Approximate Analytical and Numerical Solution of Three-dimensional MHD Flow of an Incompressible Fluid through Porous Media over a Stretching Sheet

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Abstract

The current study focuses on approximate analytical solution of three-dimensional MHD flow of an incompressible fluid through porous media over a stretching sheet using Exponentially decreasing series. The governing equations are transformed into a system of nonlinear ordinary differential equations with boundary conditions using the similarity transformation. It has been attempted to show the reliability and performance of the Dirichlet series in comparison with Direct Numerical Method (DNM). For various flow parameter values, the resulting quantities such as velocity profiles and skin friction coefficient are also geometrically presented.

Keywords: Stretching ratio, Porous, MHD, Suction/Injection, Dirichlet Series, Shooting technique, Runge-Kutta method..

1 Introduction

The flow past a stretching sheet has many applications such as manufacturing industry, petroleum industries, geothermal energy extractions, glass fiber production metal and polymer processing industries. Sakiadis [1] was first to study flow over a stretching sheet. Many researchers have studied the

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various aspects of stretching sheet, ever since the notable works of Crane [2]. The effect of MHD along with the suction over stretching sheet flow was discussed by Gupta and Gupta [3]. Later many papers were published based on Crane's problem with suction, magnetic-field, visco-elasticity of the fluid (Andersson [4], Ariel [5] etc.). Fang and Zhang [6] proposed the exact solution of MHD viscous flow over a Shrinking sheet. Later Ellahi and Hammed [7] discussed the numerical solution of steady, flows with heat transfer, MHD and nonlinear slip effects.

The results so far discussed are of two dimensional flow. If the flow becomes axisymmetric, the flow will be three dimensional flow. The three dimensional stretching sheet problem was studied numerically by Wang [8]. Later Ariel [9] obtained semi analytical solution of generalized three dimensional flow over a stretching sheet. A novel work has been carried out by Arriel([10], [11]) on three-dimensional stretching sheet problems. An analytical solution of three-dimensional flow over a stretching surface in a visco-elastic fluid is given by Hayat et. al. [12]. Nazar and Latip [13] solved the three-dimensional boundary layer flow due to a stretching surface in a visco-elastic fluid numerically.

Later the numerical study of three-dimensional MHD flow over a stretching sheet is carried out using Legendre pseudo-spectral method by Heydari, Loghmani and Dehghan [14]. Our attempt in this present paper is to discuss the effect of MHD on three-dimensional flow of an incompressible fluid through porous media over a stretching sheet.

However, to the best of author's knowledge, no attempt has been made to investigate the effects of porosity on three-dimensional MHD flow of an incompressible fluid through porous media over a stretching sheet. Being motivated by the wide range of applications, this paper analyzes the effect of suction/injection and stretching ratio on three-dimensional MHD flow of an incompressible fluid through porous media over a stretching sheet.

The remaining part of the paper is structured out as follows: In Section 2, we describe the mathematical modelling of the problem and derive its governing equations. The Dirichlet series is introduced and is applied to the coupled nonlinear ordinary differential equation in Section 3. In Section 4, the numerical algorithm is described. The solution nature has also been discussed briefly in Section 5. Final Section summarizes the method's relevance as well as the impact of various physical factors on velocity profiles and skin friction coefficient.

2 Formulation of the problem

We consider the steady, laminar flow of an incompressible electrically conducting fluid through porous media over a stretching surface in plane $z = 0$. Let u, v and w be the velocity components along x, y and z direction respectively. A uniform magnetic field B_0 is applied in the $z-$ direction. The magnetic Reynolds number is taken to be small, so that induced magnetic field is neglected. Under usual boundary layer approximations, the continuity and momentum equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1a}$$

$$\frac{1}{\epsilon^2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho\epsilon}u - \frac{\nu}{k_0}u, \tag{1b}$$

$$\frac{1}{\epsilon^2}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \nu\frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho\epsilon}v - \frac{\nu}{k_0}v, \tag{1c}$$

Where (u, v, w) are velocity components along (x, y, z) directions respectively. ρ is the density of the fluid, μ is the dynamic viscosity, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, ϵ is the porosity, σ is the electrical conductivity, B_0 is the magnetic induction and k_0 is porous medium permeability.

The boundary conditions are given by

$$\begin{aligned} &\text{at} \quad z = 0: \quad u = U(x), \quad v = V(y), \quad w = -w_0, \quad \text{and} \\ &\text{as} \quad z \to \infty: \quad u \to 0 \quad \text{and} \quad v \to 0, \end{aligned} \tag{2}$$

where $U(x) = U_\infty x$ and $V(y) = V_\infty y$ are the velocities at the surface along x and y directions respectively, w_0 is the suction or injection parameter.

Using the similarity transformations

$$u = U_\infty x f'(\eta), \qquad v = U_\infty y g'(\eta), \qquad w = -\sqrt{U_\infty \nu \epsilon^2}[f(\eta) + g(\eta)], \tag{3}$$

with $\eta = \sqrt{\frac{U_\infty}{\nu\epsilon^2}}z$, the system (1) and (2) reduces to coupled nonlinear ordinary differential equations:

$$f''' + (f+g)f'' - f'^2 - M^2 f' - \Omega f' = 0, \tag{4a}$$

$$g''' + (f+g)g'' - g'^2 - M^2 g' - \Omega g' = 0, \tag{4b}$$

with the boundary conditions

$$f(0)+g(0) = R, \quad f'(0) = 1, \quad g'(0) = \frac{V_\infty}{U_\infty} = \beta \quad f'(\infty) = g'(\infty) = 0. \tag{5}$$

Here the parameter $M \left(= B_0\sqrt{\sigma\epsilon/\rho U_\infty}\right)$ is the magnetic (Hartman number) parameter, $\beta \left(= \frac{V_\infty}{U_\infty}\right)$ is the dimensionless stretching ratio, $\Omega \left(= \frac{\epsilon^2\nu}{U_\infty k_0}\right)$ is the porosity parameter and $R \left(= \frac{w_0}{\sqrt{U_\infty \nu \epsilon^2}}\right)$ is the suction and injection parameter. The above system of ordinary differential equations (4) with the boundary conditions (5) are solved using shooting technique with fourth order Runge-Kutta integration scheme.

3 Dirichlet Series Solution

Following a thorough examination of the derivative boundary condition, we seek the Dirichlet series solution, which ideally assures us for such a boundary condition. The Dirichlet series process is an efficient way to tackle certain types of boundary value problems. In comparison to a Direct Numerical Method (DNM), the suggested approach is more flexible and efficient in computer implementation. The procedure used here takes very little computer time and storage to find the values of the unknowns, whereas the usual numerical methods are used to take much more computer time. Kravenchenko and Yablonskii [15] were the first to use the Dirichlet series to solve boundary value problems where the derivative boundary condition is zero at infinity. Many researchers have used the Dirichlet series process to solve Stretching sheet type problems due to its widespread application. Kudenatti et al. [16] applied the Dirichlet series to investigate a class of boundary layer equations over a nonlinear stretching surface. N Mahesha [19] has proposed a new exponentially decreasing series solution for the coupled nonlinear boundary value problem (BVP). Exploring the Dirichlet series, they successfully solved it and compared it to the numerical results, discovering that the results agree. A broad description of the Dirichlet series and its convergence may be found in Riesz [17] and, Sachdev et al [18]. We choose the base function for the above equations (4a) and (4b) in the form,

$$f = \sum_{n=0}^{\infty} a_n e^{-nh\eta} \tag{6a}$$

$$g = \sum_{n=0}^{\infty} b_n e^{-nh\eta} \tag{6b}$$

where a_n and b_n are unknown constants to be determined. Also the above base functions automatically satisfy the last condition in (5). Substituting functions (6a) & (6b) in the Boundary value problem (4a) & (4b) we get the following recurrence relations,

$$
\begin{aligned}
&-h^3\sum_{n=1}^{\infty} n^3 a_n e^{-nh\eta} + (a_0+b_0)h^2\sum_{n=1}^{\infty} n^2 a_n e^{-nh\eta} + h^2\sum_{n=2}^{\infty}\sum_{i=1}^{n-1} i^2 a_i a_{n-i} e^{-nh\eta}\\
&+h^2\sum_{n=2}^{\infty}\sum_{i=1}^{n-1} i^2 a_i b_{n-i} e^{-nh\eta} - h^2\sum_{n=2}^{\infty}\sum_{i=1}^{n-1} i(n-i) a_i a_{n-i} e^{-nh\eta} + M^2 h\sum_{n=1}^{\infty} n a_n e^{-nh\eta}\\
&+\Omega h\sum_{n=1}^{\infty} n a_n e^{-nh\eta} = 0
\end{aligned} \tag{7a}
$$

$$
\begin{aligned}
&-h^3\sum_{n=1}^{\infty} n^3 b_n e^{-nh\eta} + (a_0+b_0)h^2\sum_{n=1}^{\infty} n^2 b_n e^{-nh\eta} + h^2\sum_{n=2}^{\infty}\sum_{i=1}^{n-1} i^2 b_i b_{n-i} e^{-nh\eta}\\
&+h^2\sum_{n=2}^{\infty}\sum_{i=1}^{n-1} i^2 b_i a_{n-i} e^{-nh\eta} - h^2\sum_{n=2}^{\infty}\sum_{i=1}^{n-1} i(n-i) b_i b_{n-i} e^{-nh\eta} + M^2 h\sum_{n=1}^{\infty} n b_n e^{-nh\eta}\\
&+\Omega h\sum_{n=1}^{\infty} n b_n e^{-nh\eta} = 0
\end{aligned} \tag{7b}
$$

Equating coefficient of $e^{-h\eta}$ to zero we get (8)

$$-h^2 + (a_0+b_0)h + M^2 + \Omega = 0 \tag{8}$$

Therefore, we rewrite the equations (7a) & (7b) as,

$$a_n = \left[\frac{h}{n[-h^2n^2 + (a_0+b_0)hn + M^2 + \Omega]}\right]\sum_{i=0}^{n-1} i a_i[(n-2i)a_{n-i} - b_{n-i}] \tag{9a}$$

$$b_n = \left[\frac{h}{n[-h^2n^2 + (a_0+b_0)hn + M^2 + \Omega]}\right]\sum_{i=0}^{n-1} i b_i[(n-2i)b_{n-i} - a_{n-i}] \tag{9b}$$

for $n = 2, 3, 4, \cdots$.

Riesz [17] provides a detailed convergence criteria for the above mentioned series. The skin friction coefficients $f''(0)$ and $g''(0)$ are essential physical parameters of importance, which are given by,

$$f''(0) = h^2 \sum_{n=1}^{\infty} n^2 a_n \tag{10a}$$

$$g''(0) = h^2 \sum_{n=1}^{\infty} n^2 b_n \tag{10b}$$

Furthermore, we are now working on determining the unknown constants a_0, b_0, a_1, b_1 & h. For this we use the following initial conditions,

$$f(0) + g(0) = \sum_{n=0}^{\infty} a_n + \sum b_n = R \tag{11a}$$

$$f'(0) = -h \sum_{n=1}^{\infty} n a_n = 1 \tag{11b}$$

$$g'(0) = -h \sum_{n=1}^{\infty} n b_n = \beta \tag{11c}$$

We utilise Newton's nonlinear system of equations technique to calculate these unknowns up to the appropriate degree of precision for all the vales of M, Ω, R and β, using few terms of Dirichlet series. The acquired outcomes are compared to those obtained by numerically solving the boundary value problems (4a) & (4b) with the conditions (5) and are presented in Table (1) and (2). As a further step, we'll compare the Dirichlet-series solution and exact solution obtained by Ariel [9].

4 Numerical Solution

Eqs. (4a) & (4b), along with their boundary conditions given by (5), form a nonlinear boundary value problem. As a result, the boundary value problem is first converted into an initial value problem by estimating the missing slopes correctly. For many sets of physical parameters, the resultant initial value problem is addressed using the Runge Kutta fourth order technique.

First we convert the above system of ordinary differential equations (4a) & (4b) with the boundary conditions (5) are converted into the system of first order differential equations by setting

$$f' = f_1, \qquad f_1' = f_2, \qquad f_2' = -(f+g)f_2 + f_1^2 + M^2 f_1 + \Omega f_1,$$
$$g' = g_1, \qquad g_1' = g_2, \qquad g_2' = -(f+g)g_2 + g_1^2 + M^2 g_1 + \Omega g_1, \qquad (12)$$

with the initial conditions

$$f(0) + g(0) = R, \qquad f_1(0) = 1, \qquad f_2(0) = \zeta_1,$$
$$g_1(0) = \beta \qquad g_2(0) = \zeta_2. \qquad (13)$$

We required two unknown initial conditions ζ_1 and ζ_2 to solve the system of equations (12) with (13) using the Runge-Kutta fourth order technique, but no such values are provided in the problem. The suitable values for $f''(0)$ and $g''(0)$ are chosen by shooting technique, and then the integration is carried out using Runge-Kutta fourth order method with $h = 0.001$. The above said procedure is repeated until it reaches the tolerance limit 10^{-6}.

5 Results and Discussion

In this work, Similarity transformation is used to convert the governing partial differential equations of three-dimensional MHD flow of an incompressible fluid through porous media over a stretching sheet into a system of ordinary differential equations. The Dirichlet Series was successfully used to provide an approximate analytical solution to the resultant system of Ordinary Differential Equations. The current approach minimises the computing challenges of previous methods, and all computations can be performed with simple manipulations. To evaluate the effectiveness of the current technique, we compare the values of f"(0) and g"(0) with those of the direct numerical solution of the problem, which are shown in Table 1-3. Table 1 shows that the results of the above technique compare favourably with the exact solution provided by Ariel.

Firstly we consider Figures 1, which shows that the skin-friction coefficient $f''(0)$ is plotted against the stretching ratio parameter β for different values of Magnetic number and Porous parameter. It is observed that the skin-friction coefficient $f''(0)$ decreases as β increases. Also $f''(0)$ decreases as Magnetic number increases. On the other hand the skin friction coefficient $f''(0)$ decreases as increase in Porous parameter Ω and β respectively.Figures 2 present the effects of stretching ratio parameter β, Magnetic number M and Porous parameter Ω on $g''(0)$. It is found from these two figures that, $|g''(0)|$ decreases as $\beta(< 0)$ increases, also $|g''(0)|$ increases as $\beta(> 0)$ increases. It is observed that $g''(0) > 0$ if $\beta < 0$, $g''(0) < 0$ if $\beta > 0$ and $g''(0)$ is zero when $\beta = 0$ for all values of Magnetic parameter M and Porous parameter Ω. Also it is observed that $g''(0)$ increases as increase in Magnetic parameter M and Porous parameter Ω respectively.

Figures 3 represents the effect of stretching ratio parameter β, Magnetic number M, suction/injection R and Porous parameter Ω on $f'(\eta)$. Figure 3a reveals that as β increases velocity gradient is also increases accordingly. But from figures 3c and 3d, velocity increases as suction/injection parameter R and Magnetic number M decreases respectively. Figure 3b it is clear that the velocity gradient slightly increases as decrease in porous parameter Ω.

From the figure 4a it is clear that, for $\beta > 0$ the velocity profiles decreases linearly and $g'(\eta)$ approaches to 0 asymptotically as η increases. But for the stretching ratio $\beta < 0$ the velocity profiles increases linearly and approaches to 0 as η increases. From figures 3b, 3c and 3d it is clear that the velocity gradient decreases as there is increase in R, M and Ω respectively.

6 Conclusion

In the present paper, we have consider the three dimensional incompressible, electrically conducting fluid through porous media over a stretching sheet. Using similarity transformation, the PDE's are transformed into system of third order ordinary differential equations. The set of nonlinear ordinary differential equations with boundary conditions are solved using Dirichlet series method and Shooting technique along with Fourth order Runge-Kutta method. The results are given in graphs for all physical parameters. It is observed that the skin friction coefficient $f''(0)$ increases as increase in stretching parameter β, Magnetic parameter M and Permeability parameter Ω. On the other hand, $g''(0)$ decreases (increases) as M and Ω decreases for $\beta < 0$ ($\beta > 0$) respectively.

β	$-f''(0)$			$-g''(0)$		
	Exact [10]	HPM [10]	Dirichlet	Exact [10]	HPM [10]	Dirichlet
0	1	1	1	0	0	0
0.1	1.020260	1.017027	1.020650	0.066847	0.073099	0.068091
0.2	1.039495	1.034587	1.039100	0.148737	0.158231	0.148328
0.3	1.057955	1.052470	1.057290	0.243360	0.254347	0.242681
0.4	1.075788	1.070529	1.074840	0.349209	0.360599	0.348255
0.5	1.093095	1.088662	1.091880	0.465205	0.476290	0.463986
0.6	1.109947	1.106797	1.108480	0.590529	0.600833	0.589059
0.7	1.126398	1.124882	1.124700	0.724532	0.733730	0.722839
0.8	1.142489	1.142879	1.140570	0.866683	0.874551	0.864764
0.9	1.158254	1.160762	1.156150	1.016539	1.022922	1.014440
1.0	1.173721	1.178511	1.171440	1.173721	1.178511	1.171440

Table 1: Illustrating the variation of Skin friction coefficients $-f''(0)$ and $-g''(0)$ with $\beta = M = \Omega = R = 0$ using Dirichlet series solution, HPM and Exact solutions

β	M	Ω	$-f''(0)$		$-g''(0)$	
			Dirichlet	Numerical	Dirichlet	Numerical
0.2	0.3	0.2	1.220840	1.221100	0.194263	0.194802
		0.8	1.452570	1.452610	0.250169	0.250250
		1	1.521890	1.521910	0.266177	0.266227
	0.7	0.2	1.379710	1.387290	0.233044	0.240915
		0.8	1.588130	1.588150	0.281261	0.281293
		1	1.651670	1.651680	0.295562	0.295584
	1	0.2	1.558680	1.558700	0.274578	0.274617
		0.8	1.745490	1.745500	0.316430	0.316442
		1	1.803380	1.803390	0.329178	0.329187
0.4	0.3	0.2	1.252890	1.253760	0.430912	0.432231
		0.8	1.478620	1.478770	0.533362	0.533595
		1	1.546550	1.546650	0.563380	0.563526
	0.7	0.2	1.407400	1.407650	0.501542	0.501929
		0.8	1.611600	1.611660	0.591865	0.591961
		1	1.674090	1.674130	0.619026	0.619092
	1	0.2	1.582660	1.582730	0.579221	0.579337
		0.8	1.766540	1.766560	0.658891	0.658930
		1	1.823660	1.823680	0.683361	0.683389

Table 2: Illustrating the variation of Skin friction coefficients $-f''(0)$ and $-g''(0)$ with β, M and Ω using Dirichlet series solution and Numerical method with $R = 0.1$

β	M	Ω	$-f''(0)$		$-g''(0)$	
			Dirichlet	Numerical	Dirichlet	Numerical
0.2	0.3	0.2	1.330620	1.330740	0.218491	0.218747
		0.8	1.560650	1.560670	0.273150	0.273196
		1	1.629560	1.629580	0.288914	0.288943
	0.7	0.2	1.488260	1.488300	0.256333	0.256408
		0.8	1.695450	1.695460	0.303798	0.303817
		1	1.758670	1.758680	0.317931	0.317944
	1	0.2	1.666150	1.666160	0.297200	0.297223
		0.8	1.852080	1.852090	0.338587	0.338595
		1	1.909740	1.909740	0.351221	0.351227
0.4	0.3	0.2	1.363650	1.364080	0.478176	0.478826
		0.8	1.587470	1.587560	0.578785	0.578916
		1	1.654950	1.655000	0.608412	0.608498
	0.7	0.2	1.516780	1.516920	0.547439	0.547651
		0.8	1.719590	1.719630	0.636570	0.636628
		1	1.781740	1.781760	0.663453	0.663494
	1	0.2	1.690830	1.690870	0.624067	0.624136
		0.8	1.873710	1.873730	0.702959	0.702984
		1	1.930570	1.930580	0.727232	0.727251

Table 3: Illustrating the variation of Skin friction coefficients $-f''(0)$ and $-g''(0)$ with β, M and Ω using Dirichlet series solution and Numerical method with $R = 0.3$

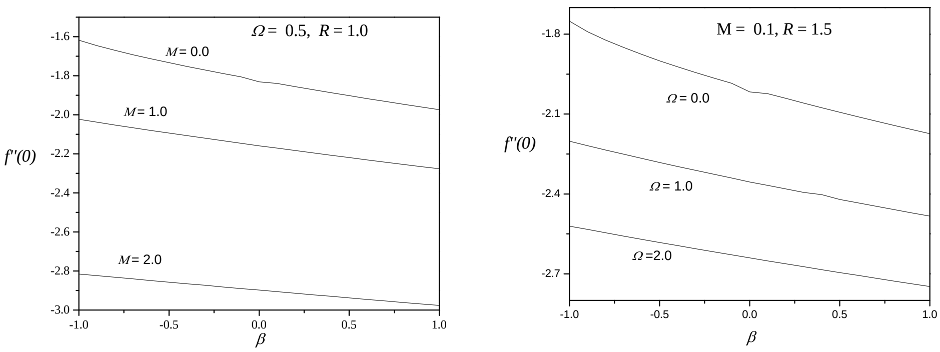

Figure 1: Skin friction coefficient $f''(0)$ against β for different values of Magnetic parameter M and Porous parameter Ω.

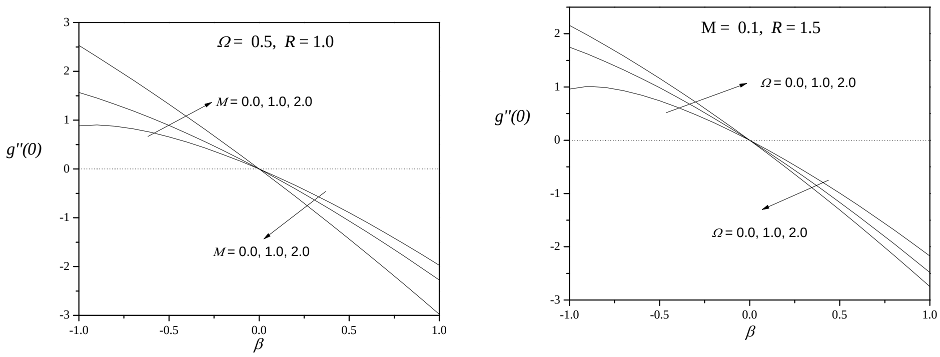

Figure 2: Skin friction coefficient $g''(0)$ against β for different values of Magnetic parameter M and Porous parameter Ω.

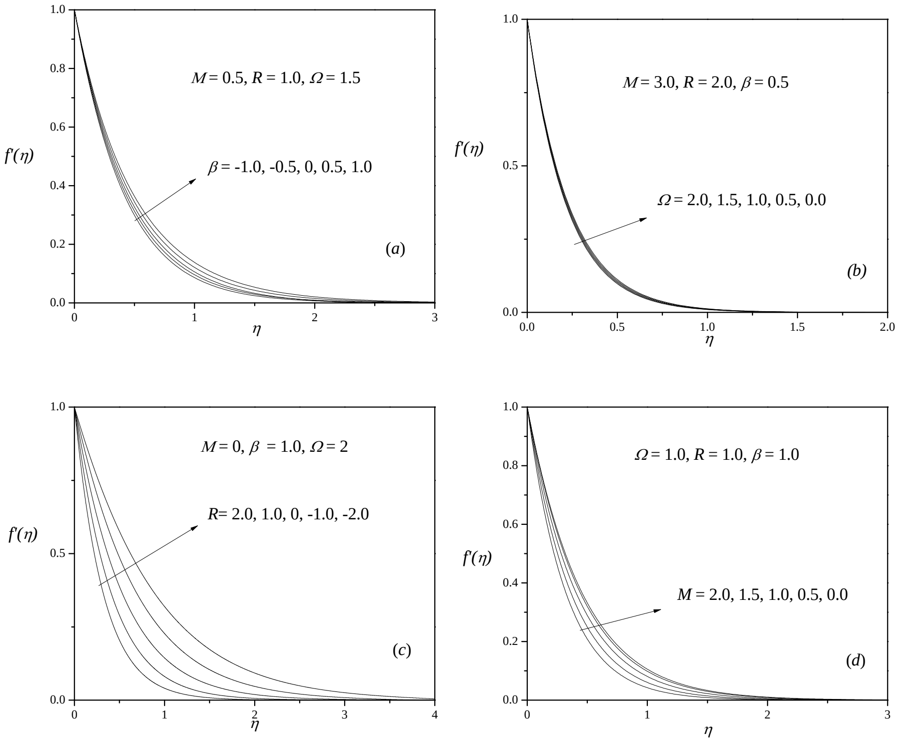

Figure 3: Velocity profiles $f'(\eta)$ for the various values of (a)Stretching ratio β (b) Porous parameter Ω, (c) Suction or Injection parameter R, (d) Magnetic parameter M.

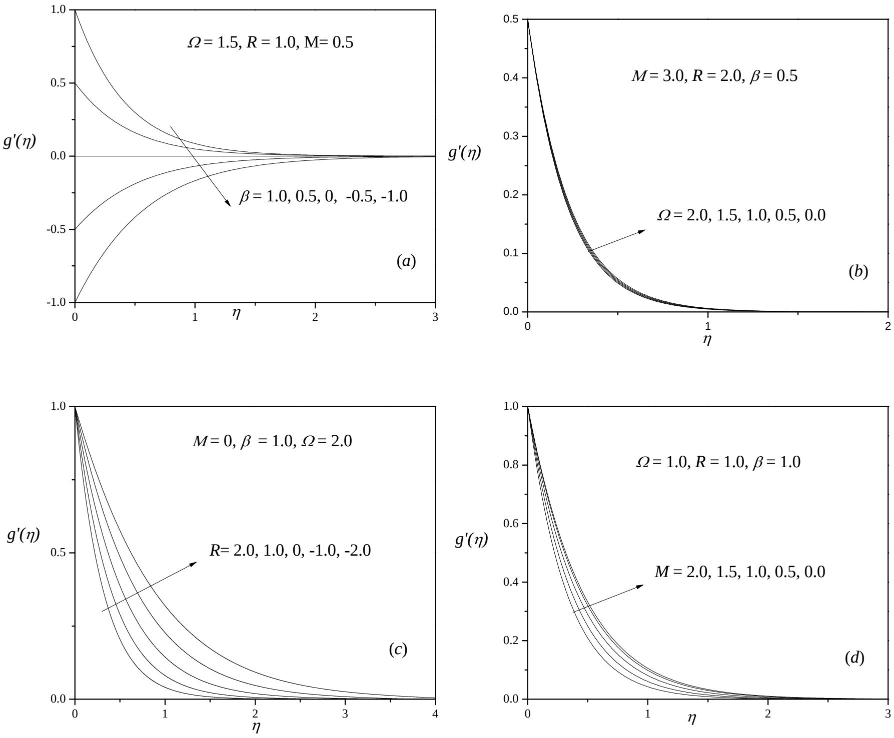

Figure 4: Velocity profiles $g'(\eta)$ for the various values of (a)Stretching ratio β (b) Porous parameter Ω, (c) Suction or Injection parameter R, (d) Magnetic parameter M.

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