

Two methods for constructing the effective lagrangian of gravitational bosons

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This paper introduces two methods for constructing the effective lagrangian of gravitational bosons: the CCWZ method and the topological method, and then uses these two methods to calculate the symmetry breaking $SO(4)/SO(3)$ of the effective lagrangian of the gravitational boson. By comparing the results, it is found that the effective lagrangian constructed by the two methods is consistent, which further proves that the topological method is useful in constructing the effective lagrangian of the gravitational boson effectiveness.

I. INTRODUCTION

Action and lagrangian: Suppose a particle or field evolves between two predetermined time points t_1 and t_2 . If it were a particle, we could trace the evolution of the particle by drawing a path extending in space, starting at time t_1 and ending at time t_2 . If it were a field, we could imagine a heatmap slowly evolving. What can we know from the behavior of these particles and fields? How can we know what path the particles will take? In physics, we start with a model that can describe a physical system, a typical one being the Lagrangian quantity. Lagrangian is a mathematical quantity, usually written as the difference between kinetic and potential energy, which can give a specific number at any time. We like to use the Laplace measure because it is independent of the observer and does not change with changes in the frame of reference. It doesn't matter whether the observer is upright or inverted or moving at nearly the speed of light. Usually, the value of the physical quantity varies with the choice of coordinates; however, the Lagrange well quantity does not change with the choice of coordinates, and its value is the same for any observer. This property of being independent of the reference frame is very useful because it allows us to perform unambiguous calculations[1–5].

Euler-Lagrange formula: The principle of least action tells us that the behavior of a field or particle is precisely the behavior that minimizes the action. So if we know this action, we can do some math to find out the behavior of the field when this action takes a minimum value. A branch of mathematics called variational methods studies the rate of change of a function. (The variational method tells us that the behavior of a field or particle can be derived using the Euler-Lagrange equation.) The particle version of the Euler-Lagrange equation is shown below. On the left side of the equation, we first take the partial derivative of the Laplace value concerning velocity and then continue taking its time derivative. On the right side of the equation, we take the derivative of the Laplace value in space. Then make the left side of the equation equal to the right side, and you get a path that minimizes the action.

Spontaneous symmetry breaking is a kind of central symmetry-breaking mechanism in physics, which has important applications in both condensed matter physics and elementary particle physics. Assuming that there is a continuous internal symmetry group G if the theoretical vacuum state is in some symmetry groups G are variable under the action, then the symmetry group G undergoes spontaneous symmetry breaking, and the symmetry transformation that keeps the vacuum state unchanged constitutes an unbroken subgroup H of G . Regarding spontaneous symmetry breaking G/H , the Goldstone theorem states that for every broken continuous symmetry transformation generator there is a massless Goldstone boson corresponding spherical paper introduces the construction of spontaneous symmetry breaking when the Goldstone boson effective Laplacian calculates the effective Laplacian of the Goldstone boson with symmetry breaking $SO(4)/SO(3)$. [1–5, 11]

Now the general formula for the Euler property of a four-dimensional Riemannian manifold given in terms of local coordinates is:[4]

$$\chi = \frac{1}{32\pi^2} \int d^4x \sqrt{g} (K_1 - 4R_{ab}R^{ab} + R^2), \quad (1)$$

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where $K_1 \equiv R_{abcd}R^{abcd}$ is the Kretschmann invariant and R_{ab} is the Ricci tensor.

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II. NEW GRAVITATIONAL COUPLING EQUATION

We can pre-set the boundary conditions $\mu = y\omega[8, 9]$.

The spherical quantum solution in a vacuum state.

In this theory, the general relativity theory's field equation is written completely.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu}. \quad (2)$$

The Ricci tensor is by $T_{\mu\nu} = 0$ in vacuum state.

$$R_{\mu\nu} = 0 \quad (3)$$

The proper time of spherical coordinates is

$$d\tau^2 = A(t, r)dt^2 - \frac{1}{c^2} [B(t, r)dr^2 + r^2 d\theta^2 + r^2 \sin\theta d\phi^2]. \quad (4)$$

If we use Eq, we obtain the Ricci-tensor equations.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = 0. \quad (5)$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = 0, \quad (6)$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = 0, R_{\phi\phi} = R_{\theta\theta} \sin^2 \theta = 0, R_{tr} = -\frac{\dot{B}}{Br} = 0, R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0. \quad (7)$$

In this time, $' = \frac{\partial}{\partial r}$, $\dot{} = \frac{1}{c} \frac{\partial}{\partial t}$,

$$\dot{B} = 0 \quad (8)$$

We see that,

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br} \left(\frac{A'}{A} + \frac{B'}{B} \right) = -\frac{(AB)'}{rAB^2} = 0. \quad (9)$$

Hence, we obtain this result.

$$A = \frac{1}{B}. \quad (10)$$

If,

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + \left(\frac{r}{B} \right)' = 0. \quad (11)$$

If we solve the Eq,

$$\frac{r}{B} = r + C \rightarrow \frac{1}{B} = 1 + \frac{C}{r} \quad (12)$$

When r tends to infinity, and we set $C=ye^{-y}$, Therefore,

$$A = \frac{1}{B} = 1 - \frac{y}{r} \Sigma, \Sigma = e^{-y} \quad (13)$$

$$d\tau^2 = \left(1 - \frac{y}{r} \Sigma\right) dt^2. \quad (14)$$

In this time, if particles' mass are m_i , the fusion energy is e ,

$$E = Mc^2 = m_1c^2 + m_2c^2 + \dots + m_nc^2 + T. \quad (15)$$

III. CALCULATION OF EFFECTIVE LAGRANGIAN FOR SYMMETRY BREAKING SO(4)/SO(3)

Calculate the effective Laplace quantity of the Goldstone boson of spontaneous symmetry breaking SO (4)/ SO(3) using the CCWZ method. There are 6 generators in the SO(4) group , its four-dimensional expression can be selected as follows:

$$\begin{aligned} \mathbf{T}_1 &= -i \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathbf{T}_2 = -i \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ \mathbf{T}_3 &= -i \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \mathbf{X}_1 = -i \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \\ \mathbf{X}_2 &= -i \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \mathbf{X}_3 = -i \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \end{aligned} \quad (16)$$

Without loss of generality, choose the vacuum state as $(0 \ 0 \ 0 \ 1)^T$, So for this in a vacuum state, the broken group generators are X_1, X_2, X_3 , and the last broken group generators are $\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3$. These three unbroken generators generate a symmetric subgroup of SO(3). 6 generators satisfy the commutation relation : $[\mathbf{T}_i, \mathbf{T}_j] = i\varepsilon_{ijk}\mathbf{T}_k, [\mathbf{T}_i, \mathbf{X}_a] = i\varepsilon_{iab}\mathbf{T}_b, [\mathbf{X}_a, \mathbf{X}_b] = i\varepsilon_{abi}\mathbf{T}_i$, where ε is the all-antisymmetric quantity. According to Goldstone's theorem, if there are 3 broken generators, 3 Goldstone boson fields must be generated, denoted as $\phi_a (a = 1, 2, 3)$. Let $\pi = \phi_a X_a = \phi_1 X_1 + \phi_2 X_2 + \phi_3 X_3$, $\phi^2 = \phi_1^2 + \phi_2^2 + \phi_3^2$, then the element in the coset SO(4)/SO(3) is $\xi = \exp(i\pi/\sqrt{1-\frac{y}{r}\Sigma})$. The Goldstone covariant derivative and the corresponding gauge field can be obtained respectively. Since the Goldstone covariant derivative is related to the broken generator, according to the generator commutation relationship, It can be known that only even commutation can have the covariant derivative of each order Goldstone calculated as follows: Therefore, the general expression for the Goldstone covariant derivative is[5-7, 10]

$$D_\mu = \frac{1}{\sqrt{1-\frac{y}{r}\Sigma}} \partial_\mu \phi_a \mathbf{X}_a + \left(\sin \frac{\phi}{\sqrt{1-\frac{y}{r}\Sigma}} - \frac{\phi}{\sqrt{1-\frac{y}{r}\Sigma}} \right) \partial_\mu \left(\frac{\phi_a}{\phi} \right) \mathbf{X}_a. \quad (17)$$

From formula , the effective amount of Goldstone boson can be obtained as:

$$\begin{aligned} \mathcal{L} &= \frac{\sqrt{1-\frac{y}{r}\Sigma}^2}{2} \text{Tr} (D^\mu D_\mu) = \\ &= \sum_a \left[\partial_\mu \phi_a + \sqrt{1-\frac{y}{r}\Sigma} \partial_\mu \left(\frac{\phi_a}{\phi} \right) \left(\sin \frac{\phi}{\sqrt{1-\frac{y}{r}\Sigma}} - \frac{\phi}{\sqrt{1-\frac{y}{r}\Sigma}} \right) \right]^2. \end{aligned} \quad (18)$$

Note:

$$A = \frac{1}{B} = 1 - \frac{y}{r} \Sigma, \Sigma = e^{-y}. \quad (19)$$

A is the special solution of ϕ_1 .

IV. THE TOPOLOGICAL METHOD FOR CONSTRUCTING THE EFFECTIVE LAGRANGIAN OF GRAVITATIONAL BOSONS

Knowing that topologically conserved quantities (such as Euler characteristic, and other conserved quantities) reflect the intrinsic geometric properties of the manifold, and the 2-dimensional quantity curvature completely determines the curvature of the Riemannian manifold, reflecting the geometric invariants of the manifold. This is analogous to the variational method of functional analysis. When the dimension is 2, the variational action can be completely determined by the pull equation, just like Euler characteristic is 2, the topological properties of matter (conserved quantities) are consistent with cases that can be completely determined by the Killing vector. However, if Euler characteristic is 4, and the dimension of the action of the analogical variation is 4, there is a situation that the pull equation cannot completely determine the motion state of the material, and the boundary conditions need to be considered.

Assuming that a physical system meets the requirements of a complete system, that is, all Huangyi coordinates are independent of each other, the Lagrange equation holds:

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}} - \frac{\partial \mathcal{L}}{\partial \mathbf{q}} = \mathbf{0} ; \quad (20)$$

Among them, $\mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}, t)$ is the Lagrangian, $\mathbf{q} = (q_1, q_2, \dots, q_N)$ are generalized coordinates, a function of time t , $\dot{\mathbf{q}} = (\dot{q}_1, \dot{q}_2, \dots, \dot{q}_N)$ is the generalized velocity. The light-like KILLING vector field has the following form:

$$\begin{aligned} l^a &= \frac{\partial x^\mu}{\partial t} \left(\frac{\partial}{\partial x^\mu} \right)^a + \frac{dz}{dt} \frac{\partial x^\mu}{\partial z} \left(\frac{\partial}{\partial x^\mu} \right)^a \\ &= \left(\frac{\partial}{\partial t} \right)^a + \frac{dz}{dt} \left(\frac{\partial}{\partial z} \right)^a . \end{aligned} \quad (21)$$

Now the general formula for the Euler property of a four-dimensional Riemannian manifold given in terms of local coordinates is:[4]

$$\chi = \frac{1}{32\pi^2} \int d^4x \sqrt{g} (K_1 - 4R_{ab}R^{ab} + R^2) . \quad (22)$$

where $K_1 \equiv R_{abcd}R^{abcd}$ is the Kretschmann invariant and R_{ab} is the Ricci tensor.

$$K_1 - 4R_{ab}R^{ab} + R^2 = \frac{-8(-337 + 2c^4)r^2 - 2696e^{-y}ry + 1062e^{-2y}y^2}{r^6} . \quad (23)$$

c is the speed of light.

Comparing the two methods, in the algebraic structure, the two methods are identical in structure.

V. SUMMARY AND DISCUSSION

This paper introduces two methods for constructing the effective lagrangian of gravitational bosons: the CCWZ method and the topological method, and then uses these two methods to calculate the symmetry breaking $SO(4)/SO(3)$ of the effective lagrangian of the gravitational boson. By comparing the results, it is found that the effective lagrangian constructed by the two methods is consistent, which further proves that the topological method is useful in constructing the effective lagrangian of the gravitational boson effectiveness.

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