# Convergence Rate of SMI Filter in Pulse Interference Environment

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This paper analyzes the convergence rate of sample matrix inversion (SMI) filter used by array receivers in pulse interference environment. Firstly, the convergence characteristics of SMI filter under pulse interference are deduced, and the relationship between the convergence speed and length of training samples and duty of interference is given. Then, the results of signal simulation are consistent with those of numerical analysis, which verifies the correctness of theoretical analysis. The results show that the convergence speed of SMI filter decreases under pulse interference, and the SMI filter needs more training samples to suppress the pulse interference effectively.

*Introduction:* Pulse interference is one type of intentional interference from jammers[1]. As the input mutation caused by pulse interference destroys the steady state of the filter and even makes the filter hardly converge to the steady state[2], the pulse interference has a great impact on the anti-jamming processing based on recursive algorithm, such as recursive least-squares (RLS) algorithm. The focus is to catch the interference samples[3, 4] when the sample matrix inversion (SMI) algorithm was used for anti-jamming processing. Even if the interference samples are caught, the convergence characteristics of the SMI filter which uses secondary samples for training in the pulse interference environment need to be further analyzed. The convergence characteristics of SMI filter in stationary environment were first given by Reed[5], in order to make the expected loss of signal interference and noise ratio (SINR) less than 3dB, the length of training samples should be greater than $2D-3$, where $D$ is the degree of freedom (DOF) of the filter. The conlusion was widely verified[6]. Some literature have also proved that the convergence speed of SMI filter based on eigenanalysis can be faster under stationary environment[7–9]. In order to solve the problem of slow convergence speed of SMI and poor anti-jamming performance with fewer samples, diagonal loading sample matrix inversion (LSMI) filter was suggested to improve the convergence speed of the filter [10, 11]. The convergence characteristics of LSMI have been analyzed and demonstrated in theory[12]. Tang [13] studied the convergence characteristics of LSMI filter in the amplitude heterogeneous clutter environment, which assumes that the amplitude of training signal is proportional to that of signal under test. At present, there is no research report on the convergence characteristics of SMI filter under pulse interference.

The main contributions of this paper include: The distribution function of SINR loss of SMI filter which uses secondary samples for training under pulse interference is derived, and the expression of expected SINR loss with different number of samples and duty of interference is given. The numerical analysis and signal simulation with typical parameters are given, and their results are consistent. The conclusion of this paper shows that the performance of SMI filter, which uses secondary samples for training, may still deteriorate even the length of training samples is greater than the pulse period and $2D-3$.

*Model of pulse interference and array anti-jamming processing:*

*Model of pulse interference:* We assume that the pulse interference is extracted from a continuous signal with rectangular pulse. The time domain waveform is described as

$$j(t) = c(t)\cos(2\pi f_0 t)p(t) = j_c(t)p(t) \tag{1}$$

where $c(t)$ is the baseband signal, $f_0$ is the carrier frequency which is the same with the interest signal, $p(t)$ is a square wave with the expression as

$$p(t) = \begin{cases} A, & -\frac{\tau}{2}+nT_s \le t \le \frac{\tau}{2}+nT_s \; n=1,2\cdots \\ 0, & \text{else.} \end{cases} \tag{2}$$

where $A$ represents the amplitude of the rectangular pulse, here we set $A$ as 1. $\tau$ is the pulse width, and $T_s$ is the pulse period.

The spectrum of pulse interference is

$$J(f) = \sum_{n=-\infty}^{+\infty} a_n J_c(f - nf_s) \tag{3}$$

where $J_c(f)$ is the spectrum of the continuous signal $j_c(t)$, $a_n = \tau f_s \,\mathrm{sa}(\pi n f_s \tau)$, where $f_s = \frac{1}{T_s}$ and $sa(x) = \sin(x)/x$.

Since the low-pass filter exist before analog-to-digital conversion, the bandwidth of the interference entering the space-time filter is limited.

*Model of array anti-jamming processing:* Taking the array satellite navigation receiver as an example, in the signal process flow, the array receiver based on digital signal processing adds the array anti-jamming processing segment compared with the single antenna receiver. This paper mainly focuses on the impact of pulse interference on anti-jamming filter.

The filter based on SMI use different optimization criteria to solve the filter weight vector. For example, minimum variance distortionless response (MVDR) filter solves the weight vector by making the output power to 0 and constraining the satellite signal gain to 1, and the optimal weigth vector is

$$\mathbf{w}_{\text{opt}} = \frac{\mathbf{R}_y^{-1}\mathbf{a}(\theta_0)}{\mathbf{a}^{\text{H}}(\theta_0)\mathbf{R}_y^{-1}\mathbf{a}(\theta_0)} \tag{4}$$

where $\mathbf{a}(\theta_0)$ is the steer vector of the interest signal, which is calculated from the incident angle of the signal $\theta_0$ and the layout of the array. $\mathbf{R}_y$ is the covariance matrix of noise and interference, which can not be obtained accurately. Because the satellite signal power is much small than noise power, it is assumed that the sampled signal is the noise and interference signal, which means that

$$\mathbf{R}_y \doteq \mathbf{R}_j + \mathbf{R}_n = \mathbf{R}_j + \sigma^2\mathbf{I} \tag{5}$$

where $\sigma^2$ is the noise power, $\mathbf{R}_j$ represents the covariance matrix of the interference signal. Under the assumption that the input signals are stationary, the covariance matrix of the samples is used to replace the real covariance matrix in the processing.

*Convergence Rate of SMI Filter Under Pulse Interference:* With the analysis of last section, it is obvious that if the covariance matrix of the interference is accurately estimated, performances of SMI filter under pulse interference and continuous interference are the same. Next, the convergence characteristics of SMI filter under pulse interference are derived. Assuming that one or more pulse cycles are in training samples, and total interest signal power of the SMI filter output is

$$P_s = \frac{1}{Kt_s}\sum_{k=1}^{K}\left(\mathbf{v}_s^{\text{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{s}(t_l+kt_s)\right)\left(\mathbf{v}_s^{\text{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{s}(t_l+kt_s)\right)^{\text{H}} \tag{6}$$

where, $\hat{\mathbf{R}}_{ptl}$ is the estimated interference signal covariance matrix used in the $l$th data cell, which is generally calculated from the samples of the $(l-1)$th data cell and $t_l$ represents the starting time of the $l$th data cell. The interference and noise signal output power of the SMI filter is

$$\begin{aligned} P_{i+n} = &\sum_{k=1}^{K\beta}\left(\mathbf{v}_s^{\text{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{j}(t_l+kt_s)\right)\left(\mathbf{v}_s^{\text{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{j}(t_l+kt_s)\right)^{\text{H}} \\ &+\sum_{k=1}^{K}\left(\mathbf{v}_s^{\text{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{n}(t_l+kt_s)\right)\left(\mathbf{v}_s^{\text{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{n}(t_l+kt_s)\right)^{\text{H}} \end{aligned} \tag{7}$$

Assuming that the pulse duty is $\beta$, the average output SINR of all the

L data cells is

$$
\begin{aligned}
\mathrm{SINR}_a &= \frac{E(P_s)}{E(P_{i+n})} = \\
&\frac{\sum\limits_{l=1}^{L}\sum\limits_{k=1}^{K}\left(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{s}(t_l+kt_s)\right)\left(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{s}(t_l+kt_s)\right)^{\mathrm{H}}}{\sum\limits_{l=1}^{L}\left[\begin{array}{l}\sum\limits_{k=1}^{\beta K}\left(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{j}(t_l+kt_s)\right)\left(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{j}(t_l+kt_s)\right)^{\mathrm{H}}+\\ \sum\limits_{k=1}^{K}\left(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{n}(t_l+kt_s)\right)\left(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_{ptl}^{-1}\mathbf{n}(t_l+kt_s)\right)^{\mathrm{H}}\end{array}\right]} \\
&\doteq E\left(\frac{(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_p^{-1})\mathbf{R}_s(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_p^{-1})^{\mathrm{H}}}{(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_p^{-1})(\beta\mathbf{R}_c+\sigma_n^2\mathbf{I})(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}_p^{-1})^{\mathrm{H}}}\right)
\end{aligned}
\tag{8}
$$

The covariance matrix of the pulse interference in a sampling period is approximate as

$$\hat{\mathbf{R}}_p \approx \beta\hat{\mathbf{R}}_c + \sigma^2\mathbf{I} = \beta(\hat{\mathbf{R}}_c + \frac{\sigma^2}{\beta}\mathbf{I}) \tag{9}$$

where $\hat{\mathbf{R}}_c$ is the covariance matrix of interference estimated from $\beta K$ pulse samples. Without affecting the conclusion, we take the noise power as the signal power unit, set $\sigma^2$ as 1, and set $\varepsilon = \frac{1}{\beta}$, record that $\hat{\mathbf{R}} = \hat{\mathbf{R}}_c + \varepsilon\mathbf{I}$, $\mathbf{R}_2 = \mathbf{R}_c + \mathbf{I}$, $\mathbf{R}_1 = \mathbf{R}_p \doteq \beta\mathbf{R}_c + \mathbf{I}$, Then the loss of SINR caused by the error of estimated covariance is

$$\rho = \frac{\mathrm{SINR}_a}{\mathrm{SINR}_o^p} = \frac{\left|\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}^{-1}\mathbf{v}_s\right|^2}{(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}^{-1})\mathbf{R}_1(\mathbf{v}_s^{\mathrm{H}}\hat{\mathbf{R}}^{-1})^{\mathrm{H}}(\mathbf{v}_s^{\mathrm{H}}\mathbf{R}_1^{-1}\mathbf{v}_s)} \tag{10}$$

Next, the theoretical distribution of $\rho$ under different training samples number and pulse interference parameters is analyzed. Let $\mathbf{y}_k = \mathbf{R}_2^{-1/2}\mathbf{j}_k$, $\hat{\mathbf{R}}_2 = \frac{1}{\varepsilon}\mathbf{R}_2^{-1/2}\hat{\mathbf{R}}\mathbf{R}_2^{-1/2}$, then

$$\hat{\mathbf{R}}_2 = \frac{1}{K}\sum_{i=1}^{\beta K}\mathbf{y}_i\mathbf{y}_i^{\mathrm{H}} + \mathbf{R}_2^{-1} \tag{11}$$

and (10) can be written as

$$\rho = \frac{\left|\mathbf{v}_s^{\mathrm{H}}\mathbf{R}_2^{-\frac{1}{2}}\hat{\mathbf{R}}_2^{-1}\mathbf{R}_2^{-\frac{1}{2}}\mathbf{v}_s\right|^2}{\mathbf{v}_s^{\mathrm{H}}\mathbf{R}_2^{-\frac{1}{2}}\hat{\mathbf{R}}_2^{-1}\mathbf{R}_2^{-\frac{1}{2}}\mathbf{R}_1\mathbf{R}_2^{-\frac{1}{2}}\hat{\mathbf{R}}_2^{-1}\mathbf{R}_2^{-\frac{1}{2}}\mathbf{v}_s\mathbf{v}_s^{\mathrm{H}}\mathbf{R}_1^{-1}\mathbf{v}_s} \tag{12}$$

Decomposing $\mathbf{R}_1$ and $\mathbf{R}_2$ into

$$\mathbf{R}_1 = \mathbf{U}_c\left(\beta\mathbf{\Sigma}_c + \mathbf{I}_{r_c}\right)\mathbf{U}_c^{\mathrm{H}} + \mathbf{U}_n\mathbf{U}_n^{\mathrm{H}} \tag{13}$$

$$\mathbf{R}_2 = \mathbf{U}_c\left(\mathbf{\Sigma}_c + \mathbf{I}_{r_c}\right)\mathbf{U}_c^{\mathrm{H}} + \mathbf{U}_n\mathbf{U}_n^{\mathrm{H}} \tag{14}$$

Assuming that the pulse interference power is much greater than the noise power and $\beta$ is not too small, which means that $\beta\mathbf{\Sigma}_c \gg \mathbf{I}_{r_c}$. Then we have the following approximate expression

$$
\begin{aligned}
&\mathbf{R}_2^{-\frac{1}{2}}\mathbf{R}_1^{-1}\mathbf{R}_2^{-\frac{1}{2}} \approx \beta\mathbf{U}_c\mathbf{U}_c^H + \mathbf{U}_n\mathbf{U}_n^H, \\
&\mathbf{R}_2^{-\frac{1}{2}} \approx \mathbf{U}_n\mathbf{U}_n^H, \mathbf{R}_2^{-1} \approx \mathbf{U}_n\mathbf{U}_n^H
\end{aligned}
\tag{15}
$$

Let $\mathbf{C} = \left(\frac{1}{K}\sum\limits_{i=1}^{K\beta}\mathbf{y}_i\mathbf{y}_i^{\mathrm{H}} + \mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\right)^{-1}$, then

$$\rho = \frac{\left|\mathbf{v}_s^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{C}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{v}_s\right|^2}{\mathbf{v}_s^{\mathrm{H}}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{C}(\beta\mathbf{U}_c\mathbf{U}_c^{\mathrm{H}} + \mathbf{U}_n\mathbf{U}_n^{\mathrm{H}})\mathbf{C}\mathbf{U}_n\mathbf{U}_n^{\mathrm{H}}\mathbf{v}_s\mathbf{v}_s^{\mathrm{H}}\mathbf{R}_1^{-1}\mathbf{v}_s} \tag{16}$$

According to the derivation of reference [13], the distribution of $\rho$ is as follows

$$f_\rho(\rho) = \frac{\varepsilon^{r_c}\rho^{K\beta-r_c}(1-\rho)^{r_c-1}}{\mathrm{B}(r_c, K\beta-r_c+1)\left[\varepsilon+(1-\varepsilon)\rho\right]^{K\beta+1}}I(\rho\geq 0) \tag{17}$$

where $\mathrm{B}(r_c, K\beta-r_c+1) = ((r_c-1)!\,(K\beta-r_c)!)/(K\beta!)$ is Beta function, and $I()$ is the indicator function, i.e.,

$$I(x\geq 0) = \begin{cases}1, & x\geq 0\\ 0, & x<0\end{cases}$$

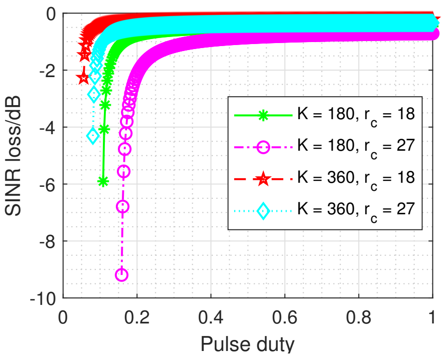


**Fig 1** *The curve of loss of SINR with different* $\beta$.

When $K\beta \geq r_c$, the expression of average SINR loss is

$$
\begin{aligned}
\mathrm{E}[\rho] &\approx \frac{\varepsilon(K\beta-r_c)}{\varepsilon K\beta-(\varepsilon-1)r_c} \\
&\times\left[1+\frac{K\beta r_c}{(\varepsilon K\beta-(\varepsilon-1)r_c)^2(K\beta-r_c-1)}\right] \\
&= \frac{K-\varepsilon r_c}{K-(\varepsilon-1)r_c}\left[1+\frac{Kr_c}{(K-(\varepsilon-1)r_c)^2(K-\varepsilon r_c-\varepsilon)}\right]
\end{aligned}
\tag{18}
$$

In the above derivation, the first assumption, i.e., $\beta\mathbf{\Sigma}_c \gg \mathbf{I}_{r_c}$, is also a necessary condition for the effectiveness of pulse interference. If $\beta\mathbf{\Sigma}_c \rightarrow \mathbf{I}_{r_c}$, which means the average power of interference is less than that of the noise, the effective jamming cannot be achieved. Then the approximation that $\mathbf{R}_p \approx \beta\mathbf{R}_c + \mathbf{I}$ is completely valid in spatial anti-jamming processing. In space-time anti-jamming processing, if the baseband of interference is wideband, according to (3) and Wiener-Khinchin theorem, it can be inferred that the approximation is also valid.

The above derivation shows that, the convergence rate is consistent with that of LSMI filter under the amplitude heterogeneous clutter environment when taking the length of interference samples as the reference variable. Combined with the implementation process of the SMI filter, the conclusion can be explained as follow. If the sampling length is long enough, the estimation of the covariance matrix of the noise signal can be approximately accurate. At this time, it can be considered that the unit matrix with amplitude $\sigma_n^2$ is loaded on the estimated interference covariance matrix, which is same as the LSMI. And the average power of the pulse interference is proportional to the power of interference samples, which reflects the amplitude heterogeneous of interference power.

*Numerical analysis and simulation:* In the numerical simulation, we set the DOF of the filter as $D = 36$, and set training samples length $K = 10D = 360$ and $K = 5D = 180$ as two cases. Fig. 1 shows the change of the expected value of SINR loss with $\beta(K\beta \geq r_c)$ when the DOF $r_c$ of the interference subspace are 18 and 27 respectively. The results show that even if $K$ is greater than $2D$, it causes a large loss of SINR. Only a large value of $K$ can reduce the loss of SINR.

Taking the 4-elements central circular array satellite navigation receiver as the simulation object, SMI space-time filtering is used[14]. The number of time-taps is 9, so the DOF of the filter is 36. The interference signal is a limited bandwidth random noise, and its bandwidth is 20MHz. Two and three interferences are set respectively, so the DOF of the interference subspace are 18 and 27. The SNR is -28dB, the INR is 92dB, the navigation signal is BPSK modulation and the spread spectrum code rate is 10.23MHz. The central frequency of all signals is 1268.42MHz. The sampling rate is set as $f_c = 40.96$MHz, so sampling interval $t_c = 1/f_c$. In the first case, we set the training samples length of SMI filter as 360 and 40960 respectively, and take the output SINR with 40960 training samples as the reference optimal SINR. For all the SMI filter tested, the training data cell is followed by the test cell. The output SINR of every test has been averaged over 5ms. And Fig. 2 shows the loss of SINR under pulse interferences with pulse period equal to $360t_c$ and pulse duty changing from $1/360$ to 1.

According to Fig. 2, it can be seen that the trend of SINR loss curve obtained by simulation is consistent with that of theoretical curve($K\beta \geq$

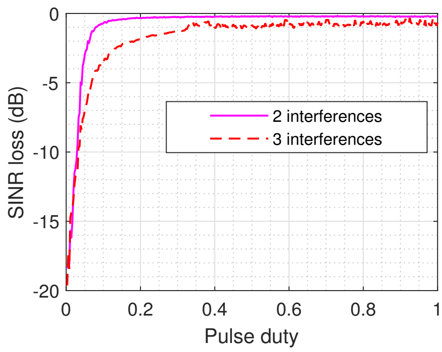


**Fig 2** *SINR loss with different pulse duty ($K = 360$).*

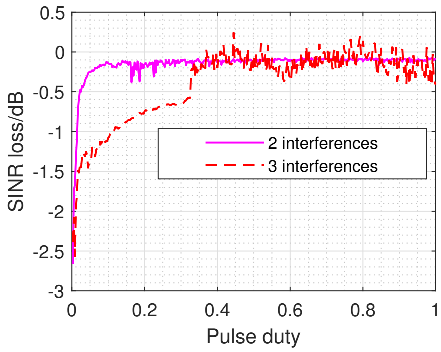


**Fig 3** *SINR loss with different pulse duty ($K = 720$).*

$r_c$). When $K\beta < r_c$ the loss of SINR is large. For another case, we set the length of training samples to 720 and pulse interference period to $360t_c$. Fig. 3 shows the loss of SINR under different duty cycles under this case. It shows that under the same conditions, the length of training samples increases and the loss of SINR decreases.

*Conclusion:* This paper studies the performance of SMI filter under pulse interference. The convergence rate of SMI filter under pulse interference is studied, and the expression of convergence rate is given. The conclusions are verified by signal simulation. The conclusions show that SMI filter needs longer training sample length under pulse interference, which provides a useful guideline for SMI filter design in pulse interference environment in satellite navigation, communication and other fields.

*Acknowledgments:* This work was supported by the National Natural Science Foundation of China(grant 62003354).



*Received:* 10 January 2022 *Accepted:* 4 March 2021

doi: 10.1049/ell2.10001

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