

# Lattice Boltzmann simulation of convection-diffusion to model viscous fingering with polymer additives



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## Introduction

Two-phase flow in porous media has a rich phenomenology with scientific and economic significance. When a fluid invades a porous matrix filled with a higher viscosity fluid above a critical capillary number, patterns of viscous fingering occur. This phenomenon has immense significance to Enhanced Oil Recovery (EOR) where water is injected into injection wells to help push out oil from an adjacent production well. This is because the viscous fingering leads to a low saturation and hence low Recovery Factor (RF) at *breakthrough* when the water reaches the production well. The degree of viscous fingering strongly depends on the viscosity ratio  $M$  between the two fluids, with strong viscous fingering for  $M \ll 1$  and minimal viscous fingering for  $M \geq 1$ . As such, polymer additives can be used to increase  $M$  and thus improve recovery rates. We adapt the Lattice Boltzmann Method to model a solute representing polymer additives in the injected fluid to enable studies of the use of polymer additives.

## The colour gradient Lattice Boltzmann Method for two phase flow

The Rothman-Keller colour gradient Lattice Boltzmann Method (LBM) can simulate immiscible two phase flow of fluids with high viscosity contrasts, variable interfacial tension and any wetting angle. It models particle distributions denoted  $f_{\alpha}^k$  of two fluids moving and colliding on a discrete lattice where  $\alpha$  is the direction of movement on a square lattice. The macroscopic density of the two fluids  $\rho_k$  ( $k = 1, 2$ ), total density  $\rho$  and macroscopic momentum  $\rho u$  are given by

$$\rho_k = \sum_{\alpha} f_{\alpha}^k, \quad \rho = \sum_k \rho_k, \quad \rho u = \sum_k \sum_{\alpha} f_{\alpha}^k c_{\alpha}.$$

There are three steps in this method: (1) streaming (movement), (2) collision and (3) recolouring.

### 1. The streaming step

The streaming step is given by

$$f_{\alpha}^k(x, t) = f_{\alpha}^k(x - c_{\alpha} \Delta t, t - \Delta t),$$

where  $c_{\alpha}$  is the velocity vector with  $\alpha = 0, \dots, 8$  which moves  $f_{\alpha}^k$  one lattice spacing per time step.

### 2. The collision step

The collision step is written as

$$f_{\alpha}^{k*} = f_{\alpha}^k + (\Delta f_{\alpha}^k)^1 + (\Delta f_{\alpha}^k)^2,$$

where  $*$  denotes post collision distributions, the first term  $(\Delta f_{\alpha}^k)^1$  accounts for particle collisions, and the second term  $(\Delta f_{\alpha}^k)^2$  adds an adhesive force to model interfacial tension. The first term is given by

$$(\Delta f_{\alpha}^k)^1 = \frac{1}{\tau} (f_{\alpha}^{k,eq} - f_{\alpha}^k)$$

where  $\tau$  is the relaxation time and  $f_{\alpha}^{k,eq}(x, t)$  is the usual LBM equilibrium distribution aside from the rest velocity factor. The relaxation time  $\tau$  relates to fluid viscosity  $\nu$  via  $\tau = \nu / (c_s^2 \Delta t) + 0.5$  where  $c_s = s / \sqrt{3}$  is the soundspeed in the lattice and  $s = \Delta x / \Delta t$ . The second term is calculated as

$$(\Delta f_{\alpha}^k)^2 = A |g| (w_{\alpha} (\cos(\lambda_{\alpha}) |c_{\alpha}|)^2 - B_{\alpha}),$$

where  $w_{\alpha}$  are the LBM weights,  $g$  is the colour gradient,  $\lambda_{\alpha}$  is the angle between  $g$  and  $c_{\alpha}$ ,  $B_{\alpha}$  are constants which obtain the correct surface tension, and  $A$  is a parameter that relates to surface tension.

### 3. The recolouring step

The recolouring step is achieves separation of the two fluids and is given by

$$f_{\alpha}^k = \frac{\rho_k}{\rho} f_{\alpha}^{k*} \pm \beta \frac{\rho_1 \rho_2}{\rho^2} f_{\alpha}^{eq}(\rho, u = 0) \cos(\lambda_{\alpha}),$$

where  $f_{\alpha}^{k*} = \sum_k f_{\alpha}^{k*}$ , the  $+$  sign is for  $k = 1$ , the  $-$  sign for  $k = 2$ ,  $\beta \in [0, 1]$  is an adjustable parameter relating to interfacial thickness, and  $f_{\alpha}^{eq}(\rho, u = 0) = w_{\alpha} \rho$ .

## The convection diffusion Lattice Boltzmann Method for two phase flow

We adapt the standard colour-gradient LBM by modelling a third number density denoted  $f_{\alpha}^c$  representing the number density for the concentration of an additive (eg. polymer) in fluid 1 ("the solute"). Namely, we model the streaming step of the fluid concentration number density using

$$f_{\alpha}^c(x, t) = f_{\alpha}^c(x - c_{\alpha} \Delta t, t - \Delta t),$$

and the collision step using

$$f_{\alpha}^{c*}(x, t) = f_{\alpha}^c(x, t) + \Delta f_{\alpha}^c, \quad (1)$$

where  $f_{\alpha}^{c*}$  is the number density after collision and  $\Delta f_{\alpha}^c$  is the collision term given by

$$\Delta f_{\alpha}^c = \frac{1}{\tau_c} (f_{\alpha}^{c,eq}(x, t) - f_{\alpha}^c(x, t)) \quad (2)$$

## Convection-diffusion LBM for two phase flow continued ...

where  $\tau_c$  is the relaxation time for the concentration and  $f^{c,eq}$  is the standard equilibrium distribution for the LBM. The relaxation time for the solute concentration is given by  $\tau_c = \nu_c / (c_s^2 \Delta t) + 0.5$  where  $\nu_c = \kappa$  is the diffusivity of the concentration and can be calculated from a knowledge of the Peclet number which relates to the rate of advection to diffusion and is given by  $\nu_c = uL / Pe$  where  $Pe$  is the Peclet number,  $u$  is the fluid velocity, and  $L$  is the scale length. The macroscopic concentration is given by  $C = \sum_{\alpha} f_{\alpha}^c$ , and in following, we use  $C \in [0, 1]$ .

The above LBM equations model the convection diffusion equation in the macroscopic limit given by

$$\frac{\partial C}{\partial t} + u \cdot \nabla C = \kappa \nabla^2 C.$$

Polymer additives are known to increase the viscosity of water. As such, these additives decrease viscous fingering which in turn increases the recovery factor (fraction of oil recovered from an oilfield) using EOR via water flooding = injecting water to help evacuate the oil within an oilfield). We set the Peclet number to be  $Pe = 50$  which is a typical value for polymer additives in water = fluid 1. In our scheme, the solute concentration is modelled everywhere within all fluid regions (fluid 1 = water and fluid 2 = oil) and we use the standard LBM bounce-back scheme at solid boundaries. To avoid leakage of the solute between disconnected regions of fluid 1 such as may occur at finger pinchouts, we set the diffusivity to be very small when there was a negligible fraction of fluid 1. Specifically, when  $\rho_1 / \rho < 0.01$  we set the diffusivity to  $\nu_c = uL / Pe^{high}$  where  $Pe^{high}$  is set to a much larger Peclet number than that of fluid 1. In the following, we used  $Pe^{high} = 1000 \gg Pe = 50$ .

The additive concentration  $C$  is then used to adjust the viscosity of fluid 1. Specifically, we set

$$\nu_1 = \nu_1^0 + (\nu_2 - \nu_1^0) C,$$

where  $\nu_1^0$  is the viscosity of pure water. We then calculate the relaxation time for fluid 1 (the solvent) using  $\tau_1 = \nu_1 / (c_s^2 \Delta t) + 0.5$ . This approach means that in our model,  $C = 1$  means we have raised the polymer concentration to a level where the viscosity of water (fluid 1) is equal to that of the oil (fluid 2). Hence, when  $C = 1$ , we can expect little viscous fingering because the viscosity ratio  $M = \nu_1 / \nu_2$  is unity, whereas for  $C = 0$ , we can expect significant viscous fingering since  $M = \nu_1 / \nu_2$  will be small, for example,  $\sim 0.01$  for oil. This is an idealized case to test the method.

## Results: viscous fingering versus stable displacement (base case)

We conducted numerical experiments in which we invade a 2D porous matrix initially saturated with fluid 2 representing oil, with fluid 1 representing water). Before we conduct experiments in which the water can have an additive (polymer) which increases its viscosity for a specified period of time, we ran reference experiments. Namely, we conducted two experiments with viscosity ratios  $M = \nu_1 / \nu_2$  of  $M = 0.01$  (cf. water injected into an oil saturated matrix), and  $M = 1$  (ie. water with a polymer additive which raises its viscosity to the same values as that of oil). Figure 1 shows snapshots for first case ( $M = 0.01$ ) and exhibits significant viscous fingering, whereas Figure 2 shows snapshots for the second case ( $M = 1$ ) and shows stable displacement as expected.

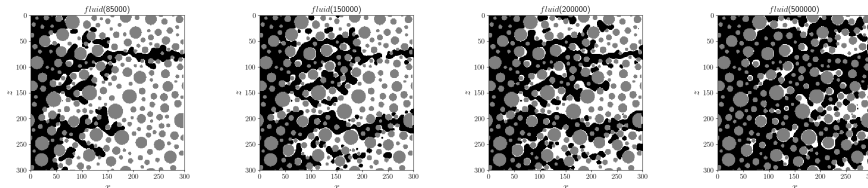


Figure 1: Snapshots for the case of a non-wetting invading fluid with a viscosity ratio of  $M = 0.01$ . Black regions are fluid 1, white regions are fluid 2, and grey regions are solid grains. Breakthrough occurred at a time of  $t = t_{br} = 84360$ .

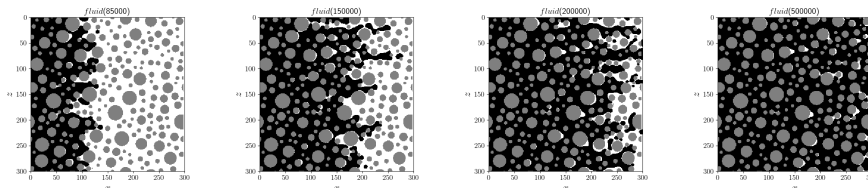


Figure 2: Snapshots at various times for the case of a non-wetting invading fluid with a viscosity ratio of  $M = 1$ . Black regions are fluid 1, white regions are fluid 2, and grey regions are solid grains.

## Results: effect on viscous fingering of polymer injection

In EOR using water flooding, polymer additives can be used to decrease viscous fingering and hence, increase the oil recovery. These tend to be used either after breakthrough when water first reaches the production well, or from the start of operations. To illustrate the two phase LBM with convection diffusion, we ran two simulations of these two cases.

Figure 3 shows polymer concentration for the case of using polymer additives **after** a specified time of  $t = t_{br} = 84360$  (the breakthrough time) for a duration of  $\Delta t = t_{br}$ . In contrast, Figure 4 shows polymer concentration for the case of using polymer additives from  $t = 0$  for the same duration.

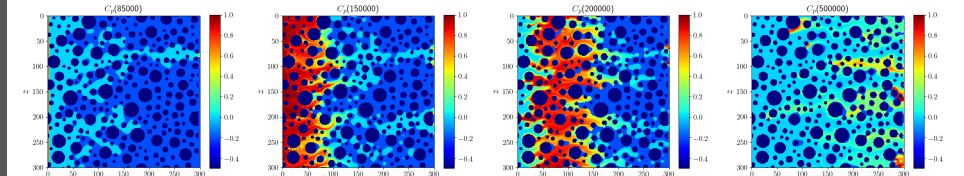


Figure 3: Snapshots at various times of the polymer concentration  $C$  for the case of the polymer being injected after a time of  $t = t_{br}$  for a duration of  $\Delta t = t_{br}$ . Light blue shows the injected fluid with  $C = 0$ , red shows the injected fluid for  $C = 1$ , dark blue depicts fluid 2 (the displaced fluid representing oil), and the darkest blue shows the grains.

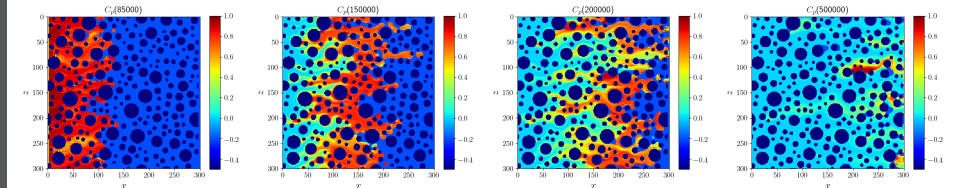


Figure 4: Snapshots at various times of the polymer concentration  $C$  for the case of the polymer being injected after a time of  $t = 0$  for a duration of  $\Delta t = t_{br}$ . Light blue shows the injected fluid with  $C = 0$ , red shows the injected fluid for  $C = 1$ , dark blue depicts fluid 2 (the displaced fluid representing oil), and the darkest blue shows the grains.

The above plots clearly show that when polymers are used from the start of the simulation, viscous fingering is greatly decreased relative to when polymers are used after the breakthrough time. As such, the case of using polymers from the start has increased sweep (flow efficiency) and hence, increased saturation = recovery factor, relative to the case of polymer injection after breakthrough.

To quantify the effect on saturation (ie. recovery factor) of using polymer additives in water flooding, Figure 5 shows plots of the saturation versus time for the four different cases studied above. One observes that the reference case of injecting pure water (cf. Figure 1), the saturation rate (= production rate) drops suddenly after breakthrough and then decreases with time. The case when a polymer additive is used after breakthrough for a duration of  $\Delta t = t_{br}$  (cf. Figure 3), the production rate increases again some time after breakthrough. In contrast, the case when a polymer additive is used before breakthrough for the same time (cf. Figure 4), the production rate remains constant until the saturation reaches about 90%.

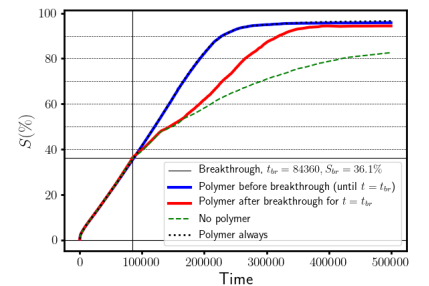


Figure 5: Plots showing saturation as a function of time for the four different cases of injection of: (1) pure water, (2) water with polymer additives always, (3) polymer additives from  $t = t_{br}$  for a duration of  $\Delta t = t_{br}$ , and (4) polymer additives from  $t = t_{br}$  for a duration of  $\Delta t = t_{br}$ .

## Conclusions

The two-phase colour gradient LBM is adapted to model convection-diffusion in fluid 1 to enable the effect on viscous fingering of polymer additives to be studied. We conduct simulations within a 2D model pore matrix initially saturated with a high viscosity fluid (cf. oil), which is injected from the left with a lower viscosity fluid (cf. water), thus representing a simplified model of EOR by water flooding. We test two cases being use of a polymer after breakthrough and before breakthrough and find that the recovery factor (RF) is much higher for the "before breakthrough" case.