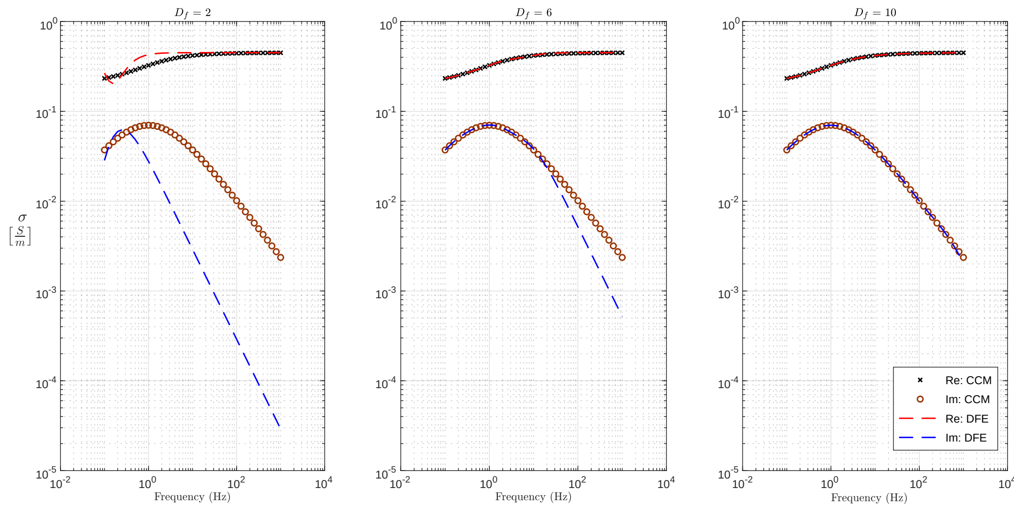

Figure 1. Approximation of a Cole-Cole model (CCM) using 2, 6 and 10 terms on the Debye function expansion (DFE, $D_f = 10$). The reference Cole-Cole conductivity model is $\sigma_0 = 0.2$, $\sigma_\infty = 0.45$, $\tau = 1$ and c= 0.65.

$$\sigma_\infty \left(1 - \frac{m}{1+(i\omega\tau)^c}\right) \approx \left[\sigma_\infty + \eta \sum_{p=1}^{D_f} \frac{\gamma_p^{Re}}{1+\left(\frac{\omega}{\omega_{r_p}}\right)^2} - i\eta \sum_{p=1}^{D_f} \gamma_p^{Im} \frac{\frac{\omega}{\omega_{r_p}}}{1+\left(\frac{\omega}{\omega_{r_p}}\right)^2}\right]. \tag{25}$$

The Cole-Cole model in the left-hand side of equation (25) must be sampled in a frequency range of interest (ω_i). We then aim to compute the corresponding real γ_p^{Re} and imaginary γ_p^{Im} weighted-parameters for the Debye Model by:

$$\gamma_p^{Re} = \left[\sum_{p=1}^{D_f} \left(\frac{1}{1+\left(\frac{\omega_i}{\omega_{r_p}}\right)^2}\right)\right]^{-1} \left\{\frac{\sigma'_{cc}(\omega_i) - \sigma_\infty}{(\sigma_0 - \sigma_\infty)}\right\}, \tag{26}$$

$$\gamma_p^{Im} = \left[\sum_{p=1}^{D_f} \left(\frac{\frac{\omega_i}{\omega_{r_p}}}{1+\left(\frac{\omega_i}{\omega_{r_p}}\right)^2}\right)\right]^{-1} \left\{\frac{\sigma''_{cc}(\omega_i)}{(\sigma_0 - \sigma_\infty)}\right\}, \tag{27}$$

whose solution is obtained by simple linear regression. The fit between the real and imaginary part of the Cole-Cole model is shown in Figure 1, and their residuals in Figure 2.

Finally, we concatenate the vector solution γ_p^{Re} and γ_p^{Im} to compute the total weighted-parameters γ_p for the equations in time (7) and (16) whose responses are shown in Figures 3 and 4, respectively. The γ_p parameters complete the solution in the coupled equation (24) to be implemented in a finite difference or finite element schemes.

As a result, we obtain a new approach to assimilate transient IP effects in an electromagnetic model based on a transient dielectric function as an alternative to the classical dispersive conductivity term which leads to a new Helmholtz equation. Since this

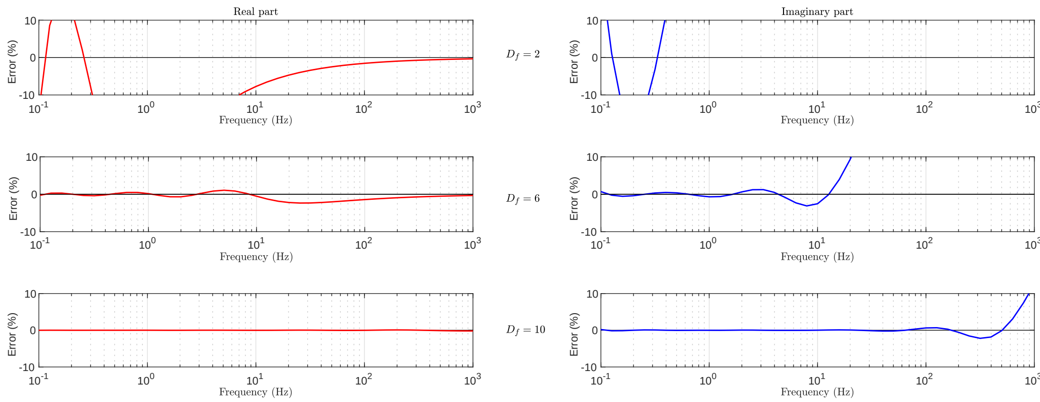

Figure 2. Residuals of real and imaginary part of the CCM approximation using 2, 6 and 10 Debye functions ($D_f = 10$). As more terms are added the residuals goes to zero.

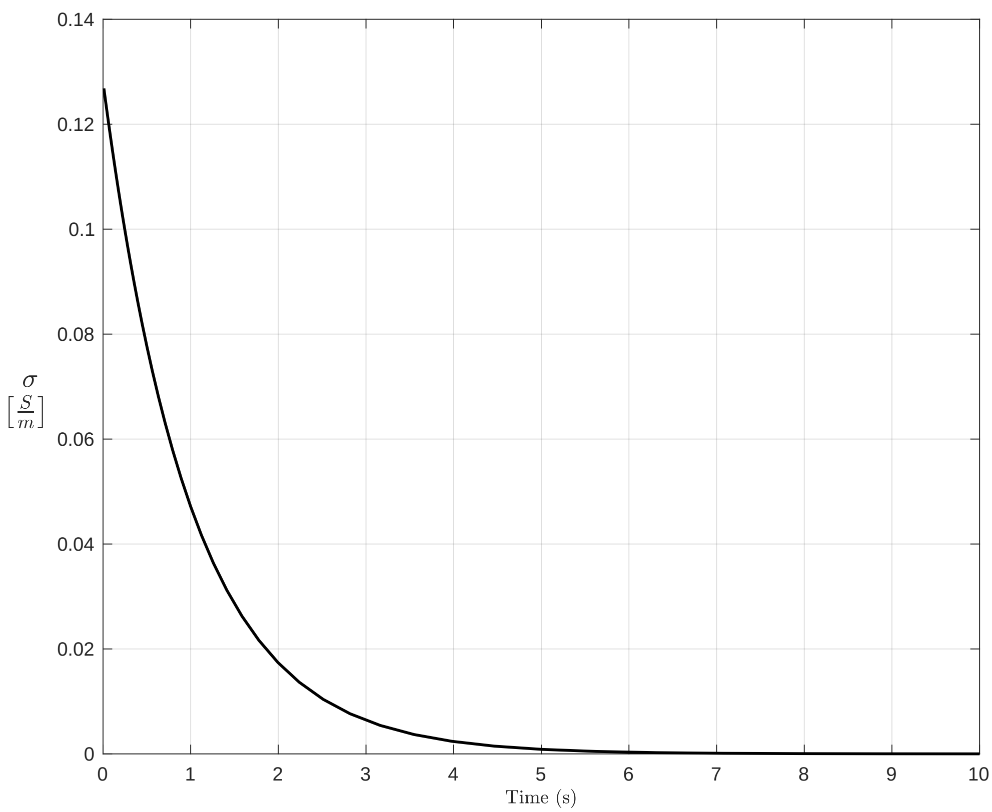

Figure 3. Time-domain response of Debye Function Expansion (equation 7).

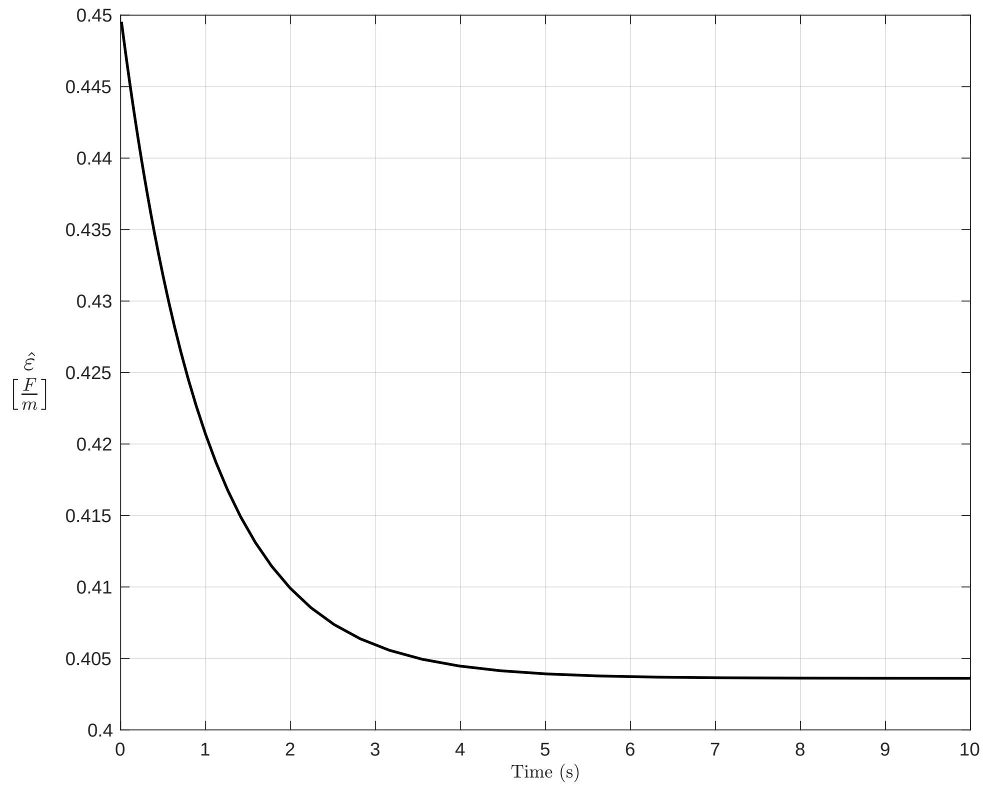

Figure 4. Time-domain response of the complex permittivity (equation 16).

approach considers both physical properties, it is theoretically feasible to identify, from the measured data, the different polarization mechanisms described in Table 1 at a relatively low frequency as applied in conventional TEM acquisitions.

5 Concluding remarks

The model developed herein, which uses the concept of the dielectric function that contains the polarization parameters of the medium and implicitly has the information of the electrical conductivity, is an alternative to analyze and obtain more information from the IP phenomenon by considering both conductivity and permittivity.

The concept of memory variables resulted in a versatile and useful mathematical tool to complete our procedure, leading to a novel PDE for geophysical electromagnetics. This new model considers both diffusion and reaction processes in a single PDE directly posed in the time domain. Furthermore, our methodology permits include any Cole-Cole model in the electromagnetic signal through the Debye Function Expansion, simplifying the mathematical procedure (in contrast to using the Cole-Cole model directly) and simultaneously, add any dispersive behavior in the electromagnetic fields.

We highlight that considering the dispersion in the electrical properties, more precisely, the dielectric function provides an entirely different view for the estimation of physical properties, such as the fluid temperature in a reservoir or the hydraulic permeability estimated from the IP parameters chargeability and relaxation time.

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