New Convex Approaches to General MVDR Robust Adaptive Beamforming Problems

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Consider general minimum variance distortionless response (MVDR) robust adaptive beamforming problems based on the optimal estimation for both the desired signal steering vector and the interference-plus-noise covariance (INC) matrix. The optimal robust adaptive beamformer design problem is an array output power maximization problem, subject to three constraints on the steering vector, namely, a (convex or nonconvex) quadratic constraint ensuring that the direction-of-arrival (DOA) of the desired signal is separated from the DOA region of all linear combinations of the interference steering vectors, a double-sided norm constraint, and a similarity constraint; as well as a ball constraint on the INC matrix, which is centered at a given data sample covariance matrix. To tackle the nonconvex problem, a new tightened semidefinite relaxation (SDR) approach is proposed to output a globally optimal solution; otherwise, a sequential convex approximation (SCA) method is established to return a locally optimal solution. The simulation results show that the MVDR robust adaptive beamformers based on the optimal estimation for the steering vector and the INC matrix have better performance (in terms of, e.g., the array output signal-to-interference-plus-noise ratio) than the existing MVDR robust adaptive beamformers by the steering vector estimation only.

Introduction: Array signal processing has wide applications in areas such as radar, sonar, communication, and speech processing [1]. Adaptive beamforming is one of the basic problems in array signal processing. The traditional adaptive beamforming techniques are not robust, meaning that small errors (such as the deviation between the estimated and actual steering vectors of the target signal, the calibration error of the array elements, etc.) can significantly reduce the performance of the beamformer [2]. In the past two decades, with the help of new techniques such as convex optimization, robust optimization and machine learning, robust adaptive beamforming techniques have made great progress [3–5].

A well-known robust adaptive beamforming technique is the minimum variance distortionless response (MVDR) robust adaptive beamforming technique, which uses the optimal/suboptimal estimate of the steering vector of the target signal to obtain the beamformer. In [6], the problem of the optimal estimate of the steering vector is transformed into a problem of maximization of array output power, satisfying the unit norm constraint and a similarity constraint. The optimization problem is a special quadratically constrained quadratic programming (QCQP) problem. The Lagrange multiplier method is leveraged to quickly obtain an optimal solution. In [7], the optimal steering vector estimation problem is formulated as a QCQP problem with two non-homogeneous constraints, and the globally optimal solution is studied using semidefinite relaxation (SDR) techniques. In [8], a maximization problem of the array output power is considered, subject to the constraints of the direction of the target signal in the actual signal angular sector and the unit norm of the target signal steering vector. Since the optimization problem is a QCQP problem with two homogeneous constraints, hence the SDR technique and a rank reduction postprocess (see, e.g., [9]) are employed to obtain an optimal solution. Extending the work in [8], another optimal steering vector estimate problem is proposed in [10], where the array output power is maximized, subject to a new nonconvex quadratic constraint to ensure the target signal direction inside the actual (predefined) signal angular sector, the double-sided norm constraint of the signal steering vector, as well as a generalized similarity constraint. This is a typical nonconvex QCQP problem with three non-homogeneous constraints, and generally it is not possible to obtain the globally optimal solution using the SDR technique. Nevertheless, several global optimality sufficient conditions are obtained to ensure that an optimal estimation of the steering vector can be found efficiently (for instance, the usual similarity constraint is imposed, rather than the generalized one).

In addition to the MVDR robust adaptive beamforming technique mentioned above, there are other techniques to get an optimal robust adaptive beamforming vector. For example, maximization the worst-case signal-to-interference-plus-noise ratio (SINR) based techniques [11, 12], random matrix theory based techniques [13], distributed robust optimization based techniques [14, 15], and reconstruction techniques for the signal steering vector and the interference-plus-noise covariance (INC) matrix [16–18].

In this paper, we consider the problem of MVDR robust adaptive beamforming based on the optimal estimation of the INC matrix and the target signal steering vector. Different from [10], we herein consider the optimal estimate of the INC matrix additionally, which leads to an improvement of the array performance, in terms of, e.g., the SINR. The optimal estimation problem is to maximize the array output power, subject to a sphere constraint on the INC matrix and some quadratic constraints over the steer vector. Then, this nonconvex problem is relaxed into an semidefinite programming (SDP) problem (namely, the SDR technique is applied to the problem). To output a rank-one optimal solution for the SDP problem, we propose a tightened SDR technique, which means that an additional second-order cone (SOC) constraint is imposed to the SDP problem without changing the optimality. By doing so, we expect that rank-one optimal solutions for the tightend SDP problem can be output and thus a globally optimal solution for the original optimal estimation problem is obtained. Then we have to design a sequential convex approximation (SCA) algorithm to solve the original optimal estimation, provided that the tightened SDP problem is solved and a high rank solution is generated. We show that the proposed approximation algorithm leads to a locally optimal solution for the optimal estimation problem (in fact, we observe in simulation that this local solution is globally optimal in most problem instances). As will be seen in simulation, the new optimal estimation problem leads to a better MVDR beamformer than the one given by a solution for the previous optimal estimation problem.

Signal Model and Problem Formulation: In many applications, the target signal steering vector $\boldsymbol{a}$ is not known accurately. Therefore, the steering vector can only be estimated by the antenna array geometry, the parameters of the target signal, etc., to obtain $\hat{\boldsymbol{a}}$. Assuming that it is the optimal estimate in some sense and that data sample covariance matrix $\hat{\boldsymbol{R}}$ is sufficiently close to the exact INC matrix $\boldsymbol{R}_{i+n}$, then the following beamvector

$$\boldsymbol{w}^\star = \frac{1}{\hat{\boldsymbol{a}}^H \hat{\boldsymbol{R}}^{-1} \hat{\boldsymbol{a}}} \hat{\boldsymbol{R}}^{-1} \hat{\boldsymbol{a}} \tag{1}$$

is called MVDR robust adaptive beamformer.

In [8], the optimal steering vector estimate $\hat{\boldsymbol{a}}$ is derived by addressing the array output power maximization problem. This approach requires two constraints: ensuring the direction-of-arrival (DOA) of the desired signal is separated from the DOA region of all linear combinations of the interference steering vectors, and maintaining the norm of the steering vector. Such a steering vector estimation problem can be modeled as follows,

$$\begin{array}{ll} \underset{\boldsymbol{a}}{\text{minimize}} & \boldsymbol{a}^H \hat{\boldsymbol{R}}^{-1} \boldsymbol{a} \\ \text{subject to} & \boldsymbol{a}^H \tilde{\boldsymbol{C}} \boldsymbol{a} \le \Delta_0 \\ & \|\boldsymbol{a}\|^2 = N, \end{array} \tag{2}$$

where $\tilde{\boldsymbol{C}}$ is defined as,

$$\tilde{\boldsymbol{C}} = \int_{\tilde{\Theta}} \boldsymbol{d}(\theta) \boldsymbol{d}^H(\theta) d\theta, \tag{3}$$

and $\boldsymbol{d}(\theta)$ is the steering vector defined by the antenna array geometry for angle θ. In (3), the angular sector $\Theta = [\theta_{\min}, \theta_{\max}]$ is the region where the DOA of the desired signal is located, $\tilde{\Theta}$ is the complement of this angular sector (assumed to be the interval of the interference signal direction), and the parameter Δ_0 is determined by the following maximum value,

$$\Delta_0 = \max_{\theta \in \Theta} \boldsymbol{d}^H(\theta) \tilde{\boldsymbol{C}} \boldsymbol{d}(\theta). \tag{4}$$

Therefore, if the steering vector $\boldsymbol{a}$ satisfies the constraint $\boldsymbol{a}^H \tilde{\boldsymbol{C}} \boldsymbol{a} \leq \Delta_0$, it means that its direction is located within Θ (for example, see [8, Figure 2]).

Extending the work in [8], a more generalized problem pertaining to optimal steering vector estimation is considered in [10],

$$\underset{\boldsymbol{a}}{\text{minimize}} \quad \boldsymbol{a}^H \hat{\boldsymbol{R}}^{-1} \boldsymbol{a} \tag{5a}$$

$$\text{subject to} \quad \boldsymbol{a}^H \tilde{\boldsymbol{C}} \boldsymbol{a} \leq \Delta_0 \tag{5b}$$

$$N(1-\eta_1) \leq \|\boldsymbol{a}\|^2 \leq N(1+\eta_2) \tag{5c}$$

$$\|\boldsymbol{Q}^H(\boldsymbol{a}-\boldsymbol{a}_0)\|^2 \leq \epsilon, \tag{5d}$$

where η_1 and η_2 set the boundaries for the variation in the norm of the steering vector, $\boldsymbol{a}_0$ is a given steering vector, and $\boldsymbol{Q}$ is a $N \times M$ matrix. Because constraint (5c) is nonconvex, problem (5) is a nonconvex problem. Nevertheless, [10] proves that when $\boldsymbol{Q}\boldsymbol{Q}^H$ is an identity matrix, the globally optimal solution of the optimization problem (5) can be obtained in polynomial-time computational complexity (by SDR and rank reduction techniques, see [9]). Otherwise, to efficiently obtain the global solution to (5), additional sufficient conditions are necessitated.

Not only does the steering vector exhibit error, but there's also a discrepancy between $\hat{\boldsymbol{R}}$ and $\boldsymbol{R}_{i+n}$. In this study, we explore MVDR robust adaptive beamforming, focusing on optimal estimation of the target signal steering vector and the INC matrix. The problem can be formulated as follows,

$$\underset{\boldsymbol{R},\boldsymbol{a},t}{\text{minimize}} \quad t \tag{6a}$$

$$\text{subject to} \quad \begin{bmatrix} \boldsymbol{R} & \boldsymbol{a} \\ \boldsymbol{a}^H & t \end{bmatrix} \succeq \boldsymbol{0} \tag{6b}$$

$$\|\boldsymbol{R}-\hat{\boldsymbol{R}}\|_F^2 \leq \delta \tag{6c}$$

$$\text{(5b)} - \text{(5d) satisfied.} \tag{6d}$$

The constraint (6c) states that the INC matrix $\boldsymbol{R}$ (for convenience, the subscript $\boldsymbol{R}_{i+n}$ is removed in the above optimization problem) is located within the sphere centered at $\hat{\boldsymbol{R}}$ with radius $\sqrt{\delta}$.

If the optimal solution $(\boldsymbol{R}^\star, \boldsymbol{a}^\star, t^\star)$ of problem (6) can be found, then the MVDR robust adaptive beamformer is defined as:

$$\boldsymbol{w}^\star = \frac{1}{\boldsymbol{a}^{\star H}(\boldsymbol{R}^\star)^{-1}\boldsymbol{a}^\star}(\boldsymbol{R}^\star)^{-1}\boldsymbol{a}^\star = \frac{(\boldsymbol{R}^\star)^{-1}\boldsymbol{a}^\star}{t^\star}. \tag{7}$$

A Tightened SDR Technique Solution to Problem (6)*:* We apply the tightened SDR technique to problem (6), aiming at that it will yield rank-one optimal solutions for the SDP relaxation problem for (6). It is known that applying the SDR technique to problem (6) yields the SDP problem,

$$\underset{\boldsymbol{R},\boldsymbol{X},\boldsymbol{a},t}{\text{minimize}} \quad t \tag{8a}$$

$$\text{subject to} \quad \begin{bmatrix} \boldsymbol{R} & \boldsymbol{a} \\ \boldsymbol{a}^H & t \end{bmatrix} \succeq \boldsymbol{0} \tag{8b}$$

$$\|\boldsymbol{R}-\hat{\boldsymbol{R}}\|_F^2 \leq \delta \tag{8c}$$

$$\text{tr}\,(\tilde{\boldsymbol{C}}\boldsymbol{X}) \leq \Delta_0 \tag{8d}$$

$$N(1-\eta_1) \leq \text{tr}\,\boldsymbol{X} \leq N(1+\eta_2) \tag{8e}$$

$$\text{tr}\,(\boldsymbol{Q}\boldsymbol{Q}^H\boldsymbol{X}) - 2\Re(\boldsymbol{a}^H\boldsymbol{Q}\boldsymbol{Q}^H\boldsymbol{a}_0) + \boldsymbol{a}_0^H\boldsymbol{Q}\boldsymbol{Q}^H\boldsymbol{a}_0 \leq \epsilon \tag{8f}$$

$$\begin{bmatrix} \boldsymbol{X} & \boldsymbol{a} \\ \boldsymbol{a}^H & 1 \end{bmatrix} \succeq \boldsymbol{0}. \tag{8g}$$

When the optimal solution $(\boldsymbol{R}^\star, \boldsymbol{X}^\star, \boldsymbol{a}^\star, t^\star)$ of the SDP problem has an element $\boldsymbol{X}^\star$ that is a rank-one matrix, i.e. $\boldsymbol{X}^\star = \boldsymbol{a}^\star\boldsymbol{a}^{\star H}$, then problem (8) is equivalent to the original problem (6). When $\boldsymbol{X}^\star$ is a high-rank solution, we seek to solve the original problem (6) using the tightened SDR technique. In fact, tightened SDR reduces the feasibility set by adding reasonable constraints to the SDP problem, making it more likely that its optimal solution will become a rank-one solution $\boldsymbol{X}^\star = \boldsymbol{a}^\star\boldsymbol{a}^{\star H}$.

Toward this end, note that the second inequality constraint in (5c) is

$$\|\boldsymbol{a}\| \leq \sqrt{N(1+\eta_2)}. \tag{9}$$

For any given non-zero vector $\boldsymbol{b} \in \mathbb{C}^N$ with $\boldsymbol{a}^H\boldsymbol{b} \neq 0$, we have

$$\|\boldsymbol{a}(\boldsymbol{a}^H\boldsymbol{b})\| \leq \sqrt{N(1+\eta_2)}|\boldsymbol{a}^H\boldsymbol{b}|. \tag{10}$$

Using phase rotation technique (i.e. letting $\boldsymbol{a} = \boldsymbol{a}e^{j\arg(\boldsymbol{a}^H\boldsymbol{b})}$), (10) can be equivalently restructured as

$$\|(\boldsymbol{a}\boldsymbol{a}^H)\boldsymbol{b}\| \leq \sqrt{N(1+\eta_2)}\Re(\boldsymbol{a}^H\boldsymbol{b}), \tag{11}$$

where SDR form is

$$\|\boldsymbol{X}\boldsymbol{b}\| \leq \sqrt{N(1+\eta_2)}\Re(\boldsymbol{a}^H\boldsymbol{b}). \tag{12}$$

In other words, $(\boldsymbol{X}, \boldsymbol{a})$ satisfies the convex form

$$\begin{bmatrix} \sqrt{N(1+\eta_2)}\Re(\boldsymbol{a}^H\boldsymbol{b}) \\ \boldsymbol{X}\boldsymbol{b} \end{bmatrix} \in \text{SOC}(N+1), \tag{13}$$

where $\text{SOC}(N+1) = \{\begin{bmatrix} t \\ \boldsymbol{x} \end{bmatrix} \in \mathbb{R}_+ \times \mathbb{C}^N \mid t \geq \|\boldsymbol{x}\|\}$.

Thus, by adding (13) to the optimization problem (8), we obtain a tightened SDP problem of (6):

$$\underset{\boldsymbol{R},\boldsymbol{X},\boldsymbol{a},t}{\text{minimize}} \quad t \tag{14a}$$

$$\text{subject to} \quad \text{(8b)} - \text{(8g), (13) satisfied.} \tag{14b}$$

Let

$$\boldsymbol{b} = \boldsymbol{Q}\boldsymbol{Q}^H\boldsymbol{a}_0. \tag{15}$$

If $(\boldsymbol{R}^\star, \boldsymbol{X}^\star, \boldsymbol{a}^\star, t^\star)$ is an optimal solution for problem (14), with $\boldsymbol{X}^\star = \boldsymbol{a}^\star\boldsymbol{a}^{\star H}$, then $(\boldsymbol{R}^\star, \boldsymbol{a}^\star, t^\star)$ is an optimal solution for problem (6).

An SCA Algorithm Solution to Problem (6)*:* If the tightened SDP problem still ouputs a high-rank solution $\boldsymbol{X}^\star$, we design an SCA algorithm to solve the original problem (6).

Note that the only nonconvex constraint in (6) is the one in (5c):

$$\|\boldsymbol{a}\|^2 \geq N(1-\eta_1). \tag{16}$$

By Cauchy's inequality,

$$\|\boldsymbol{a}\| \geq \frac{\|\boldsymbol{a}^H\boldsymbol{a}^l\|}{\|\boldsymbol{a}^l\|} \geq \frac{\Re(\boldsymbol{a}^H\boldsymbol{a}^l)}{\|\boldsymbol{a}^l\|}, \tag{17}$$

where $\boldsymbol{a}^l$ is the current steering vector, it can be seen that when the linear condition

$$\frac{\Re(\boldsymbol{a}^H\boldsymbol{a}^l)}{\|\boldsymbol{a}^l\|} \geq \sqrt{N(1-\eta_1)} \tag{18}$$

holds, then the nonconvex constraint (16) must be satisfied. Therefore, consider using (18) in place of (16). To solve (6), in the lth iteration, one only needs to solve the following SDP problem,

$$\underset{\boldsymbol{R},\boldsymbol{a},t}{\text{minimize}} \quad t \tag{19a}$$

$$\text{subject to} \quad \frac{\Re(\boldsymbol{a}^H\boldsymbol{a}^l)}{\|\boldsymbol{a}^l\|} \geq \sqrt{N(1-\eta_1)} \tag{19b}$$

$$\|\boldsymbol{a}\| \leq \sqrt{N(1+\eta_2)} \tag{19c}$$

$$\text{(6b), (6c), (5b), (5d) satisfied,} \tag{19d}$$

obtain the optimal solution $(\boldsymbol{R}^{l+1}, \boldsymbol{a}^{l+1}, t^{l+1})$, update l by setting $l = l+1$, and continue solving (19) until a stopping condition is met. The procedure is summarized as Algorithm 1. It holds that the optimal values $\{t^l\}$ for SDP problem (19) in the iterative process are descend, namely

$$t^l \geq t^{l+1}, l = 1, \cdots. \tag{20}$$

Hence, the process for solving the optimal estimation problem for the steering vector and INC matrix (6) is as follows: solve the tightened SDP problem (14) (where $\boldsymbol{b}_0$ is taken according to (15)), obtain the optimal solution $(\boldsymbol{R}^\star, \boldsymbol{X}^\star, \boldsymbol{a}^\star, t^\star)$, if the rank of $\boldsymbol{X}^\star = \boldsymbol{a}^\star\boldsymbol{a}^{\star H}$ is one, then end, otherwise, use the SCA algorithm 1 to solve the optimal estimation problem (6).

Algorithm 1 SCA Algorithm for Solving Problem (6)

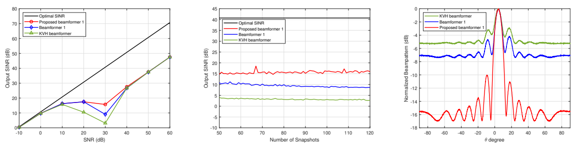
Require: $\hat{\boldsymbol{R}}, \tilde{\boldsymbol{C}}, \boldsymbol{Q}, \delta, \Delta_0, \epsilon, \eta_1, \eta_2, \xi$;
Ensure: A solution $(\boldsymbol{R}^\star, \boldsymbol{a}^\star, t^\star)$ of problem (6);
Assume $(\boldsymbol{R}^0, \boldsymbol{a}^0, \boldsymbol{a}^{0H}(\boldsymbol{R}^0)^{-1}\boldsymbol{a}^0)$ is a feasible solution of problem (6); set $l = 0$;
repeat
  Solve the SDP problem (19) to get the optimal solution $(\boldsymbol{R}^\star, \boldsymbol{a}^\star, t^\star)$;
  Set $\boldsymbol{R}^{l+1} = \boldsymbol{R}^\star, \boldsymbol{a}^{l+1} = \boldsymbol{a}^\star, t^{l+1} = t^\star$;
  $l = l + 1$;
until $|t^l - t^{l-1}| \le \xi$

Simulation results: In this section, we present several simulations aiming to evaluate the performance of the proposed MVDR robust adaptive beamformers. In all cases, results are averaged over 200 simulation runs for consistency and precision.

Simulation 1. Signal Look Direction Mismatch: A uniform linear array setup, featuring $N = 12$ omni-directional antennas half a wavelength apart, is used. The array noise is a spatiotemporally white Gaussian vector with zero mean and covariance $\boldsymbol{I}$. Interfering signals from angles $\theta_1 = -5^\circ$ and $\theta_2 = 15^\circ$ possess an INR of 30 dB, while the consistent desired signal appears in the training data cell. With a sample size of 100 for the covariance matrix, T, each sample encompasses the target signal, interference, and array noise.

The angular sector of interest, denoted as Θ, ranges from 0° to 10°, and the presumed direction θ_0 set at 5°. However, the actual direction is $\theta = 4^\circ$, demonstrating a uniform direction mismatch. For the proposed beamformer, parameters η_1 and η_2 are both 0.3, with ϵ at $0.25N$. The condition $\sqrt{\delta} = 10^{-5}\|\hat{\boldsymbol{R}}\|_F$ is applied for parameter δ.

In each simulation run, three optimization problems are solved, resulting in three MVDR robust adaptive beamformers. In this case, for the optimization problem (6) in this paper, which consider $\boldsymbol{Q}$ as the identity matrix (i.e. $\boldsymbol{Q} = \boldsymbol{I}$), is named as "Proposed beamformer 1" . The optimization problems (8) and (16) in reference [10] are named as "KVH Beamformer" and "Beamformer 1", respectively.



(a) (b) (c)

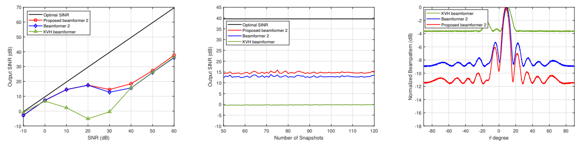
Fig 1 *(a) display the average beamformer output SINR versus SNR; (b) showcase average beamformer output SINR versus training sample size T, with SNR = 30 dB; (c) present the comparison of normalized beampatterns of the beamformers, with $T = 100$, and SNR = 30 dB.*

Figure 1(a) reveals that the beamformer proposed in this study exhibits higher output SINR in the medium SNR range. Figure 1(b) demonstrates that regardless of the training sample size, the beamformer proposed in this work yields a superior output SINR. Furthermore, as seen in Figure 1(c), the normalized beampattern of our beamformer, while maintaining the same main lobe, features reduced sidelobes. In summary, the beamformer proposed in this paper offers the best performance.

Simulation 2. Beamforming Based on an Ellipsoid Constraint: In this simulation, both the nominal and actual target signal directions are set at 9°. Interference signals come from angles $\theta_1 = -15^\circ$ and $\theta_2 = 15^\circ$. Unlike simulation 1, we consider wavefront distortion in a non-homogeneous medium [8], causing irregular distortions across the steering vector. This distortion, attributed to wave propagation, introduces phase increments, modeled as independent Gaussian variables with zero mean and a 0.01 standard deviation, which are consistent across simulations. The parameters η_1 and η_2 for the proposed beamformer are set as 0.45, and the similarity constraint parameter ϵ satisfies $\epsilon = 0.15\|\boldsymbol{Q}\boldsymbol{a}_0\|_F^2$.

In contrast to the previous simulation, the $\boldsymbol{Q}$ and $\boldsymbol{a}_0$ in constraint (5d) are defined the same as in the third example of reference [10].

The three beamformers resulted from the optimization problem (6) in this paper, and the optimization problems (8), (15) in reference [10]. They are termed "Proposed beamformer 2", "KVH beamformer" and "Beamformer 2", respectively.



(a) (b) (c)

Fig 2 *(a) display the average beamformer output SINR versus SNR; (b) showcase average beamformer output SINR versus training sample size T, with SNR = 30 dB; (c) present the comparison of normalized beampatterns of the beamformers, with $T = 100$, and SNR = 30 dB.*

In Figure 2, it is evident that the beamformer proposed in this study still achieves higher output SINR and lower sidelobes. Thus, even when adopting the ellipsoid constraint, the beamformer proposed in this paper further enhances performance.

Conclusion: In this paper, the MVDR robust adaptive beamforming problem based on the optimal estimation of the INC matrix and the steering vector is studied. The optimal estimation problem is to maximize the array output power, subject to a constraint on the direction of the target signal restricted within a given angular sector, the double-sided constraint on the norm of the steering vector, the general similarity constraint, and the spherical constraint on the INC matrix. To tackle this non-convex problem, a tightened SDR method is proposed to obtain a globally optimal solution of the optimal estimation problem, and an SCA algorithm is designed to obtain a locally optimal solution of the optimal estimation problem. Simulation results show that the MVDR robust adaptive beamformers proposed in this paper outperform the other MVDR robust adaptive beamformers by solutions for those problems estimating only the steering vector.

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