

# On the seasonal cycle of the statistical properties of Sea Surface Temperature

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## Key Points:

- The intensity of SST fronts is quantified using singularity exponents, which measure the continuity of the field
- Anomalous scaling of SST structure functions is correlated to the intensity of the strongest fronts
- The variability of the strongest fronts depends on the seasonal variability of the coastal upwelling in the area of study

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## 14 **Abstract**

15 The contribution of ocean fronts to the properties and temporal evolution of Sea Sur-  
16 face Temperature (SST) structure functions have been investigated using a numerical  
17 model of the California Current system. First, the intensity of fronts have been quan-  
18 tified by using singularity exponents. Then, leaning on the multifractal theory of tur-  
19 bulence, we show that the departure of the scaling of the structure functions from a straight  
20 line, known as anomalous scaling, depends on the intensity of the strongest fronts. These  
21 fronts, at their turn, are closely related to the seasonal change of intensity of the coastal  
22 upwelling characteristics of this area. Our study points to the need to correctly repro-  
23 duce the intensity of the strongest fronts and, consequently, properly model processes  
24 such as coastal upwelling in order to reproduce SST statistics in ocean models.

## 25 **Plain Language Summary**

26 Forecasting the evolution of Earth Climate requires to predict the evolution of the  
27 statistical characteristics of essential climate variables such as the Sea Surface Temper-  
28 ature. In this study, it has been found that some of such statistical properties depend  
29 on the intensity of the strongest fronts in the ocean. This implies that those ocean, or  
30 climate, models that fail to correctly predict their intensity won't be able to correctly  
31 reproduce the statistical characteristics of key variables such as temperature. The area  
32 analyzed in this study is the California Current system, where the strongest fronts are  
33 modulated by the seasonal evolution of the upwelling. Therefore, our results imply that  
34 such a system has to be correctly modeled in order to properly reproduce the statistics  
35 of ocean temperatures.

## 36 **1 Introduction**

37 Sea Surface Temperature (SST) is a fundamental variable of the Earth climate sys-  
38 tem due to its role in regulating climate and weather (Deser et al., 2010) and its dynam-  
39 ical connection to ocean currents (Isern-Fontanet et al., 2014). Moreover, the availabil-  
40 ity of long time series of global high resolution satellite measurements of SST (Merchant  
41 et al., 2019) makes it well suited for addressing a wide range of problems such as mon-  
42 itoring the Climate Change (Gulev et al., 2021); retrieving ocean currents (Isern-Fontanet  
43 et al., 2017); or calibrating and validating ocean and climate models (Skákala et al., 2019).  
44 It is, therefore, of major importance to understand how ocean processes contribute to

45 SST statistics to exploit such a wealth of data and get insight into the functioning of the  
46 ocean and climate.

47 A prominent feature of SST is the presence of fronts, which are known to be sinks  
48 of energy (D'Asaro et al., 2011; Isern-Fontanet & Turiel, 2021) and significantly contribute  
49 to the vertical transport of nutrients and, thus, to primary production (Mahadevan, 2016).  
50 The variability of the characteristics of fronts, such as the density of fronts or their in-  
51 tensity, are expected to be mirrored by the variability of some SST statistics. A popu-  
52 lar approach is based on the spectral slope of SST because it can be connected to the-  
53 ories of turbulence. Nevertheless, they provide an incomplete framework, if only because  
54 different theories may predict the same slope (Callies & Ferrari, 2013) and the under-  
55 lying turbulence regime may not change in spite of the seasonal changes in the proper-  
56 ties of fronts.

57 The structure functions of a turbulent variable, i.e. the moments of the differences  
58 between two points, are also at the core of theories of turbulence (Pope, 2000) and ex-  
59 tend the information provided by spectral slopes (Yu et al., 2017; Sukhatme et al., 2020).  
60 Moreover, the anomalous scaling of the power laws deduced from the structure functions,  
61 i.e. its deviation from a straight line, can be related to the geometry of gradients mak-  
62 ing use of the multifractal framework (Isern-Fontanet & Turiel, 2021). The relevance of  
63 this approach has already been demonstrated in the oceanic context. (Isern-Fontanet et  
64 al., 2007) and it has been used to develop metrics for model validation (Ivanov et al.,  
65 2009; Skákala et al., 2016), although it has not been yet exploited to investigate the con-  
66 tribution of fronts to SST statistics.

67 Here, we introduce a metric to measure the intensity of SST fronts in numerical  
68 simulations of the California Current System (Capet, McWilliams, et al., 2008), which  
69 is dominated by cross-shore gradients generated by the coastal upwelling (Chenillat et  
70 al., 2018). This metric is then connected to the scaling of the structure functions using  
71 the multifractal framework (Frisch, 1995) and used to investigate how the temporal vari-  
72 ability of front intensity contribute to the variability of anomalous scaling and spectral  
73 slopes. The paper is organized as follows: section 2 puts the multifractal theory of tur-  
74 bulance in the context of oceanography; section 3 describes the numerical simulations  
75 and the algorithms used for this study; sections 4 and 5 describe results and discuss them,  
76 respectively; and section 6 list the conclusions.

## 2 Theoretical framework

Coarse-grained Sea Surface Temperature (SST) gradients are built by filtering the module of the thermal gradient as

$$\overline{|\nabla T|}_\ell(\vec{x}) \equiv \int_{\mathbf{R}^d} \ell^{-d} G(\ell^{-1}\vec{x}) |\nabla T|(\vec{x} + \vec{x}') d\vec{x}', \quad (1)$$

where  $d = 2$  is the geometrical dimension;  $\ell$  is the scale of the filter;  $G(\vec{x})$  is a normalized positive function that decays fast to zero as  $|\vec{x}| \rightarrow \infty$ ;  $T(\vec{x})$  is the SST and  $\nabla = (\partial_x, \partial_y)$ . These SST gradients are known to possess a range of scaling exponents  $h(\vec{x})$  that verify

$$\overline{|\nabla T|}_\ell(\vec{x}) \sim \left(\frac{\ell}{\ell_0}\right)^{h(\vec{x})} \quad (2)$$

as  $\ell/\ell_0 \rightarrow 0$ , where  $\ell_0$  is the integral scale of the flow (Isern-Fontanet et al., 2007). The scaling exponents  $h(\vec{x})$ , known as singularity or Hölder exponents, quantify the degree of continuity of SST. Indeed, if  $h(\vec{x}) \in (n, n+1)$  with  $n$  being a positive integer,  $\overline{|\nabla T|}_\ell(\vec{x})$  is derivable  $n$  times but not  $n+1$  (Arneodo et al., 1995). Consequently, we propose the use of singularity exponents as a proxy measure for the intensity of fronts, on the basis that the strongest fronts are those with the most marked singularity, hence also those with the smallest singularity exponents. .

The domain of the turbulent flow can be, then, divided into subsets according to their singularity exponent. This gives rise to the singularity spectrum, a concave function of  $h$  defined as

$$D(h') \equiv d_F(\{\vec{x} | h(\vec{x}) = h'\}), \quad (3)$$

where  $d_F(A)$  is the fractal dimension of set  $A$ . It follows that, the singularity spectrum  $D(h)$  characterizes the 'volume' occupied by fronts with intensity  $h$ . Moreover, the singularity spectrum  $D(h)$  provides information about the statistical properties of SST. Indeed, the scaling properties of the moments of SST gradients are defined by a continuous function  $\tau(p)$  of the moment order  $p$

$$\langle \overline{|\nabla T|}_\ell^p \rangle \sim \left(\frac{\ell}{\ell_0}\right)^{\tau(p)}, \quad (4)$$

which is related to the singularity spectrum by a Legendre transform pair

$$\tau(p) = ph + d - D(h), \quad \text{with} \quad p = \frac{dD}{dh} \quad (5)$$

and

$$D(h) = ph + d - \tau(p), \quad \text{with} \quad h = \frac{d\tau}{dp}, \quad (6)$$

107 as shown by Parisi and Frisch (1985). It's worth mentioning that equation 4 implies that  
 108 the Probability Density Functions (PDF) of thermal gradients are dependent on the anal-  
 109 ysis scale  $\ell$ , which is a signature of intermittency (Frisch, 1995). As a consequence, care  
 110 must be taken when analysing PDF and kurtosis and when comparing PDF from data  
 111 with different resolutions.

112 The singularity spectrum can also be related to the scaling of the structure func-  
 113 tions of temperature, which are defined as

$$114 \quad S_p(\ell) \equiv \langle |T_s(\vec{x} + \vec{\ell}) - T_s(\vec{x})|^p \rangle \quad (7)$$

115 and scale according to the continuous function  $\zeta(p)$ , i.e.

$$116 \quad S_p(\ell) \sim \left( \frac{\ell}{\ell_0} \right)^{\zeta(p)}. \quad (8)$$

117 Using that, at small scales,  $|T_s(\vec{x} + \vec{\ell}) - T_s(\vec{x})| \sim \ell |\nabla T|$  it follows that,

$$118 \quad \frac{1}{\ell_0^p} \langle |T_s(\vec{x} + \vec{\ell}) - T_s(\vec{x})|^p \rangle \sim \left( \frac{\ell}{\ell_0} \right)^p \langle |\nabla T|_\ell^p \rangle \sim \left( \frac{\ell}{\ell_0} \right)^p \left( \frac{\ell}{\ell_0} \right)^{\tau(p)} \sim \frac{1}{\ell_0^p} \left( \frac{\ell}{\ell_0} \right)^{\zeta(p)}, \quad (9)$$

119 with  $\ell_0$  being a constant, and, consequently, both scaling functions are related by

$$120 \quad \zeta(p) = p + \tau(p). \quad (10)$$

121 Recall that, the scaling of the structure function of order  $p = 2$  gives the spectral slope  
 122 of SST,

$$123 \quad E(k) \propto k^{-\zeta(2)-1} = k^{-\tau(2)-3}, \quad (11)$$

124 where  $E(k)$  is the energy spectrum and  $k$  the wavenumber (Frisch, 1995).

125 Guided by the recent work of Isern-Fontanet and Turiel (2021), here, we focus on  
 126 two properties of the singularity spectrum: the most singular exponent  $h_\infty$ ,

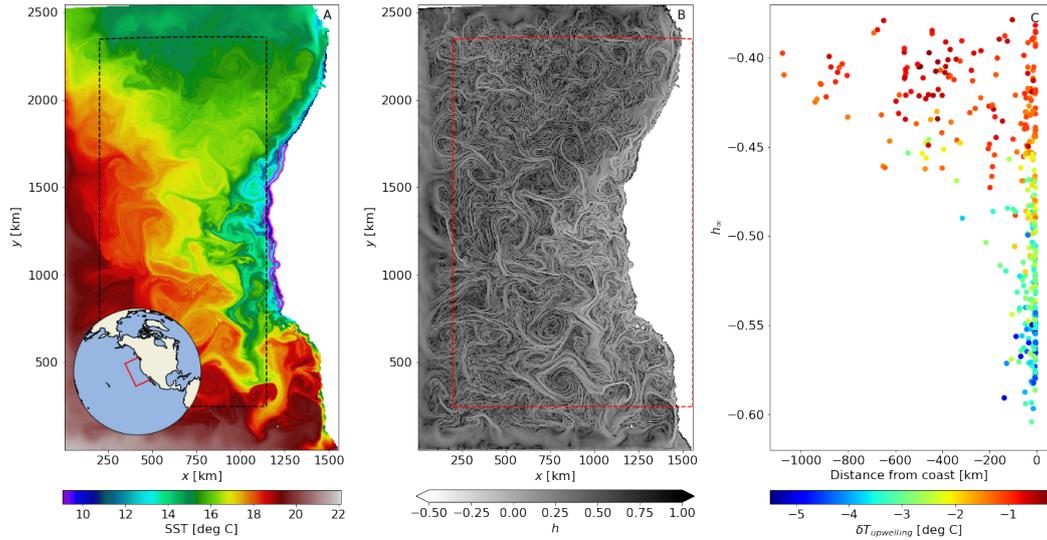
$$h_\infty \equiv \min(h), \quad (12)$$

127 which is a measure of the intensity of the strongest fronts; and the width of the singu-  
 128 larity spectrum defined as

$$\Delta h^- \equiv h_d - h_\infty, \quad (13)$$

129 where  $h_d$  is the mode. This quantity corresponds to the difference of slopes of  $\zeta(p)$  be-  
 130 tween the origin ( $p = 0$ ) and large orders ( $p \rightarrow \infty$ ). Indeed, using equation (6) and  
 131 equation (10), it can be seen that

$$\frac{d\zeta}{dp} \Big|_{p=0} - \frac{d\zeta}{dp} \Big|_{p \rightarrow \infty} = \frac{d\tau}{dp} \Big|_{p=0} - \frac{d\tau}{dp} \Big|_{p \rightarrow \infty} = h_d - h_\infty = \Delta h^-. \quad (14)$$

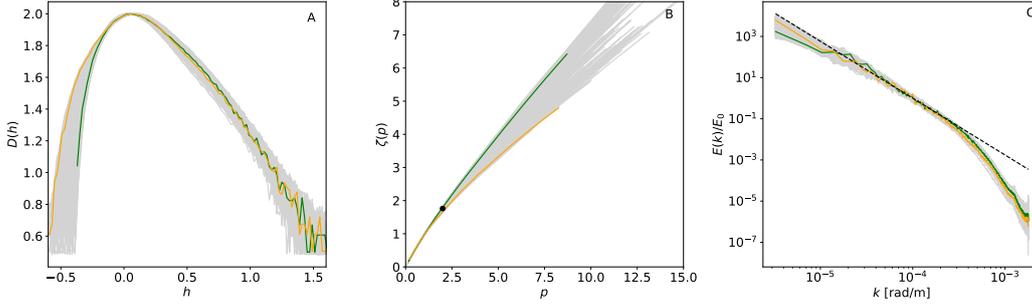


**Figure 1.** A: Example of instantaneous SST corresponding to July 10th of the first year analyzed ( $t=20$  days) with the area used to compute Fourier spectra (black, dashed) and  $\delta T_{upwelling}$  (purple, solid). The inset globe shows the geographical limits of the numerical simulations. B: singularity exponents for the SST image with the area used to compute singularity spectra (red, dashed). C: distance from coast of the  $h_\infty$  observed for the whole analyzed period with the color corresponding to  $\delta T_{upwelling}$ .

132 Therefore, the anomalous scaling, i.e. the departure from a straight line, increases with  
 133 growing  $\Delta h^-$ .

### 134 3 Data and procedures

135 SST fields were taken from numerical simulations of the circulation in the Califor-  
 136 nia Current System (see figure 1A) generated with the ROMS oceanic model (Shchepetkin  
 137 & McWilliams, 2005). The model was configured with a horizontal resolution of  $\sim 2.5$   
 138 km ( $1025 \times 625$  grid points) and 32 vertical levels with higher resolution in the upper lay-  
 139 ers. The boundary and initial conditions, as well as the forcing at the air-sea interface  
 140 (wind stress, heat and freshwater fluxes) were derived from climatologies as in Capet,  
 141 Colas, et al. (2008). Singularity analysis was applied to snapshots of SST taken every  
 142 two days of simulation spanning a period of two years.



**Figure 2.** A: Singularity spectra  $D(h)$ . B: scaling of the structure functions of temperature  $\zeta(p)$  derived from the singularity spectra. The black dot corresponds to  $\langle\zeta(2)\rangle$ . C: normalized Fourier spectra. The black dashed line has a slope given by  $-\langle\zeta(2)\rangle - 1$ . Energy spectra are normalized by  $E_0 \equiv E(k_0)$ , where  $k = 10^{-4}$  rad/m. Grey lines correspond to the observations for the full period, while orange and green lines correspond the examples of two particular days: March 4th (green,  $t = 246$  days) and August 23rd (orange,  $t = 418$  days) of the second year.

143 Although very appealing, equation (2) can not be used directly to compute singu-  
 144 larity exponents due to long-range correlations and discretization effects (Turiel et al.,  
 145 2008). To avoid these difficulties, we used the method proposed by Pont et al. (2013) to  
 146 compute singularity exponents (see figure 1B). Then, the singularity spectrum of each  
 147 snapshot of SST was computed within the domain of analysis (see red rectangle in fig-  
 148 ure 1B) using the histogram method

$$D(h_i) \approx d - \frac{\log N_i - \log N_{max}}{\log(\ell/\ell_0)}, \quad (15)$$

149 where  $N_i$  is the number of grid cells having a singularity exponent in the range  $[h_i -$   
 150  $\frac{\delta h}{2}, h_i + \frac{\delta h}{2})$ ,  $N_{max}$  the number of valid ocean grid cells in the analysis domain, and  $\ell/\ell_0 =$   
 151  $(\sum_i N_i)^{\frac{1}{d}}$  (Turiel et al., 2006). The grid points surrounding the land mask were removed  
 152 to avoid spurious values due to the land-sea transition and we used  $d = 2$  and  $\delta h =$   
 153  $0.02$  in the range from  $h = -1$  to  $h = 3$  for computing  $D(h)$ . Translational invariance  
 154 was imposed to each singularity spectrum to correct for any shift that may exist in the  
 155 singularity exponents (Isern-Fontanet & Turiel, 2021). This invariant condition consists  
 156 in imposing that the  $\langle|\nabla T|_\ell\rangle$  does not depends on  $\ell$ , i.e.  $\tau(1) \equiv 0$ . Finally, the mode  
 157  $h_d$  was estimated by locally adjusting a parabola around the maximum of  $D(h)$  and, then,  
 158 analytically calculating its maximum.

159 The function  $\zeta(p)$  was derived from the instantaneous  $D(h)$  by first applying the  
 160 Legendre transform equations (5) and, then, equation (10). To reduce the spurious os-  
 161 cillations due to noise, we used a similar approach to the computation of  $h_d$ , i.e. the Leg-  
 162 endre transform was obtained by locally fitting a second order polynomial to the sur-  
 163 roundings of each value of  $D(h_i)$  and, then, analytically inverting it. On the contrary,  
 164 the SST spectrum was computed independently from  $D(h)$  using SST anomalies in the  
 165 black box shown in figure 1A, i.e.

$$\delta T(\vec{x}, t) = T(\vec{x}, t) - \tilde{T}(\vec{x}, t), \quad (16)$$

166 where  $\tilde{T}(\vec{x}, t) \equiv a_x(t)x + a_y(t)y + a_{xy}(t)xy + a_0(t)$  was estimated by least-squares fit-  
 167 ting to SST in the whole domain. With the aim of having a simple measure of the in-  
 168 tensity of the coastal upwelling, the temperature anomaly associated to it was defined  
 169 as the mean temperature anomaly close to the coast, i.e.

$$\delta T_{upwelling}(t) \equiv \langle \delta T(\vec{x}, t) \rangle_{upwelling}. \quad (17)$$

170 This area was taken as the area between the coast and 10 grid points seawards ( $\sim 25$   
 171 km) and between  $y = 810$  km and  $y = 1720$  km, which corresponds the purple area marked  
 172 in figure 1A.

## 173 4 Results

174 Figure 1B unveils the complex structure of thermal fronts observable in a snapshot  
 175 of SST, with the most intense fronts, bright lines in the figure, being those with smaller  
 176 singularity exponents. The intensity of fronts has some spatial variability. On one side,  
 177 the areas with blurred fronts found in the North, West and South limits of the domain  
 178 are due to the low-resolution information imposed at the model open boundary condi-  
 179 tions. On the other, the intensity of fronts tend to be higher within the area strongly  
 180 influenced by coastal upwelling (the area within  $[600 \text{ km}, 1150 \text{ km}] \times [700 \text{ km}, 1700 \text{ km}]$   
 181 approximately). Moreover, results shown in figure 1C reveal that the smallest values of  
 182  $h_\infty$  are concentrated close to the coast and correspond to large values of  $|\delta T_{upwelling}|$ ,  
 183 while larger values of  $h_\infty$  can be found away from the coast for and correspond to small  
 184 values of  $|\delta T_{upwelling}|$ .

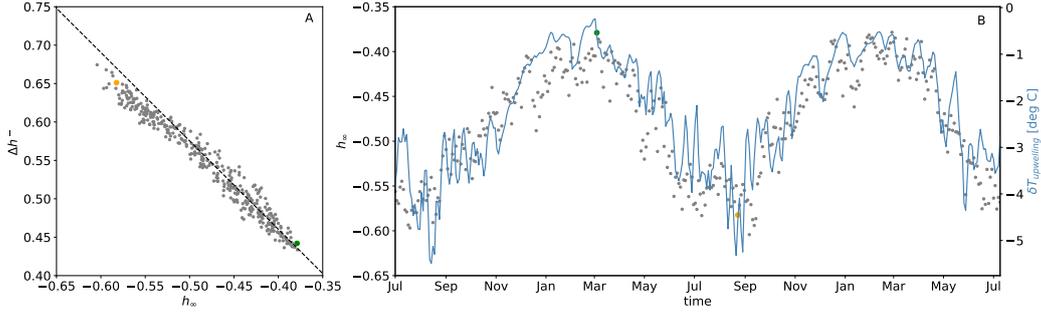
185 Singularity spectra  $D(h)$  are asymmetric functions of  $h$ , whose properties change  
 186 over time as revealed by figure 2A. Indeed, the value of  $h_\infty$  ranges between -0.6 and -  
 187 0.35 for the two years of simulation and the width  $\Delta h^-$  between 0.4 and 0.7. The changes

188 in the width of  $D(h)$  are related to changes in the anomalous scaling of the structure func-  
 189 tions (figure 2B) as expected from equation 14. Such changes are more pronounced for  
 190 moments larger than  $p = 2$  (which provides the spectral slope of SST; equation 11).  
 191 Moreover, the slope of the instantaneous spectra of SST for  $k < 10^{-4}$  rad/m, which  
 192 has been computed independently, is close to the value given by the average  $\langle \zeta(2) \rangle$  com-  
 193 puted from the singularity spectra  $D(h)$  (figure 2C). The observed spectral slope is some-  
 194 what steeper than  $k^{-2}$ , in contrast to Capet, McWilliams, et al. (2008). A shallower spec-  
 195 tral slope can be recovered by reducing the spectral analysis to the area dominated by  
 196 the upwelling, where fronts are stronger and more energy is present at the smaller re-  
 197 solved scales (not shown).

198 The two properties analyzed in this study,  $h_\infty$  and  $\Delta h^-$  are not independent but  
 199 are strongly correlated with a linear correlation of -0.98 and a slope between them of -  
 200 1.13 (figure 3A). A closer look, however, shows that the snapshots with  $h_\infty < -0.5$  have  
 201 weaker slopes between  $h_\infty$  and  $\Delta h^-$  (-1.11) and a tendency to have larger values of  $|\delta T_{upwelling}|$   
 202 (3.56 deg C on average) than snapshots with  $h_\infty > -0.5$  (-1.14 and 1.66 deg C on av-  
 203 erage, respectively). Besides, the temporal evolution of  $h_\infty$  follows a seasonal cycle (fig-  
 204 ure 3B), which has associated a seasonal variation of the width of the singularity spec-  
 205 trum of SST gradients and, thus, a seasonal variation of the anomalous scaling of the  
 206 structure functions of temperature. Moreover, the close relation between the spatial lo-  
 207 cation of  $h_\infty$  and  $\delta T_{upwelling}$  shown in figure 1C suggests a strong relationship between  
 208 them, which is confirmed statistically: the Pearson correlation coefficient between the  
 209 temporal evolution of  $h_\infty$  and  $\delta T_{upwelling}$  (figure 3B) is 0.87.

## 210 5 Discussion

211 In this study we have proposed, for the first time, to measure the intensity of fronts  
 212 in SST using the singularity exponents of thermal gradients. Singularity exponents char-  
 213 acterise the scaling at small scales, are independent of the gradient magnitude and mea-  
 214 sure the degree of continuity of the field. Moreover, singularity exponents have the ad-  
 215 vantage over other popular approaches for detecting fronts (Chang & Cornillon, 2015;  
 216 Kirches et al., 2016) that they can be easily connected to statistical quantities that are  
 217 central to turbulence theories. Indeed, the singularity spectrum, which gives the frac-  
 218 tural dimension of those points with the same exponents, emerges as a fundamental prop-  
 219 erty of the ocean providing the link between anomalous scaling and the intensity of fronts.



**Figure 3.** A: Scatter plot between  $h_\infty$  and  $\Delta h^-$ . The black line corresponds to a slope of -1.13. B: temporal evolution of  $h_\infty$  and  $\delta T_{upwelling}$ . Grey points correspond to the observations for the full period, while orange and green points correspond to the examples of two particular days: March 4th (green,  $t=246$  days) and August 23rd (orange,  $t=418$  days) of the second year.

220 Here, we have exploited this relationship to understand the seasonal variability of the  
 221 scaling of the structure functions in the California Current System.

222 Two main results have been reported in this study. First, there is a seasonal vari-  
 223 ability in the value of the most singular (the smallest) singularity exponent  $h_\infty$ , which  
 224 is well correlated with the evolution of the temperature anomaly associated with the up-  
 225 welling  $\delta T_{upwelling}$ . Moreover, it has been observed that, for strong upwelling events, the  
 226 strongest fronts are located close to the coast, while for weak upwelling events they can  
 227 also be located offshore, confirming then, that the strongest fronts are generated by the  
 228 upwelling process. The second main result is the existence of a linear correlation between  
 229 the anomalous scaling of the structure functions measured by  $\Delta h^-$  and the most sin-  
 230 gular exponents  $h_\infty$ . With the interpretation of singularity exponents as normalized mea-  
 231 sures of front intensity in mind, our results imply that anomalous scaling are linearly anti-  
 232 correlated to the intensity of the strongest fronts. Putting these two results together, it  
 233 implies that some statistical properties of the flow in the area under study, including the  
 234 spectral slopes of SST, are correlated to the intensity of the upwelling.

235 An important question that emerges is whether the correlation and slope between  
 236  $h_\infty$  and  $\Delta h^-$  is universal. A preliminary answer would be positive for two main reasons:  
 237 the same correlation between  $h_\infty$  and  $\Delta h^-$  has been found for different variables, SST  
 238 and velocities; and it has been found in regions with different dynamical regimes, the Cal-  
 239 ifornia Current System and the Gulf stream (see Isern-Fontanet & Turiel, 2021). To con-

240 firm this answer it would be necessary to analyse global numerical simulations (Su et al.,  
241 2020) or observations (Merchant et al., 2019). However, before using SST measurements,  
242 it is necessary to address the problems generated by data gaps due to cloud coverage (Isern-  
243 Fontanet et al., 2021); the masking out of strong fronts by the failure of cloud mask al-  
244 gorithms (Kilpatrick et al., 2019); and the changes in  $\Delta h^-$  induced by noise (Isern-Fontanet  
245 & Hascoët, 2014). Among them, the most critical problem is the masking of strong fronts  
246 because it has a direct impact on the estimation of  $h_\infty$  and, thus,  $\Delta h^- = h_d - h_\infty$ .  
247 Besides,  $h_\infty$ ,  $\Delta h^-$  and the slope between them could be used to validate ocean models  
248 and compare models with data. These variables are, in principle, independent of the res-  
249 olution and the algorithms for computing singularity exponents are robust (Pont et al.,  
250 2013).

## 251 **6 Conclusions**

252 Singularity exponents provide a measure of the intensity of SST fronts that can be  
253 connected to the scaling of the structure function and the spectral slope of SST through  
254 the singularity spectrum. When analysing the numerical simulations of the California  
255 Current System, results show that the intensity of the most singular fronts is correlated  
256 to the anomalous scaling of the structure functions. These fronts, at their turn, are closely  
257 related to the seasonal change of intensity of the coastal upwelling characteristic of this  
258 area. Our study points to the need to correctly reproduce the intensity of the strongest  
259 fronts and, consequently, properly model processes such as coastal upwelling in order to  
260 reproduce correctly SST statistics in ocean models.

## 261 **7 Open Research**

262 The details of the model configuration, as well as, the simulated Sea Surface Tem-  
263 peratures generated for this study and the singularity analysis described in Section 3 are  
264 available in <https://doi.org/10.20350/digitalCSIC/14487> (Isern-Fontanet et al., 2022).

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