Semi-analytic approach to the background evolution of $f(R)$-gravity theories in the metric formalism

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Abstract

Context: ... 

Aims: An approximation of the modified Friedmann equation in the metric $f(R)$-gravity context is proposed. 

Methods: It is applied the Differential Transformation Method to obtain the approximation to the modified Friedmann equations. 

Results: ... 

Conclusions: ... 

Introduction

... 

Background evolution for $f(R)$-gravity in the metric formalism

... 

The Differential Transformation Method

The differential transform of some function $f(x)$ is given by

\begin{equation}
F(k) \equiv D[f(x)](k) = \frac{1}{k!} \left. \frac{d^k f(x)}{dx^k} \right|_{x_i}.
\end{equation}

(1)
The inverse differential transform is then

\[
f(x) = D^{-1}[F(k)](x) = \sum_{k=0}^{\infty} F(k)(x-x_i)^k = \sum_{k=0}^{\infty} \frac{(x-x_i)^k}{k!} \frac{d^k f(x)}{dx^k} |_{x_i}.
\]  

(2)

It can be clearly seen that the DTM is the Taylor expansion of a function \( f(x) \) around \( x_i \). In practice, since it is impossible to extend the summation to \( \infty \), one must break the summation at some desired \( N \) terms so that the expansion above converges. Table 1 shows some properties and particular cases of the differential transform.

<table>
<thead>
<tr>
<th>Function ( f(x) )</th>
<th>Transform ( F(k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha u(x) ) + ( \beta v(x) )</td>
<td>( \alpha U(k) + \beta V(k)^1 )</td>
</tr>
<tr>
<td>( \frac{d^n u(x)}{dx^n} )</td>
<td>( \frac{(k+n)!}{(k+n)^n} U(k+n) )</td>
</tr>
<tr>
<td>( u_1(x) u_2(x) \ldots u_{n-1}(x) u_n(x) )</td>
<td>( \sum_{k_{n-1}=0}^{\infty} \sum_{k_{n-2}=0}^{k_{n-1}} \ldots \sum_{k_2=0}^{k_3} \sum_{k_1=0}^{k_2} U(k_1) U_2(k_2-k_1) \ldots U_{n-2}(k_{n-2}-k_{n-3}) U_n(k-k_{n-1}) )</td>
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<td>( x^n )</td>
<td>( \frac{1}{(n-k)!} x^{n-k} ) for ( n, k = 0, 1, \ldots ) and ( n = k = n ) and ( k &gt; n ), respectively(^2)</td>
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Apply DTM to the modified Friedmann equation

The transformed terms up to \( k = 3 \) are:

\[
U(0) = u_i
\]  

(3)

\[
U(1) = u_i^{(4)}
\]

\[
U(2) = -\frac{1}{108x_0^8 U(0) D[f_v u](0)} \left\{ 4x_0^3 D[f](0) + x_0^3 D[f](1) - 3x_0^3 \left[ 8U(0) + 7x_0 U(1) + 2x_0^2 U(2) \right] D[f_v u](0) - 3x_0^4 [2U(0) + x_0 U(1)] \right\}
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\[
V(k) = 12U(k) + 3 \sum_{k_i=0}^{k} \frac{x_i^{1-k_i}}{(1-k_i)!k_i!} (k - k_i + 1) U(k - k_i + 1)
\]  

(7)

\[
V(1) = 15U(1) + 6x_0 U(2)
\]  

(8)

\(^1\)\( \alpha \) and \( \beta \) are constants.

\(^2\)The result is \( \delta(k) \) when \( n = 0 \), where \( \delta(k) \) is the Kronecker delta symbol.
\[ D[f](1) = V(1)f_v(v_i) \]  
\[ D[f_v](1) = V(1)f_{vv}(v_i) \]  
\[ D[f_{vv}](1) = V(1)f_{vve}(v_i) \]  

Some \( f(R) \) examples

...  

Conclusions

...