Semi-analytic approach to the background evolution of $f\,(R)$-gravity theories in the metric formalism

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Abstract

Context: ...

Aims: An approximation of the modified Friedmann equation in the metric $f(R)$-gravity context is proposed.

Methods: It is applied the Differential Transformation Method to obtain the approximation to the modified Friedmann equations.

Results: ...

Conclusions: ...

Introduction

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Background evolution for $f(R)$-gravity in the metric formalism

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The Differential Transformation Method

The differential transform of some function $f(x)$ is given by

$$F(k) \equiv D[f(x)](k) = \frac{1}{k!}\left.\frac{\mathrm{d}^k f(x)}{\mathrm{d}x^k}\right|_{x_i}. \tag{1}$$

The inverse differential transform is then

$$f(x) = D^{-1}[F(k)](x) = \sum_{k=0}^{\infty} F(k)(x - x_i)^k = \sum_{k=0}^{\infty} \frac{(x - x_i)^k}{k!} \left. \frac{\mathrm{d}^k f(x)}{\mathrm{d}x^k} \right|_{x_i} . \tag{2}$$

It can be clearly seen that the DTM is the Taylor expansion of a function $f(x)$ around x_i. In practice, since it is impossible to extend the summation to ∞, one must break the summation at some desired N terms so that the expansion above converges. Table 1 shows some properties and particular cases of the differential transform.

Table 1: Some DTM properties.

Function $f(x)$	Transform $F(k)$
$\alpha u(x) + \beta v(x)$	$\alpha U(k) + \beta V(k)$[1]
$\frac{\mathrm{d}^n u(x)}{\mathrm{d}x^n}$	$\frac{(k+n)!}{k!} U(k+n)$
$u_1(x) u_2(x) \dots u_{n-1}(x) u_n(x)$	$\sum_{k_{n-1}=0}^{k} \sum_{k_{n-2}=0}^{k_{n-1}} \dots \sum_{k_2=0}^{k_3} \sum_{k_1=0}^{k_2} U_1(k_1) U_2(k_2 - k_1) \dots U_{n-2}(k_{n-2} - k_{n-1}) U_n(k - k_{n-1})$
x^n	$\frac{n!}{(n-k)!k!} x_i^{n-k} (= x_i, 1 \text{ and } 0 \text{ for } k = 0, k = n \text{ and } k > n, \text{ respectively})$[2]

Applying DTM to the modified Friedmann equation

The transformed terms up to $k = 3$ are:

$$U(0) \quad = u_i \tag{3}$$

U(1) = $u_i^{\prime}$(4)

U(2) = - $\frac{1}{36 x_i^6 U(0) D[f_{vv}](0)} \left\{ x_i^4 D[f](0) - 3 x_i^4 \left[x_i U(1) + 2U(0) \right] D[f_v](0) + 90 x_i^5 U(0) U(1) D[f_{vv}](0) - 6(\Omega_{\mathrm{m},0} x_i + \Omega_{\mathrm{r},0}) \right\}$ (5)

U(3) = - $\frac{1}{108 x_i^6 U(0) D[f_{vv}](0)} \left\{ 4 x_i^3 D[f](0) + x_i^4 D[f](1) - 3 x_i^3 \left[8U(0) + 7 x_i U(1) + 2 x_i^2 U(2) \right] D[f_v](0) - 3 x_i^4 \left[2U(0) + x_i U(1) \right] \right.$

$$V(k) = 12U(k) + 3 \sum_{k_1=0}^{k} \frac{x_i^{1-k_1}}{(1-k_1)! k_1!} (k - k_1 + 1) U(k - k_1 + 1) \tag{7}$$

$$V(1) = 15U(1) + 6 x_i U(2) \tag{8}$$

[1] α and β are constants.
[2] The result is $\delta(k)$ when $n = 0$, where $\delta(k)$ is the Kronecker delta symbol.

$$D[f](1) = V(1) f_v(v_i) \tag{9}$$
$$D[f_v](1) = V(1) f_{vv}(v_i) \tag{10}$$
$$D[f_{vv}](1) = V(1) f_{vvv}(v_i) \tag{11}$$

Some $f(R)$ examples

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Conclusions

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