



Motivations

- We require the accurate model of the D-region ionization rate altitude profiles;
- It should include an IGRF magnetic field;
- The model should be self-efficient and fast compared to general MC models.

Model overview

During enhanced magnetic activities, massive ejections of energetic electrons from radiation belts ionize the upper polar atmosphere, which affects VLF propagation.

We develop a model to calculate ion productions in the D-Region of the ionosphere caused by energetic (10 keV-1 MeV) electrons using a general Monte Carlo approach implemented in the latest version of the MCNP6 code for electron tracking in magnetic fields.

By expressing those results using the ionization yield functions, the pre-calculated results are extended to cover arbitrary magnetic field inclinations and atmospheric density profiles in the range of altitudes from 20 to 200 km, at any geographic point of interest and date/time by adopting results from an external atmospheric density model (e.g. NRLMSISE-00).

The pre-calculated MCNP6 results are supplied in a CDF (Common Data Format) file, and both IDL routines library and testing software are included to provide an end-user interface to the model.

Electron transport validation

We validate MCNP electron transport based on direct comparison with experimental results.

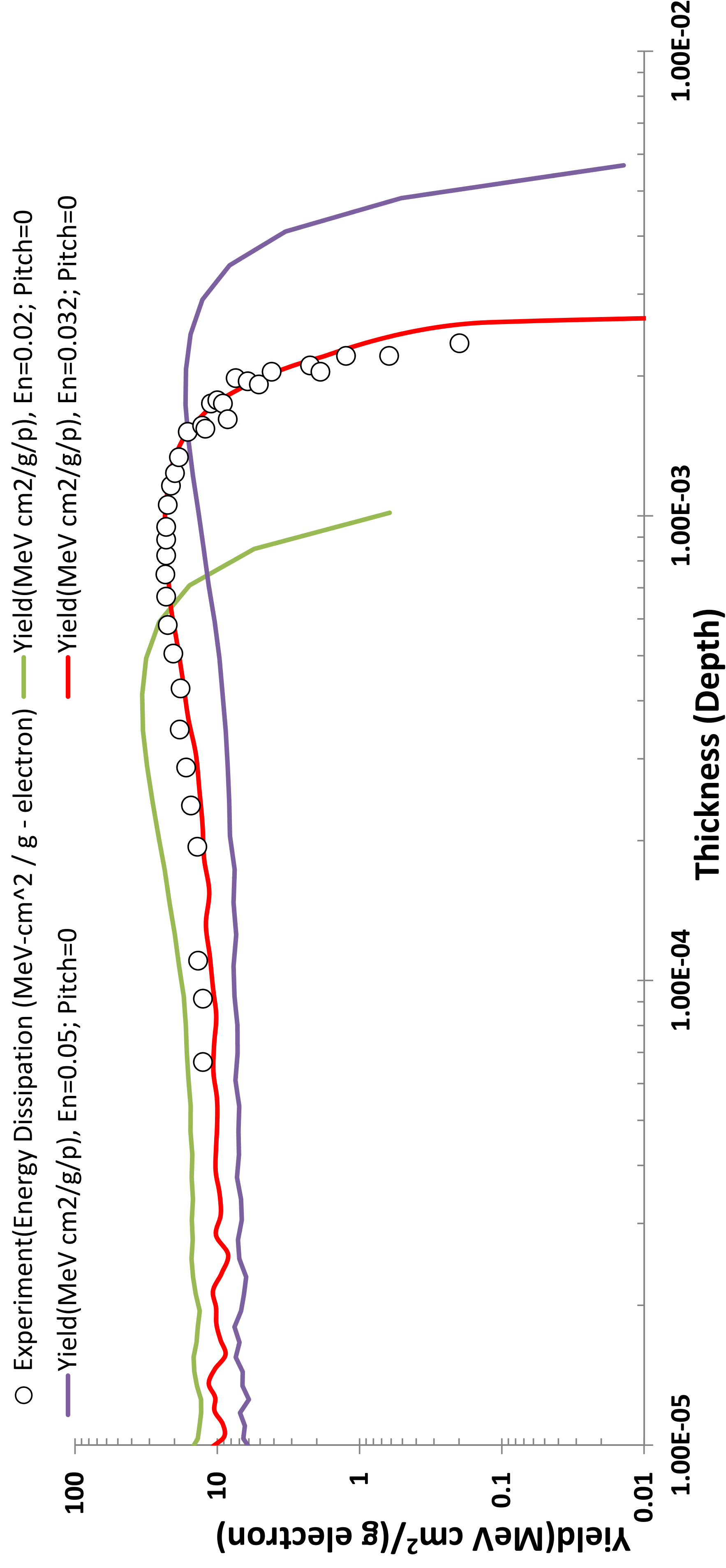


Fig. 1. MCNP 6 energy dissipation (yield function) calculation results for mono-energetic (20, 32 and 50 keV) electrons in the air are compared with experimental results from the [Kobetich et al., 1968] paper

Yield functions

Electron ionization yield function $Y(d, E_e)$ (ion pairs·cm²/g) at a given atmospheric depth $d(z)$ (g/cm²) is proportional to the differential energy loss, so $Y(d, E_e) = \frac{1}{E_{ion}} \frac{\partial E}{\partial d}$, and the yield function is normalized on a single (mono-energetic) electron from the source.

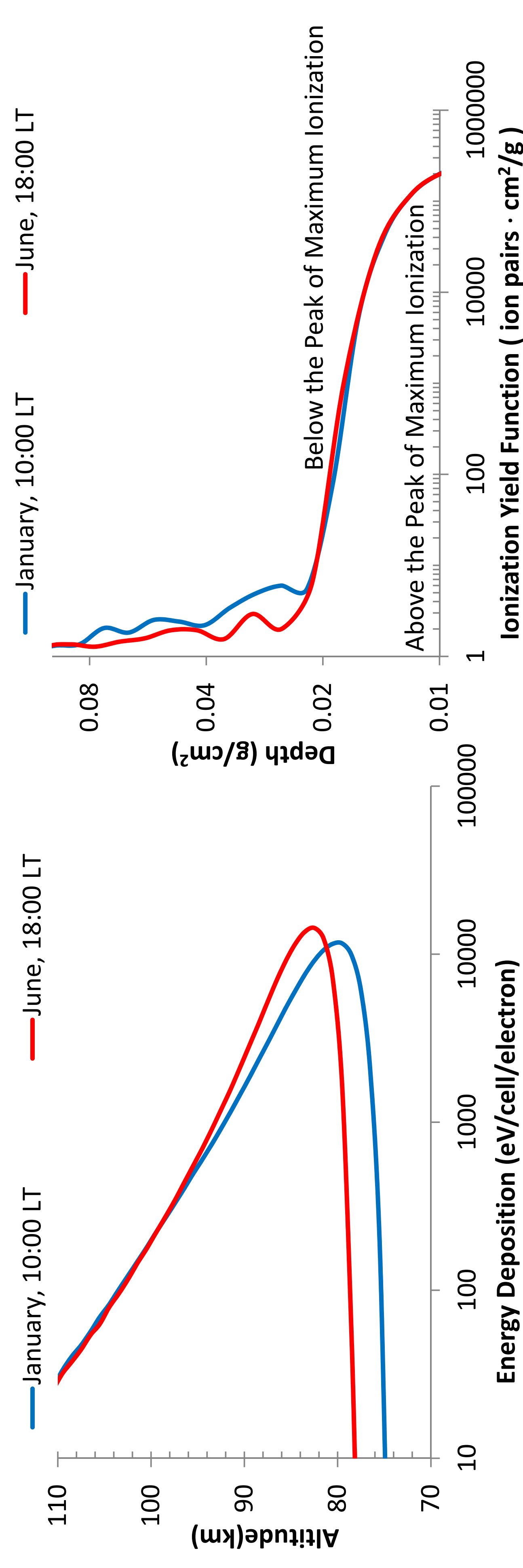


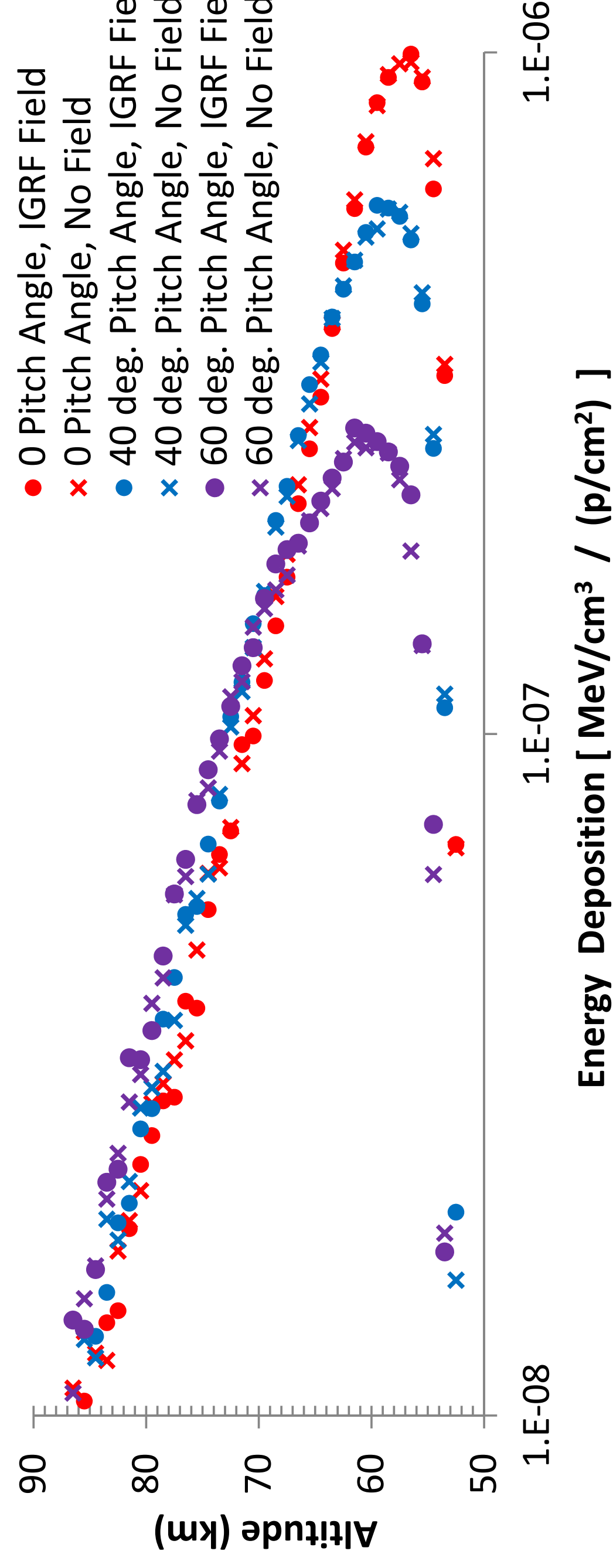
Fig. 2. The sensitivity of the energy deposition to the extreme atmosphere illumination for 100 keV electrons for two high-latitude (64°) atmospheres, corresponding to the highest and lowest solar illuminations. The left panel shows energy deposition altitude profiles; The right panel is a plot of corresponding atmospheric depths and Yield functions.

Collisional transport: No mirror force

Electron trajectory in a plane slab with const magnetic field is a helix, which is the shortest path between two points on a cylindrical surface. As a result, electron trajectory has the same path length with and without constant magnetic field applied:

$$T_e = L \sqrt{1 + \left(\frac{v_z}{v_\perp}\right)^2} = H \sqrt{1 + \tan^2(\theta)} = \frac{H}{\mu}$$

Therefore, average over the cell volume un-scattered electron flux, which is proportional to the path length T_e , is not affected by a constant magnetic field applied for the infinite slab geometry and large field inclinations: $\bar{\Phi}_e = \frac{1}{V} \int dV \int d\hat{\Omega} \varphi(\hat{r}, E, \hat{\Omega}) \square \frac{T_e}{V}$



Collisionless transport: Mirror force

Primary flux increases toward the mirror point during electron propagation through a gradient magnetic field. A collisionless drift kinetic equation for electrons propagation in an arbitrary magnetic field in the plane 1-D geometry written based on the steady-state Liouville equation is:

$$\mu \frac{df(z, \mu)}{dz} + (1 - \mu^2) \frac{1}{2B(z)} \frac{\partial B(z)}{\partial z} \frac{\partial f(z, \mu)}{\partial \mu} = 0 \quad (1)$$

There are three important features of the above Equation:

- **Density** of electrons (electron trajectories) does depend on electrons energy, but electron **flux** doesn't (Fig. 5)
- **Flux** of electrons does not change during propagation in a constant magnetic field
- **Flux gradient** increases abruptly for small angles when electrons move almost in parallel to the surface of the earth
- **Integral flux** estimation derived from the 1st adiabatic invariant is

$$F(z, \mu_0) = \frac{\sqrt{1 - B(z_0)/B_{MP}(z_0, \mu_0)}}{\sqrt{1 - B(z)/B_{MP}(z, \mu_0)}} F(z_0, \mu_0), \quad (2)$$

where both the incoming flux of mono-directional electrons $F(z_0, \mu_0)$ and the magnetic field magnitude at the mirror point $B_{MP}(z_0, \mu_0)$ are defined for the reference altitude z_0 and pitch angle θ_0 , so $\mu_0 = \cos(\theta_0)$

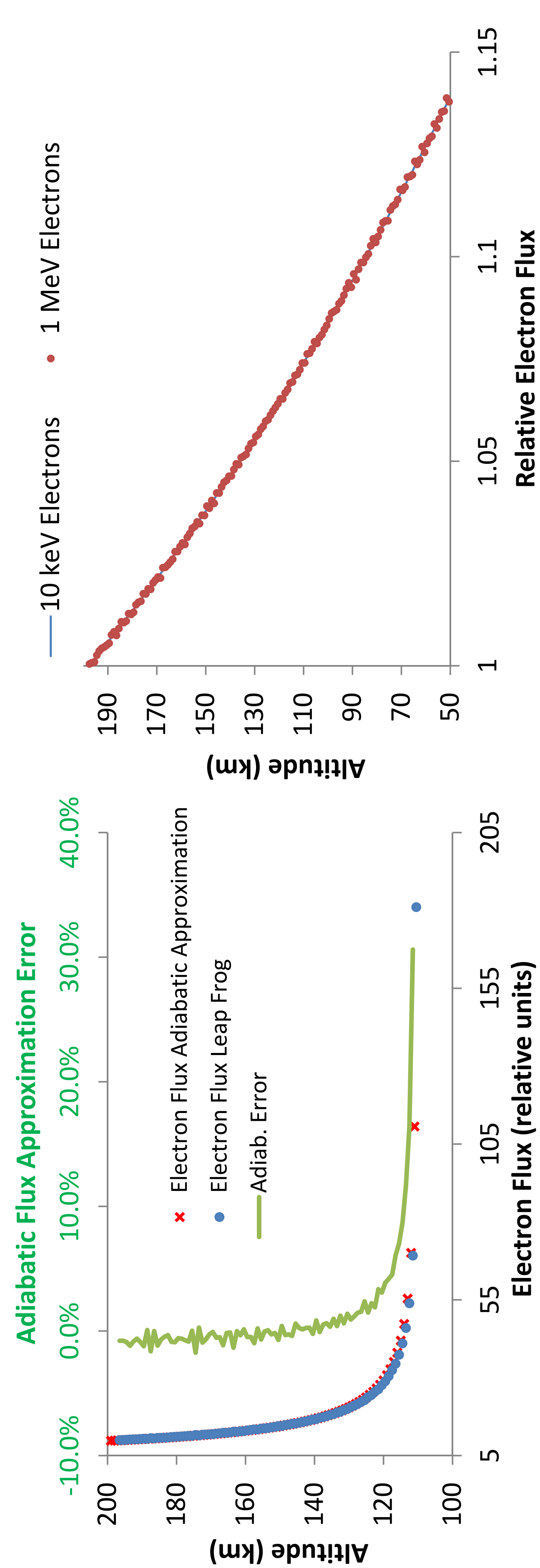


Fig. 4. 3-D calculations of un-scattered cell-averaged fluxes of 1 MeV electrons propagating through IGRF magnetic field based on Leap-Frog technique [Kouznetsov and Knudsen, 2013] (in red) and fluxes obtained from the adiabatic approximation (Eq. 2) are close to each other, with errors (green line) increasing toward the mirror point

Fig. 5. Comparison of cell-averaged electron fluxes (relative units) for two limiting (10 keV and 1 MeV) electron energies and the same 60° pitch angle calculated based on 3-D Leap-Frog technique [Kouznetsov and Knudsen, 2013]. An MeV electrons (red dots) have gyroradii comparable to the cell size which explains points scattering.

Model validation

At a small fraction of the computational cost, the model results improve direct MCNP calculations:

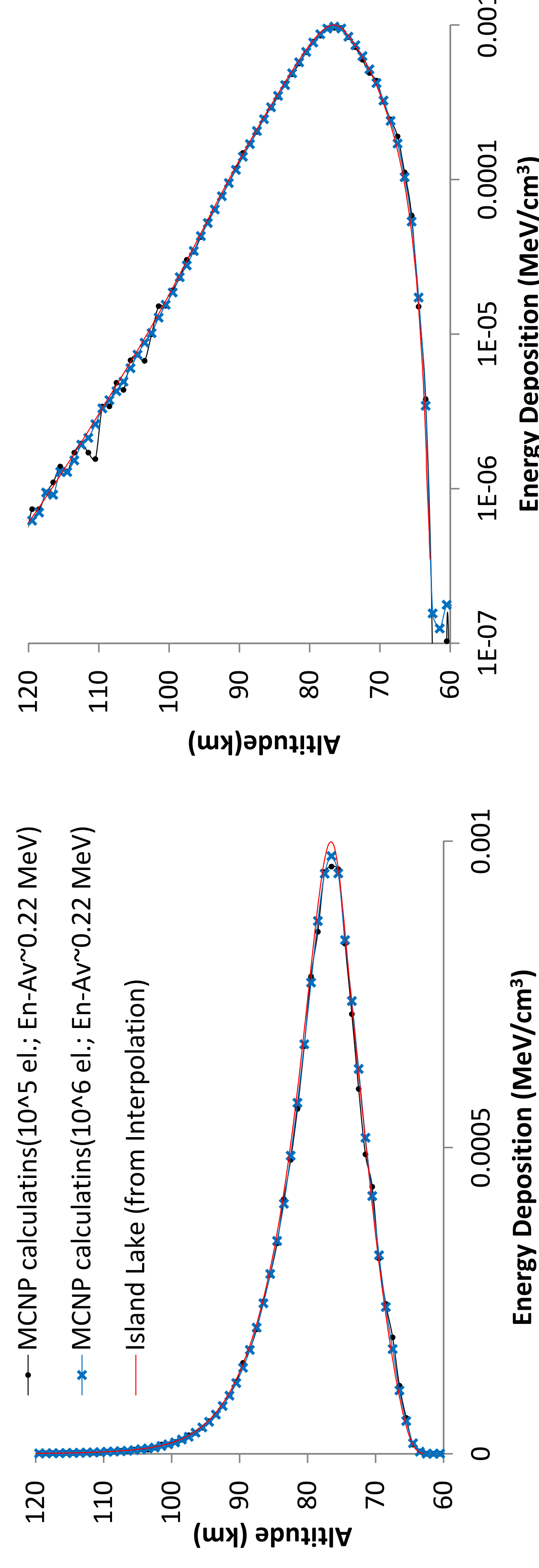


Fig. 6. We define a testing source of precipitating electrons from radiation belts by its energy-angular distribution estimated on August 25, 2016, at 53° North, 108° West. We calculate a testing atmosphere altitude density profile with NRLMSISE-00 neutral atmosphere model for the same date-time and location. We use the source and altitude density profile to validate the model by comparing with direct MCNP calculations of energy deposition altitude profiles for small (10°) and large (10°) electron numbers emitted from the source. The model is based on pre-calculated energy deposition altitude profiles obtained from 126 multi-hours MCNP runs for mono-energetic electrons. Overall model calculation results (red lines) enhance calculation precision compare to direct MCNP calculations (blue lines, dots) both in the peak of maximum ionization (left panel, linear scale) and at high altitudes (right panel, log scale) because of essentially larger statistics accumulated in the model.

Acknowledgments

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