SOLUTIONS OF THE NONLINEAR INTEGRAL EQUATION USING THE FIXED POINT TECHNIQUE IN MODIFIED INTUITIONISTIC FUZZY SOFT METRIC SPACE

VISHAL GUPTA[1], AANCHAL[2], ADESH KUMAR TRIPATHI[3]

[1,2,3]*Department of Mathematics, Maharishi Markandeshwar, Deemed to be University, Mullana, Ambala, Haryana, India*
[2]*Department of Mathematics, M.D.S.D. Girls College, Ambala City-134003, Haryana, India*

ABSTRACT. This paper introduces coupled fixed points of maps in MIFSMS. The notion of compatible maps and weakly compatible maps is also defined. Common coupled fixed point theorem for weakly compatible maps have been proved in the setting of MIFSMS. Moreover, some corollaries and an example is given to validate our new result. Also, to show the usability of our findings, we have applied our new result to get the solution of nonlinear Fredholm integral equation.

1. INTRODUCTION

To deal with uncertainty Zadeh [1] presented fuzzy sets. Further K.T. Atanassov [2] gave intuitionistic fuzzy set that includes membership as well as non membership function. Moreover, in case of data consisting of parameters, Moldostov [3] gave the concept of Soft Sets to deal with the uncertainties. Das and Samanta [9, 10] applied the concept of soft sets to metric spaces and hence presented Soft Metric Spaces utilizing soft points of soft sets. Maji et al [4] in 2001 introduced Fuzzy Soft Sets. Beaula and Gunaseli [5] applied the metric space concept to Fuzzy Soft Sets and hence introduced Fuzzy Soft Metric Spaces using fuzzy soft point and defined some of its properties. Saadiat et al [6] gave another important concept of Modified Intuitionistic Fuzzy Metric Spaces by using continuous t-representable norm. Vishal and Aanchal [13, 14] introduced Modified intuitionistic fuzzy soft metric space and proved the fixed point theorems in its settings, they have also introduced weakly compatible maps in MIFSMS. Bhaskar and and Lakshmikantham [15] introduced coupled fixed point. Bhavana Deshpande and Amrish Handa [16] gave coupled fixed point theorem for weakly compatible maps in modified intuitionistic fuzzy metric spaces.

2. Preliminaries

In the following section X is taken as universe, U represents the parameter set, $\bar{U}$ is taken as the absolute soft set and $SP(\bar{X})$ denotes the collection of all the soft points of $\bar{X}$.

Definition 2.1. [3] *Soft set is a pair (S, U) on a universe X where U represents the parameter set and S defines the map from U to power set of X i.e. $S : U \to P(X)$.*

Definition 2.2. [9,10] *A Soft Metric Space is a 3-tuple $(\bar{X}, \bar{\mu}, U)$, where the soft metric $\bar{\mu} : SP(\bar{X}) \times SP(\bar{X}) \to R(U)$ here $R(U)$ is the set containing non-negative soft real numbers and $\bar{\mu}$ satisfies the given conditions for all $\bar{u}_{e_1}, \bar{v}_{e_2}, \bar{w}_{e_3} \in SP(\bar{X})$:*

(i) $\bar{\mu}(\bar{u}_{e_1}, \bar{v}_{e_2}) > \bar{0}$,
(ii) $\bar{\mu}(\bar{u}_{e_1}, \bar{v}_{e_2}) = \bar{0}$ iff $\bar{u}_{e_1} = \bar{v}_{e_2}$,
(iii) $\bar{\mu}(\bar{u}_{e_1}, \bar{v}_{e_2}) = \bar{\mu}(\bar{v}_{e_2}, \bar{u}_{e_1})$,
(iv) $\bar{\mu}(\bar{u}_{e_1}, \bar{v}_{e_2}) \leq \bar{\mu}(\bar{u}_{e_1}, \bar{w}_{e_3}) + \bar{\mu}(\bar{w}_{e_3}, \bar{v}_{e_2})$.

Definition 2.3. [5] *A soft fuzzy metric space is the 3-tuple $(\bar{X}, S, *)$ where soft fuzzy metric on $\bar{X}$ is given by map $S : SP(\bar{X}) \times SP(\bar{X}) \times (0, \infty) \to [0, 1]$ satisfying the given condition $\bar{u}_{e_1}, \bar{v}_{e_2}, \bar{w}_{e_3} \in SP(\bar{X}), s, t > 0$:*

(i) $S(\bar{u}_{e_1}, \bar{v}_{e_2}, t) > 0$,
(ii) $S(\bar{u}_{e_1}, \bar{v}_{e_2}, t) = 1$ iff $\bar{u}_{e_1} = \bar{v}_{e_2}$,
(iii) $S(\bar{u}_{e_1}, \bar{v}_{e_2}, t) = S(\bar{v}_{e_2}, \bar{u}_{e_1}, t)$,
(iv) $S(\bar{u}_{e_1}, \bar{v}_{e_2}, t + s) \geq S(\bar{u}_{e_1}, \bar{w}_{e_3}, t) * S(\bar{w}_{e_3}, \bar{v}_{e_2}, s)$,
(v) $S(\bar{u}_{e_1}, \bar{v}_{e_2}, .) : (0, \infty) \to [0, 1]$ *is continuous.*

Definition 2.4. [6] *A Modified Intuitionistic Fuzzy Metric Space is given by 3-tuple $(\bar{X}, \varpi_{M,N}, \Theta)$, where X is any arbitrary set, M, N are fuzzy sets from $X^2 \times (0, \infty)$ to $[0, 1]$ satisfying $M(u, v, t) + N(u, v, t) \leq 1$ for all $u, v \in X$ and $t > 0$, continuous t-representable norm is given as Θ and $\varpi_{M,N}$ is a function $X^2 \times (0, \infty) \to L^*$ fulfilling the given conditions for all $u, v \in X$ and $t, s > 0$:*

(i) $\varpi_{M,N}(u, v, t) >_{L^*} 0_{L^*}$;
(ii) $\varpi_{M,N}(u, v, t) = 1_{L^*}$ iff $u = v$;
(iii) $\varpi_{M,N}(u, v, t) = \varpi_{M,N}(v, u, t)$;
(iv) $\varpi_{M,N}(u, v, t + s) \geq_{L^*} \Theta(\varpi_{M,N}(u, w, t), \varpi_{M,N}(w, v, s))$;
(v) $\varpi_{M,N}(u, v, .) : (0, \infty) \to L^*$ *is continuous.*

Here modified intuitionistic fuzzy metric $\varpi_{M,N}$ is given as
$\varpi_{M,N}(u, v, t) = (M(u, v, t), N(u, v, t))$.

Definition 2.5. [8] *A map $S : X \to IF^U$, where X is an arbitrary set and IF^U is the set consisting of all the intuitionistic fuzzy subsets of U, then S is a function defined for every $a \in X$ as $S(a) = \{< u, \mu_{S(a)}, \nu_{S(a)} >: u \in U\}$, where degree of association and non association is given by $\mu_{S(a)}$ and $\nu_{S(a)}$ respectively.*

Definition 2.6. [13] *A MIFSMS is a 3-tuple $(\bar{X}, \varpi_{M,N}, \Theta)$ where $\bar{X}$ is any set, M and N are soft fuzzy sets, Θ is a continuous t-representable norm, $\varpi_{M,N}$ is a mapping from $SP(\bar{X}) \times SP(\bar{X}) \times (0, \infty)$ to L^* so that for all $\bar{u}_{e_1}, \bar{v}_{e_2}, \bar{w}_{e_3} \in SP(\bar{X})$ and $s, t > 0$:*

(i) $\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t) >_{L^*} 0_{L^*}$,
(ii) $\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t) = 1_{L^*}$ iff $\bar{u}_{e_1} = \bar{v}_{e_2}$,
(iii) $\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t) = \varpi_{M,N}(\bar{v}_{e_2}, \bar{u}_{e_1}, t)$,

(iv) $\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t+s) \geq_{L^*} \Theta(\varpi_{M,N}(\bar{u}_{e_1}, \bar{w}_{e_3}, t), \varpi_{M,N}(\bar{w}_{e_3}, \bar{v}_{e_2}, s))$,
(v) $\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, .) : (0, \infty) \to L^*$ is continuous.

Then $\varpi_{M,N}$ is called MIFSM on $\bar{X}$. Here level of closeness, non closeness between $\bar{u}_{e_1}, \bar{v}_{e_2}$ w.r.t. t is given by the maps $M(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$ and $N(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$ respectively and metric $\varpi_{M,N}$ is given as

$$\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t) = (M(\bar{u}_{e_1}, \bar{v}_{e_2}, t), N(\bar{u}_{e_1}, \bar{v}_{e_2}, t)).$$

Remark 2.1. [13] The function $M(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$ is increasing and the function $N(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$ is decreasing in a MIFSMS $\bar{X}$, for all $\bar{u}_{e_1}, \bar{v}_{e_2} \in SP(\bar{X})$.

Lemma 2.1. [13] Consider $(\bar{X}, \varpi_{M,N}, \Theta)$ be MIFSMS. Then $\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$ is increasing with respect to $t > 0$ in $(L^*, \leq_{L^*})$ for all $\bar{u}_{e_1}, \bar{v}_{e_2} \in SP(\bar{X})$.

Definition 2.7. [13] Consider $(\bar{X}, \varpi_{M,N}, \Theta)$ be MIFSMS. $\varpi_{M,N}$ is called continuous on $SP(\bar{X}) \times SP(\bar{X}) \times (0, \infty)$ if
$\lim_{n \to \infty} \varpi_{M,N}(\bar{u}_{e_n}, \bar{v}_{e_n}, t_n) = \varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$ where $\{(\bar{u}_{e_n}, \bar{v}_{e_n}, t_n)\}$ is a sequence converging to $(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$.

Lemma 2.2. [13] For $(\bar{X}, \varpi_{M,N}, \Theta)$ be MIFSMS, then $\varpi_{M,N}$ is continuous on $SP(\bar{X}) \times SP(\bar{X}) \times (0, \infty)$.

Definition 2.8. [13] Consider $(\bar{X}, \varpi_{M,N}, \Theta)$ be MIFSMS and $\{\bar{u}_{e_n}\}$ be any sequence in $\bar{X}$, then

1. the sequence is Cauchy iff for every $t > 0$, there exist $n_o \in N$ so that

$$lt_{n_o \to \infty} \varpi_{M,N}(\bar{u}_{e_n}, \bar{u}_{e_{n+m}}, t) = 1_{L^*}$$

for each $n, m \geq n_o$.
2. the sequence converges to $\bar{u}$ iff for every $t > 0$

$$lt_{n \to \infty} \varpi_{M,N}(\bar{u}_{e_n}, \bar{u}, t) = 1_{L^*}$$

Definition 2.9. [13] A MIFSMS $(\bar{X}, \varpi_{M,N}, \Theta)$ is called complete iff every Cauchy sequence is convergent.

Lemma 2.3. [13] Consider $(\bar{X}, \varpi_{M,N}, \Theta)$ be MIFSMS and for all $\bar{u}_{e_1}, \bar{v}_{e_2} \in SP(\bar{X})$, $t > 0$; $0 < k < 1$ (2.1) holds

(2.1) $$\varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, kt) \geq_{L^*} \varpi_{M,N}(\bar{u}_{e_1}, \bar{v}_{e_2}, t)$$

then $\bar{u}_{e_1} = \bar{v}_{e_2}$.

3. Main Results

In this section we are going to define coupled fixed points for maps in the setting of MIFSMS as follows:

Definition 3.1. An element $(\bar{p}_{e_1}, \bar{q}_{e_2}) \in \bar{X} \times \bar{X}$ is called a coupled fixed point of map

$$\alpha : \bar{X} \times \bar{X} \to \bar{X}$$

if

$$\alpha(\bar{p}_{e_1}, \bar{q}_{e_2}) = \bar{p}_{e_1}, \alpha(\bar{q}_{e_2}, \bar{p}_{e_1}) = \bar{q}_{e_2}.$$

Definition 3.2. *An element $(\bar{p}_{e_1}, \bar{q}_{e_2}) \in \bar{X} \times \bar{X}$ is called a coupled fixed point of maps*

$$\alpha : \bar{X} \times \bar{X} \to \bar{X}, \beta : \bar{X} \to \bar{X}$$

if

$$\alpha(\bar{p}_{e_1}, \bar{q}_{e_2}) = \beta(\bar{p}_{e_1}), \alpha(\bar{q}_{e_2}, \bar{p}_{e_1}) = \beta(\bar{q}_{e_2}).$$

Definition 3.3. *An element $\bar{p}_{e_1} \in \bar{X}$ is called a coupled fixed point of maps*

$$\alpha : \bar{X} \times \bar{X} \to \bar{X}, \beta : \bar{X} \to \bar{X}$$

if

$$\bar{p}_{e_1} = \alpha(\bar{p}_{e_1}, \bar{p}_{e_1}) = \beta(\bar{p}_{e_1}).$$

Definition 3.4. *The maps $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ and $\beta : \bar{X} \to \bar{X}$ are called commutative if for all $(\bar{p}_{e_1}, \bar{q}_{e_2}) \in \bar{X}$ (3.1) holds*

$$(3.1) \qquad\qquad \beta\alpha(\bar{p}_{e_1}, \bar{q}_{e_2}) = \alpha(\beta\bar{p}_{e_1}, \beta\bar{q}_{e_2}).$$

Definition 3.5. *Consider $(\bar{X}, \varpi_{M,N}, \Theta)$ be MIFSMS. The maps $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ and $\beta : \bar{X} \to \bar{X}$ are called compatible if (3.2) holds for all $\varrho > 0$*

$$(3.2) \qquad \begin{aligned} &lt_{n\to\infty}(\beta\alpha(\bar{p}_{e_n}, \bar{q}_{e_n}), \alpha(\beta\bar{p}_{e_n}, \beta\bar{q}_{e_n}), \varrho) = 1_{L^*}, \\ &lt_{n\to\infty}(\beta\alpha(\bar{q}_{e_n}, \bar{p}_{e_n}), \alpha(\beta\bar{q}_{e_n}, \beta\bar{p}_{e_n}), \varrho) = 1_{L^*}, \end{aligned}$$

whenever $\{\bar{p}_{e_n}\}$ and $\{\bar{q}_{e_n}\}$ are sequences in $\bar{X}$ so that

$$\begin{aligned} lt_{n\to\infty}\alpha(\bar{p}_{e_n}, \bar{q}_{e_n}) = lt_{n\to\infty}\beta(\bar{p}_{e_n}) = \bar{p}_{e_1}, \\ lt_{n\to\infty}\alpha(\bar{q}_{e_n}, \bar{p}_{e_n}) = lt_{n\to\infty}\beta(\bar{q}_{e_n}) = \bar{q}_{e_2}, \end{aligned}$$

for all $\bar{p}_{e_1}, \bar{q}_{e_2} \in \bar{X}$.

Definition 3.6. *The maps $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ and $\beta : \bar{X} \to \bar{X}$ are called weakly compatible if for all $\bar{p}_{e_1}, \bar{q}_{e_2} \in \bar{X}$ $\alpha(\bar{p}_{e_1}, \bar{q}_{e_2}) = \beta(\bar{p}_{e_1})$, $\alpha(\bar{q}_{e_2}, \bar{p}_{e_1}) = \beta(\bar{q}_{e_2})$ implies that $\beta\alpha(\bar{p}_{e_1}, \bar{q}_{e_2}) = \alpha(\beta\bar{p}_{e_1}, \beta\bar{q}_{e_2})$ and $\beta\alpha(\bar{q}_{e_2}, \bar{p}_{e_1}) = \alpha(\beta\bar{q}_{e_2}, \beta\bar{p}_{e_1})$.*

Definition 3.7. *A t-representable norm Θ which is continuous is known as of H-type if the family of function $\{\Theta^m(\varrho)\}_{m=1}^{\infty}$ is equicontinuous at $\varrho = 1_{L^*}$.*
It is trivial that Θ is H-type t- representable if and only if for any $0 < \nu < 1$ implies the existence of $0 < \xi < 1$ so that (3.3) holds for all $m \in N$ whenever $\varrho >_{L^*} (N_s(\xi), \xi)$,

$$(3.3) \qquad\qquad \Theta^m(\varrho) >_{L^*} (N_s(\nu), \nu).$$

Remark 3.1. *In MIFSMS $(\bar{X}, \varpi_{M,N}, \Theta)$ if for $\bar{p}_{e_1}, \bar{q}_{e_2} \in \bar{X}$, $\varrho > 0$ and $0 < \nu < 1$ we have $\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho) >_{L^*} (N_S(\nu), \nu)$, then there exist a ϱ_o, lying between 0 and ϱ so that $\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho_o) >_{L^*} (N_S(\nu), \nu)$.*

Remark 3.2. *We will represent $[\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho)]^n = \Theta^{n-1}(\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho))$ for all $n \in N$, for the sake of simplicity.*

Definition 3.8. *Consider $(\bar{X}, \varpi_{M,N}, \Theta)$ be MIFSMS, then $\varpi_{M,N}$ satisfies p-property if (3.4) holds for every $\bar{p}_{e_1}, \bar{q}_{e_2} \in \bar{X}$ and $p \in N$,*

$$(3.4) \qquad\qquad lt_{p\to\infty}[\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, k^p\varrho)]^p = 1_{L^*}.$$

Lemma 3.1. *Consider* $(\bar{X}, \varpi_{M,N}, \Theta)$ *be MIFSMS with* $\varpi_{M,N}$ *satisfying p-property, then* (3.5) *holds for every* $\bar{p}_{e_1}, \bar{q}_{e_2} \in \bar{X}$,

$$(3.5) \qquad\qquad lt_{\varrho \to \infty} \varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho) = 1_{L^*}.$$

Proof. Consider (3.5) does not hold, since $\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, .)$ is non decreasing, there exist $\bar{p}_{e_o}, \bar{q}_{e_o} \in \bar{X}$ so that

$$lt_{\varrho \to \infty} \varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho) = (\nu, N_s(\nu)) < 1_{L^*},$$

and for $k > 1$, $k^p \varrho \to \infty$ when $p \to \infty$ as $\varrho > 0$, thus we get

$$lt_{p \to \infty} [\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, k^p \varrho)]^p = 1_{L^*},$$

which is absurd. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Remark 3.3. *Condition* (3.5) *does not guarantees the p-property.*

Definition 3.9. *Consider* $\vartheta = \{v : v : R^+ \to R^+\}$ *so that* $v \in \vartheta$ *satisfies the conditions given below:*

  (i) v *is non decreasing,*
  (ii) v *is continuous,*
  (iii) $\Sigma_{n=0}^{\infty} v^n(\varrho) < \infty$ *for all* $\varrho > 0$, *where* $v^{n+1}(\varrho) = v^n(v(\varrho))$, $n \in N$.
It is trivial that $v(\varrho) < \varrho$ *for all* $\varrho > 0$ *and* $v \in \vartheta$.

Lemma 3.2. *Consider* $(\bar{X}, \varpi_{M,N}, \Theta)$ *be MIFSMS where* Θ *is continuous t-representable norm of H-type. If there exists* $v \in \vartheta$, *so that* (3.6) *holds for all* $\varrho > 0$

$$(3.6) \qquad\qquad \varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \vartheta(\varrho)) \geq_{L^*} \varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho)$$

then $\bar{p}_{e_1} = \bar{q}_{e_2}$.

Proof. Since for $\varrho > 0$, $\vartheta(\varrho) < \varrho$. As $\varpi_{M,N}$ is non- decreasing, we have

$$(3.7) \qquad\qquad \varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \vartheta(\varrho)) \geq_{L^*} \varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho).$$

Utilizing (3.6) we have $\bar{p}_{e_1} = \bar{q}_{e_2}$. $\qquad\qquad\qquad\qquad\qquad\qquad\square$

Now, we are going to prove a coupled fixed point theorem for weakly compatible maps in the setting of MIFSMS.

Theorem 3.3. *Consider* $(\bar{X}, \varpi_{M,N}, \Theta)$ *be MIFSMS where* Θ *H-type continuous t-representable norm that satisfies* (3.5). *Consider* $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ *and* $\beta : \bar{X} \to \bar{X}$ *be two weakly compatible maps and* $v \in \vartheta$ *so that* (3.8) *holds for all* $\bar{p}_{e_1}, \bar{q}_{e_2}, \bar{r}_{e_3}, \bar{s}_{e_4} \in \bar{X}$ *and* $\varrho > 0$

(3.8)
$$\varpi_{M,N}(\alpha(\bar{p}_{e_1}, \bar{q}_{e_2}), \alpha(\bar{r}_{e_3}, \bar{s}_{e_4}), \vartheta(\varrho)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}_{e_1}, \beta\bar{r}_{e_3}, \varrho), \varpi_{M,N}(\beta\bar{q}_{e_3}, \beta\bar{s}_{e_4}, \varrho)).$$

Let $\alpha(\bar{X} \times \bar{X}) \subseteq \beta(\bar{X})$ *and* $\alpha(\bar{X} \times \bar{X})$ *or* $\beta(\bar{X})$ *is complete, then* α *and* β *possess a unique coupled fixed point, that is there exist* $\bar{p} \in \bar{X}$ *so that* $\bar{p} = \beta(\bar{p}) = \alpha(\bar{p}, \bar{p})$.

Proof. Consider $\bar{p}_{e_o}, \bar{q}_{e_o} \in \bar{X}$. Since $\alpha(\bar{X} \times \bar{X}) \subseteq \beta(\bar{X})$, take $\bar{p}_{e_1}, \bar{q}_{e_1} \in \bar{X}$ so that

$$\beta(\bar{p}_{e_1}) = \alpha(\bar{p}_{e_o}, \bar{q}_{e_o}), \beta(\bar{q}_{e_1}) = \alpha(\bar{q}_{e_o}, \bar{p}_{e_o}).$$

By Induction, we have two sequences $\{\bar{p}_{e_n}\}$ and $\{\bar{q}_{e_n}\}$ in $\bar{X}$ so that

$$\beta(\bar{p}_{e_{n+1}}) = \alpha(\bar{p}_{e_n}, \bar{q}_{e_n}), \beta(\bar{q}_{e_{n+1}}) = \alpha(\bar{q}_{e_n}, \bar{p}_{e_n}),$$

for all $n \geq 0$. We have divided the proof into 4 parts.
Part 1. Claim that sequences $\{\beta\bar{p}_{e_n}\}$ and $\{\beta\bar{q}_{e_n}\}$ are Cauchy.

Since Θ H-type is continuous t-representable norm, thus for any $\nu > 0$, there exists a $\xi > 0$ so that

$$(3.9) \qquad \Theta^{m-1}(N_s(\xi), \xi) \geq_{L^*} (N_s(\nu), \nu)$$

for all $m \in N$.

Since $lt_{\varrho \to \infty} \varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho) = 1_{L^*}$ for all $\bar{p}_{e_1}, \bar{q}_{e_2} \in \bar{X}$, there exist $\varrho_o > 0$ so that

$$(3.10) \qquad \begin{aligned} \varpi_{M,N}(\beta \bar{p}_{e_o}, \beta \bar{p}_{e_1}, \varrho_o) &\geq_{L^*} (N_s(\xi), \xi), \\ \varpi_{M,N}(\beta \bar{q}_{e_o}, \beta \bar{q}_{e_1}, \varrho_o) &\geq_{L^*} (N_s(\xi), \xi). \end{aligned}$$

Also $\upsilon \in \vartheta$, thus we have $\sum_{n=1}^{\infty} \upsilon^n(\varrho_o) < \infty$, then for $\varrho > 0$, there exists $n_o \in N$ so that

$$(3.11) \qquad \varrho > \sum_{k=n_o}^{\infty} \upsilon^n(\varrho_o).$$

From condition (3.8), we have

$$\begin{aligned} \varpi_{M,N}(\beta \bar{p}_{e_1}, \beta \bar{p}_{e_2}, \upsilon(\varrho_o)) &= \varpi_{M,N}(\alpha(\bar{p}_{e_o}, \bar{q}_{e_o}), \alpha(\bar{p}_{e_1}, \bar{q}_{e_1}), \upsilon(\varrho_o)), \\ &\geq_{L^*} \Theta(\varpi_{M,N}(\beta \bar{p}_{e_o}, \beta \bar{p}_{e_1}, \varrho_o), \varpi_{M,N}(\beta \bar{q}_{e_o}, \beta \bar{q}_{e_1}, \varrho_o)) \end{aligned}$$

$$(3.12)$$
$$\begin{aligned} \varpi_{M,N}(\beta \bar{q}_{e_1}, \beta \bar{q}_{e_2}, \upsilon(\varrho_o)) &= \varpi_{M,N}(\alpha(\bar{q}_{e_o}, \bar{p}_{e_o}), \alpha(\bar{q}_{e_1}, \bar{p}_{e_1}), \upsilon(\varrho_o)) \\ &\geq_{L^*} \Theta(\varpi_{M,N}(\beta \bar{q}_{e_o}, \beta \bar{q}_{e_1}, \varrho_o), \varpi_{M,N}(\beta \bar{p}_{e_o}, \beta \bar{p}_{e_1}, \varrho_o)). \end{aligned}$$

Similarly, we also get

$$\begin{aligned} \varpi_{M,N}(\beta \bar{p}_{e_2}, \beta \bar{p}_{e_3}, \upsilon^2(\varrho_o)) &= \varpi_{M,N}(\alpha(\bar{p}_{e_1}, \bar{q}_{e_1}), \alpha(\bar{p}_{e_2}, \bar{q}_{e_2}), \upsilon^2(\varrho_o)) \\ &\geq_{L^*} \Theta(\varpi_{M,N}(\beta \bar{p}_{e_1}, \beta \bar{p}_{e_2}, \upsilon(\varrho_o)), \varpi_{M,N}(\beta \bar{q}_{e_1}, \beta \bar{q}_{e_2}, \upsilon(\varrho_o))) \\ &\geq_{L^*} \Theta([\varpi_{M,N}(\beta \bar{p}_{e_o}, \beta \bar{p}_{e_1}, \varrho_o)]^2, [\varpi_{M,N}(\beta \bar{q}_{e_o}, \beta \bar{q}_{e_1}, \varrho_o)]^2), \end{aligned}$$

$$(3.13)$$
$$\begin{aligned} \varpi_{M,N}(\beta \bar{q}_{e_2}, \beta \bar{q}_{e_3}, \upsilon^2(\varrho_o)) &= \varpi_{M,N}(\alpha(\bar{q}_{e_1}, \bar{p}_{e_1}), \alpha(\bar{q}_{e_2}, \bar{p}_{e_2}), \upsilon^2(\varrho_o)) \\ &\geq_{L^*} \Theta(\varpi_{M,N}(\beta \bar{q}_{e_1}, \beta \bar{q}_{e_2}, \upsilon(\varrho_o)), \varpi_{M,N}(\beta \bar{p}_{e_1}, \beta \bar{p}_{e_2}, \upsilon(\varrho_o))) \\ &\geq_{L^*} \Theta([\varpi_{M,N}(\beta \bar{q}_{e_o}, \beta \bar{q}_{e_1}, \varrho_o)]^2, [\varpi_{M,N}(\beta \bar{p}_{e_o}, \beta \bar{p}_{e_1}, \varrho_o)]^2). \end{aligned}$$

By Induction, we get

$$\varpi_{M,N}(\beta \bar{p}_{e_n}, \beta \bar{p}_{e_{n+1}}, \upsilon^n(\varrho_o)) \geq_{L^*} \Theta([\varpi_{M,N}(\beta \bar{p}_{e_o}, \beta \bar{p}_{e_1}, \varrho_o)]^{2^{n-1}}, [\varpi_{M,N}(\beta \bar{q}_{e_o}, \beta \bar{q}_{e_1}, \varrho_o)]^{2^{n-1}}),$$

$$(3.14)$$
$$\varpi_{M,N}(\beta \bar{q}_{e_n}, \beta \bar{q}_{e_{n+1}}, \upsilon^n(\varrho_o)) \geq_{L^*} \Theta([\varpi_{M,N}(\beta \bar{q}_{e_o}, \beta \bar{q}_{e_1}, \varrho_o)]^{2^{n-1}}, [\varpi_{M,N}(\beta \bar{p}_{e_o}, \beta \bar{p}_{e_1}, \varrho_o)]^{2^{n-1}}).$$

Thus from (3.9), (3.10) and (3.11), for $m > n \geq n_o$, we have

(3.15)

$$\varpi_{M,N}(\beta\bar{p}_{e_n}, \beta\bar{p}_{e_m}, \varrho) \geq_{L^*} \varpi_{M,N}(\beta\bar{p}_{e_n}, \beta\bar{p}_{e_m}, \sum_{k=n_o}^{\infty} \upsilon^k(\varrho_o))$$

$$\geq_{L^*} \varpi_{M,N}(\beta\bar{p}_{e_n}, \beta\bar{p}_{e_m}, \sum_{k=n}^{m-1} \upsilon^k(\varrho_o))$$

$$\geq_{L^*} \Theta^{m-n-1}(\varpi_{M,N}(\beta\bar{p}_{e_n}, \beta\bar{p}_{e_{n+1}}, \upsilon^n(\varrho_o)), \varpi_{M,N}(\beta\bar{p}_{e_{n+1}}, \beta\bar{p}_{e_{n+2}}, \upsilon^{n+1}(\varrho_o)),$$

$$..., \varpi_{M,N}(\beta\bar{p}_{e_{m-1}}, \beta\bar{p}_{e_m}, \upsilon^{m-1}(\varrho_o)))$$

$$\geq_{L^*} \Theta^{m-n-1}(\Theta([\varpi_{M,N}(\beta\bar{p}_{e_o}, \beta\bar{p}_{e_1}, \varrho_o)]^{2^{n-1}}, [\varpi_{M,N}(\beta\bar{q}_{e_o}, \beta\bar{q}_{e_1}, \varrho_o)]^{2^{n-1}}),$$

$$\Theta([\varpi_{M,N}(\beta\bar{p}_{e_o}, \beta\bar{p}_{e_1}, \varrho_o)]^{2^n}, [\varpi_{M,N}(\beta\bar{q}_{e_o}, \beta\bar{q}_{e_1}, \varrho_o)]^{2^n}),$$

$$..., \Theta([\varpi_{M,N}(\beta\bar{p}_{e_o}, \beta\bar{p}_{e_1}, \varrho_o)]^{2^{m-2}}, [\varpi_{M,N}(\beta\bar{q}_{e_o}, \beta\bar{q}_{e_1}, \varrho_o)]^{2^{m-2}})$$

$$\geq_{L^*} \Theta([\varpi_{M,N}(\beta\bar{p}_{e_o}, \beta\bar{p}_{e_1}, \varrho_o)]^{2^{m-1}-2^{n-1}}, [\varpi_{M,N}(\beta\bar{q}_{e_o}, \beta\bar{q}_{e_1}, \varrho_o)]^{2^{m-1}-2^{n-1}})$$

$$\geq_{L^*} \Theta^{2^m-2^n-1}(N_s(\xi), \xi)$$

$$\geq_{L^*} (N_s(\nu), \nu).$$

Thus, we have

(3.16)
$$\varpi_{M,N}(\beta\bar{p}_{e_n}, \beta\bar{p}_{e_m}, \varrho) >_{L^*} (N_s(\nu), \nu)$$

for all $m, n \in N$ with $m, n \geq n_o$ and $\varrho > 0$. Thus $\{\beta\bar{p}_{e_n}\}$ is a Cauchy sequence. Similarly, we can get $\{\beta\bar{q}_{e_n}\}$ is also a Cauchy sequence.

Part 2. Claim that β and α have a coupled coincidence point.
Consider that $\beta(\bar{X})$ is complete, the there exist $\bar{p}, \bar{q} \in \beta(\bar{X})$ and $\bar{a}, \bar{b} \in \bar{X}$ so that

(3.17)
$$lt_{n\to\infty}\beta\bar{p}_{e_n} = lt_{n\to\infty}\alpha(\bar{p}_{e_n}, \bar{q}_{e_n}) = \beta\bar{a} = \bar{p},$$
$$lt_{n\to\infty}\beta\bar{q}_{e_n} = lt_{n\to\infty}\alpha(\bar{q}_{e_n}, \bar{p}_{e_n}) = \beta\bar{b} = \bar{q}.$$

From equation (3.8), we get

(3.18)
$$\varpi_{M,N}(\alpha(\bar{p}_{e_n}, \bar{q}_{e_n}), \alpha(\bar{a}, \bar{b}), \upsilon(\varrho)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}_{e_n}, \beta\bar{a}, \varrho), \varpi_{M,N}(\beta\bar{q}_{e_n}, \beta\bar{b}, \varrho)).$$

As $\varpi_{M,N}$ is continuous, taking $n \to \infty$, we have

(3.19)
$$\varpi_{M,N}(\beta\bar{a}, \alpha(\bar{a}, \bar{b}), \upsilon(\varrho)) = 1_{L^*},$$

which implies

(3.20)
$$\alpha(\bar{a}, \bar{b}) = \beta\bar{a} = \bar{p}.$$

Similarly, we can claim that

(3.21)
$$\alpha(\bar{b}, \bar{a}) = \beta\bar{b} = \bar{q}.$$

Since α and β are weakly compatible, we have

(3.22)
$$\beta\alpha(\bar{a}, \bar{b}) = \alpha(\beta\bar{a}, \beta\bar{b}),$$
$$\beta\alpha(\bar{b}, \bar{a}) = \alpha(\beta\bar{b}, \beta\bar{a}),$$

which implies

$$(3.23) \qquad \begin{aligned} \beta\bar{p} &= \alpha(\bar{p}, \bar{q}), \\ \beta\bar{q} &= \alpha(\bar{q}, \bar{p}). \end{aligned}$$

Part 3. Claim that $\beta\bar{p} = \bar{q}$ and $\beta\bar{q} = \bar{p}$.
As Θ is continuous t-representable norm of H-type. Thus for any $\nu > 0$, there exist $\xi > 0$ so that

$$(3.24) \qquad \theta^{m-1}(N_s(\xi), \xi) \geq_{L^*} (N_s(\nu), \nu),$$

for all $m \in N$.
Since $lt_{\varrho \to \infty} \varpi_{M,N}(\bar{p}, \bar{q}, \varrho) = 1_{L^*}$ for all $\bar{p}, \bar{q} \in \bar{X}$, there exist $\varrho_o > 0$ so that

$$(3.25) \qquad \begin{aligned} \varpi_{M,N}(\beta\bar{p}, \bar{q}, \varrho_o) &\geq_{L^*} (N_s(\xi), \xi), \\ \varpi_{M,N}(\beta\bar{q}, \bar{p}, \varrho_o) &\geq_{L^*} (N_s(\xi), \xi). \end{aligned}$$

Also $\upsilon \in \vartheta$, so we have $\sum_{n=1}^{\infty} \upsilon^n(\varrho_o) < \infty$. Then for any $\varrho > 0$, there exist $n_o \in N$ so that

$$(3.26) \qquad \sum_{k=n_o}^{\infty} \upsilon^k(\varrho_o) < \varrho.$$

Since

$$(3.27) \qquad \begin{aligned} \varpi_{M,N}(\beta\bar{p}, \beta\bar{q}_{e_{n+1}}, \upsilon(\varrho_o)) &= \varpi_{M,N}(\alpha(\bar{p}, \bar{q}), \alpha(\bar{q}_{e_n}, \bar{p}_{e_n}), \upsilon(\varrho_o)) \\ &\geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}, \beta\bar{q}_{e_n}, \varrho_o), \varpi_{M,N}(\beta\bar{q}, \beta\bar{p}_{e_n}, \varrho_o)). \end{aligned}$$

thus

$$(3.28) \qquad \varpi_{M,N}(\beta\bar{p}, \beta\bar{q}_{e_{n+1}}, \upsilon(\varrho_o)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}, \beta\bar{q}_{e_n}, \varrho_o), \varpi_{M,N}(\beta\bar{q}, \beta\bar{p}_{e_n}, \varrho_o)).$$

Taking $n \to \infty$, we get

$$(3.29) \qquad \varpi_{M,N}(\beta\bar{p}, \bar{q}, \upsilon(\varrho_o)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}, \bar{q}, \varrho_o), \varpi_{M,N}(\beta\bar{q}, \bar{p}, \varrho_o)).$$

Similarly, we can get

$$(3.30) \qquad \varpi_{M,N}(\beta\bar{q}, \bar{p}, \upsilon(\varrho_o)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}, \bar{q}, \varrho_o), \varpi_{M,N}(\beta\bar{q}, \bar{p}, \varrho_o)).$$

By equations (3.29) and (3.30) we get

$$(3.31)$$
$$\Theta(\varpi_{M,N}(\beta\bar{p}, \bar{q}, \upsilon(\varrho_o)), \varpi_{M,N}(\beta\bar{q}, \bar{p}, \upsilon(\varrho_o))) \geq_{L^*} \Theta([\varpi_{M,N}(\beta\bar{p}, \bar{q}, \varrho_o)]^2, [\varpi_{M,N}(\beta\bar{q}, \bar{p}, \varrho_o)]^2).$$

By induction we have, for all $n \in N$

$$(3.32)$$
$$\begin{aligned} \Theta(\varpi_{M,N}(\beta\bar{p}, \bar{q}, \upsilon^n(\varrho_o)), \varpi_{M,N}(\beta\bar{q}, \bar{p}, \upsilon^n(\varrho_o))) &\geq_{L^*} \Theta([\varpi_{M,N}(\beta\bar{p}, \bar{q}, \upsilon^{n-1}(\varrho_o))]^2, [\varpi_{M,N}(\beta\bar{q}, \bar{p}, \upsilon^{n-1}(\varrho_o))]^2) \\ &\geq_{L^*} \Theta([\varpi_{M,N}(\beta\bar{p}, \bar{q}, \varrho_o)]^{2^n}, [\varpi_{M,N}(\beta\bar{q}, \bar{p}, \varrho_o)]^{2^n}). \end{aligned}$$

Thus

$$(3.33)$$
$$\Theta(\varpi_{M,N}(\beta\bar{p}, \bar{q}, \upsilon^n(\varrho_o)), \varpi_{M,N}(\beta\bar{q}, \bar{p}, \upsilon^n(\varrho_o))) \geq_{L^*} \Theta([\varpi_{M,N}(\beta\bar{p}, \bar{q}, \varrho_o)]^{2^n}, [\varpi_{M,N}(\beta\bar{q}, \bar{p}, \varrho_o)]^{2^n}).$$

By equations (3.24), (3.25), (3.26) and (3.33), we have
(3.34)

$$\Theta(\varpi_{M,N}(\beta\bar{p},\bar{q},\varrho),\varpi_{M,N}(\beta\bar{q},\bar{p},\varrho)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p},\bar{q},\sum_{k=n_o}^{\infty} \upsilon^k(\varrho_o)),\varpi_{M,N}(\beta\bar{q},\bar{p},\sum_{k=n_o}^{\infty}\upsilon^k(\varrho_o)))$$

$$\geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p},\bar{q},\upsilon^{n_o}(\varrho_o)),\varpi_{M,N}(\beta\bar{q},\bar{p},\upsilon^{n_o}(\varrho_o)))$$

$$\geq_{L^*} \Theta([\varpi_{M,N}(\beta\bar{p},\bar{q},\varrho_o)]^{2^{n_o}},[\varpi_{M,N}(\beta\bar{q},\bar{p},\varrho_o)]^{2^{n_o}})$$

$$\geq_{L^*} \Theta^{2^{n_o+1}-1}(N_s(\xi),\xi)$$

$$\geq_{L^*} (N_s(\nu),\nu).$$

Thus for any $\nu > 0$, we have
(3.35)
$$\Theta(\varpi_{M,N}(\beta\bar{p},\bar{q},\varrho),\varpi_{M,N}(\beta\bar{q},\bar{p},\varrho)) \geq_{L^*} (N_s(\nu),\nu)$$

for all $\varrho > 0$. So we get $\beta\bar{p} = \bar{q}$ and $\beta\bar{q} = \bar{p}$.
Part 4. Claim that $\bar{p} = \bar{q}$.
For any $\varrho_o > 0$, we have
(3.36)
$$\varpi_{M,N}(\beta\bar{p}_{e_{n+1}},\beta\bar{q}_{e_{n+1}},\upsilon(\varrho_o)) = \varpi_{M,N}(\alpha(\bar{p}_{e_n},\bar{q}_{e_n}),\alpha(\bar{q}_{e_n},\bar{p}_{e_n}),\upsilon(\varrho_o))$$

$$\geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}_{e_n},\beta\bar{q}_{e_n},\varrho_o),\varpi_{M,N}(\beta\bar{q}_{e_n},\beta\bar{p}_{e_n},\varrho_o)),$$

thus
(3.37)
$$\varpi_{M,N}(\beta\bar{p}_{e_{n+1}},\beta\bar{q}_{e_{n+1}},\upsilon(\varrho_o)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}_{e_n},\beta\bar{q}_{e_n},\varrho_o),\varpi_{M,N}(\beta\bar{q}_{e_n},\beta\bar{p}_{e_n},\varrho_o)).$$

Taking $n \to \infty$, we get
(3.38)
$$\varpi_{M,N}(\bar{p},\bar{q},\upsilon(\varrho_o)) \geq_{L^*} \Theta(\varpi_{M,N}(\bar{p},\bar{q},\varrho_o),\varpi_{M,N}(\bar{q},\bar{p},\varrho_o)).$$

By equations (3.24), (3.25), (3.26) and (3.38), we get

$$\varpi_{M,N}(\bar{p},\bar{q},\upsilon(\varrho_o)) \geq_{L^*} \varpi_{M,N}(\bar{p},\bar{q},\sum_{k=n_o}^{\infty}\upsilon^k(\varrho_o))$$

$$\geq_{L^*} \varpi_{M,N}(\bar{p},\bar{q},\upsilon^{n_o}(\varrho_o))$$

(3.39)
$$\geq_{L^*} \Theta([\varpi_{M,N}(\bar{p},\bar{q},\varrho_o)]^{2^{n_o-1}},[\varpi_{M,N}(\bar{q},\bar{p},\varrho_o)]^{2^{n_o-1}})$$

$$\geq_{L^*} \Theta^{2^{n_o+1}-3}(N_s(\xi),\xi)$$

$$\geq_{L^*} (N_s(\nu),\nu),$$

that implies $\bar{p} = \bar{q}$. Thus, α and β possess unique coupled fixed point. This completes the proof. $\square$

Following are the two corollaries of our main result (3.3):

Corollary 3.1. *Consider* $(\bar{X},\varpi_{M,N},\Theta)$ *be complete MIFSMS where* Θ *is continuous t-representable norm of H-type that satisfies (3.5). Consider* $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ *and* $\upsilon \in \vartheta$ *so that (3.40) holds for all* $\bar{p}_{e_1},\bar{q}_{e_2},\bar{r}_{e_3},\bar{s}_{e_4} \in \bar{X}$ *and* $\varrho > 0$
(3.40)
$$\varpi_{M,N}(\alpha(\bar{p}_{e_1},\bar{q}_{e_2}),\alpha(\bar{r}_{e_3},\bar{s}_{e_4}),\vartheta(\varrho)) \geq_{L^*} \Theta(\varpi_{M,N}(\bar{p}_{e_1},\bar{r}_{e_3},\varrho),\varpi_{M,N}(\bar{q}_{e_2},\bar{s}_{e_4},\varrho)).$$

Then there exist $\bar{p} \in \bar{X}$ *so that* $\bar{p} = \alpha(\bar{p},\bar{p})$.

Proof. Take $\beta = I$ in Theorem (3.3), we get the result. $\square$

Corollary 3.2. *Consider* $(\bar{X}, \varpi_{M,N}, \Theta)$ *be MIFSMS so that* $\varpi_{M,N}$ *has p-property. Let* $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ *and* $\beta : \bar{X} \to \bar{X}$ *be two functions so that (3.41) holds for all* $\bar{p}_{e_1}, \bar{q}_{e_2}, \bar{r}_{e_3}, \bar{s}_{e_4} \in \bar{X}$ *and* $\varrho > 0$*, where* $0 < k < 1$

(3.41)
$$\varpi_{M,N}(\alpha(\bar{p}_{e_1}, \bar{q}_{e_2}), \alpha(\bar{r}_{e_3}, \bar{s}_{e_4}), k\varrho) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}_{e_1}, \beta\bar{r}_{e_3}, \varrho), \varpi_{M,N}(\beta\bar{q}_{e_2}, \beta\bar{s}_{e_4}, \varrho)).$$

Let $\alpha(\bar{X} \times \bar{X}) \subseteq \beta(\bar{X})$ *and* $\alpha(\bar{X} \times \bar{X})$ *or* $\beta(\bar{X})$ *is complete, then* α *and* β *possess a unique coupled fixed point, that is there exist a unique* $\bar{p} \in \bar{X}$ *so that* $\bar{p} = \beta(\bar{p}) = \alpha(\bar{p}, \bar{p})$.

Following is an example validating Theorem 3.3:

Example 3.4. *Consider* $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{n}, ...\}$ *and* $\Theta(\bar{a}, \bar{b}) = (min\{\bar{a}_1, \bar{b}_1\}, max\{\bar{a}_2, \bar{b}_2\})$, *for all* $\bar{a} = (\bar{a}_1, \bar{a}_2)$ *and* $\bar{b} = (\bar{b}_1, \bar{b}_2) \in L^*$. *Define*

$$\varpi_{M,N}(\bar{p}_{e_1}, \bar{q}_{e_2}, \varrho) = (\frac{\varrho}{\varrho + |\bar{p}_{e_1} - \bar{q}_{e_2}|}, \frac{|\bar{p}_{e_1} - \bar{q}_{e_2}|}{\varrho + |\bar{p}_{e_1} - \bar{q}_{e_2}|}),$$

for all $\bar{p}_{e_1}, \bar{q}_{e_2} \in \bar{X}$ *and* $\varrho > 0$. *Then* $(\bar{X}, \varpi_{M,N}, \Theta)$ *is a MIFSMS. Consider* $\upsilon(\varrho) = \frac{\varrho}{2}$, $\beta : \bar{X} \to \bar{X}$ *and* $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ *as*

(3.42) $\quad \beta(\bar{p}) = \begin{cases} 0 & if \bar{p} = 0, \\ 1 & if \bar{p} = \frac{1}{2n+1}, \\ \frac{1}{2n+1} & if \bar{p} = \frac{1}{2n}. \end{cases} \qquad \alpha(\bar{p}, \bar{q}) = \begin{cases} \frac{1}{(2n+1)^4} & if (\bar{p}, \bar{q}) = (\frac{1}{2n}, \frac{1}{2n}), \\ 0 & otherwise. \end{cases}$

Consider $\bar{p}_{e_n} = \bar{q}_{e_n} = \frac{1}{2n}$. *Taking* $n \to \infty$, *we have*

(3.43)
$$\beta\bar{p}_{e_n} = \frac{1}{2n+1} \to 0,$$
$$\alpha(\bar{p}_{e_n}, \bar{q}_{e_n}) = \frac{1}{2n+1} \to 0,$$

but

(3.44) $\qquad \varpi_{M,N}(\alpha(\beta\bar{p}_{e_n}, \beta\bar{q}_{e_n}), \beta\alpha(\bar{p}_{e_n}, \bar{q}_{e_n}), \varrho) = \varpi_{M,N}(0, 1, \varrho) \not\to 1_{L^*}.$

Thus β *and* α *are not compatible. As* $\alpha(\bar{p}, \bar{q}) = \beta(\bar{p})$ *and* $\alpha(\bar{q}, \bar{p}) = \beta(\bar{q})$, *we get* $(\bar{p}, \bar{q}) = (0, 0)$ *and thus* $\beta\alpha(0, 0) = \alpha(\beta0, \beta0)$. *Hence,* α *and* β *are weakly compatible. It is trivial that*

(3.45)
$$\frac{\varrho}{P + \varrho} \geq min\{\frac{\varrho}{Q + \varrho}, \frac{\varrho}{R + \varrho}\},$$
$$\frac{P}{P + \varrho} \geq max\{\frac{Q}{Q + \varrho}, \frac{R}{R + \varrho}\},$$

if and only if $P \leq max\{Q, R\}$, *for all* $P, Q, R \geq 0$, $\varrho > 0$. *Thus, we get the following inequality:*

(3.46) $\quad \varpi_{M,N}(\alpha(\bar{p}, \bar{q}), \alpha(\bar{u}, \bar{v}), \upsilon(\varrho)) \geq_{L^*} \Theta(\varpi_{M,N}(\beta\bar{p}, \beta\bar{u}, \varrho), \varpi_{M,N}(\beta\bar{q}, \beta\bar{v}, \varrho)),$

that is equivalent to the following

(3.47) $\qquad 2 |\alpha(\bar{p}, \bar{q}) - \alpha(\bar{u}, \bar{v})| \leq max\{|\beta\bar{p} - \beta\bar{u}|, |\beta\bar{q} - \beta\bar{v}|\}.$

Thus by (3.46), if $\bar{p} = \bar{q}$ and $\bar{u} = \bar{v}$, consider $\bar{p} = \bar{q} = \frac{1}{2l}$, $\bar{u} = \bar{v} = \frac{1}{2m}$. Then

$$
\begin{aligned}
2 \mid \alpha(\bar{p}, \bar{q}) - \alpha(\bar{u}, \bar{v}) \mid &= 2 \mid \frac{1}{(2l+1)^4} - \frac{1}{(2m+1)^4} \mid \\
&\leq max\{\mid \beta\bar{p} - \beta\bar{u} \mid, \mid \beta\bar{q} - \beta\bar{v} \mid\} \\
&= \frac{1}{2l+1} - \frac{1}{2m+1}.
\end{aligned}
$$

(3.48)

Hence all the conditions of theorem (3.3) are satisfied and β and α possess 0 as their unique coupled fixed point.

4. APPLICATION

In this section we are going to give an application on Fredholm nonlinear integral equation utilizing our newly developed results.
Consider the following integral equation:

(4.1) $$\bar{\zeta}(\bar{\sigma}) = \int_{\bar{r}}^{\bar{s}} (F_1(\bar{\sigma}, \bar{\varsigma}) + F_2(\bar{\sigma}, \bar{\varsigma})) \times [\Gamma(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma}))d\bar{\varsigma} + \Lambda(\bar{\sigma}),$$

for all $\bar{\sigma} \in K = [\bar{r}, \bar{s}]$.
Consider Ψ be the set consisting of all functions $\psi : R(U)^* \to R(U)^*$, where $R(U)^*$ is the set of non negative soft real numbers, that satisfies the following conditions:

(i) ψ is increasing,
(ii) $\psi(\bar{\sigma}) \leq \bar{\sigma}$.

Let F_1, F_2, f, g satisfies the following conditions.

(i) For all $\bar{\sigma}, \bar{\varsigma} \in K$

(4.2) $$F_1(\bar{\sigma}, \bar{\varsigma}) \geq \bar{0}, F_2(\bar{\sigma}, \bar{\varsigma}) \leq \bar{0}.$$

(ii) For all $\bar{\zeta}, \bar{\eta} \in R$ such that $\bar{\zeta} \geq \bar{\eta}$ there exist positive soft real numbers $\bar{\lambda}, \bar{\mu}$ and $\psi \in \Psi$ so that

(4.3)
$$
\begin{aligned}
\bar{0} \leq \Gamma(\bar{\varsigma}, \bar{\zeta}) - \Gamma(\bar{\varsigma}, \bar{\eta}) \leq \bar{\lambda}\psi(\bar{\zeta} - \bar{\eta}), \\
-\bar{\mu}\psi(\bar{\zeta} - \bar{\eta}) \leq \Delta(\bar{\varsigma}, \bar{\zeta}) - \Delta(\bar{\varsigma} - \bar{\eta}) \leq \bar{0}.
\end{aligned}
$$

(iii) Suppose

(4.4) $$max\{\bar{\lambda}, \bar{\mu}\}sup_{\bar{\sigma} \in K} \int_{\bar{r}}^{\bar{s}} [F_1(\bar{\sigma}, \bar{\varsigma}) - F_2(\bar{\sigma}, \bar{\varsigma})] \leq \frac{\bar{1}}{8}$$

Theorem 4.1. *Consider the integral equation (4.1) with $F_1, F_2 \in C(K \times K, R(U)^*)$, $\alpha, \beta \in C(K \times R(U)^*, R(U)^*)$ and $\Lambda \in C(K, R(U)^*)$. Consider (4.2), (4.3) and (4.4) holds, then the integral equation (4.1) possess a unique solution in $C(K, R)$*

Proof. Consider $\bar{X} = C(K, R)$, then $(\bar{X}, \varpi_{M,N}, \Theta)$ is a complete MIFSMS, where $\varpi_{M,N}$ is defined as follows:

$$\varpi_{M,N}(\bar{\zeta}, \bar{\eta}, \varrho) = \left(\frac{\varrho}{\varrho + \mid \bar{\zeta} - \bar{\eta} \mid}, \frac{\mid \bar{\zeta} - \bar{\eta} \mid}{\varrho + \mid \bar{\zeta} - \bar{\eta} \mid} \right)$$

for all $\bar{\zeta}, \bar{\eta} \in \bar{X}$, $\varrho > 0$ and $\Theta(\bar{a}, \bar{b}) = (min\{\bar{a}_1, \bar{b}_1\}, max\{\bar{a}_2, \bar{b}_2\})$, for all $\bar{a} = (\bar{a}_1, \bar{a}_2)$ and $\bar{b} = (\bar{b}_1, \bar{b}_2) \in L^*$. Consider map $\alpha : \bar{X} \times \bar{X} \to \bar{X}$ defined as

$$
(4.5) \quad
\begin{aligned}
\alpha(\bar{\zeta}, \bar{\eta})(\bar{\sigma}) &= \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma}))]d\bar{\varsigma} \\
&+ \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma}))]d\bar{\varsigma} + \Lambda(\bar{\sigma})
\end{aligned}
$$

for all $\bar{\sigma} \in K$ and $\upsilon(\varrho) = \frac{\varrho}{2}$. Thus, for all $\bar{\zeta}, \bar{\eta}, \bar{\gamma}, \bar{\delta} \in \bar{X}$.

$$
\begin{aligned}
\alpha(\bar{\zeta}, \bar{\eta})(\bar{\sigma}) - \alpha(\bar{\gamma}, \bar{\delta})(\bar{\sigma}) &= \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma}))]d\bar{\varsigma} + \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma}))]d\bar{\varsigma} \\
&- \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\gamma}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\delta}(\bar{\varsigma}))]d\bar{\varsigma} - \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\delta}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\gamma}(\bar{\varsigma}))]d\bar{\varsigma} \\
&= \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma})) - \Gamma(\bar{\varsigma}, \bar{\gamma}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma})) - \Delta(\bar{\varsigma}, \bar{\delta}(\bar{\varsigma}))]d\bar{\varsigma} \\
&+ \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma})) - \Gamma((\bar{\varsigma}, \bar{\delta}(\bar{\varsigma})) + \Delta(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma})) - \Delta(\bar{\varsigma}, \bar{\gamma}(\bar{\varsigma}))]d\bar{\varsigma} \\
&= \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma})) - \Gamma(\bar{\varsigma}, \bar{\gamma}(\bar{\varsigma})) - (\Delta(\bar{\varsigma}, \bar{\delta}(\bar{\varsigma})) - \Delta(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma})))]d\bar{\varsigma} \\
&+ \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\Gamma(\bar{\varsigma}, \bar{\delta}(\bar{\varsigma})) - \Gamma(\bar{\varsigma}, \bar{\eta}(\bar{\varsigma})) - (\Delta(\bar{\varsigma}, \bar{\zeta}(\bar{\varsigma})) - \Delta(\bar{\varsigma}, \bar{\gamma}(\bar{\varsigma})))]d\bar{\varsigma} \\
&\leq \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\bar{\lambda}\psi(\bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma})) + \bar{\mu}\psi(\bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}))]d\bar{\varsigma} \\
&- \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\bar{\lambda}\psi(\bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma})) + \bar{\mu}\psi(\bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}))]d\bar{\varsigma}.
\end{aligned}
$$

Hence, we have

$$
(4.6) \quad
\begin{aligned}
\alpha(\bar{\zeta}, \bar{\eta})(\bar{\sigma}) - \alpha(\bar{\gamma}, \bar{\delta})(\bar{\sigma}) &\leq \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\bar{\lambda}\psi(\bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma})) + \bar{\mu}\psi(\bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}))]d\bar{\varsigma} \\
&- \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\bar{\lambda}\psi(\bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma})) + \bar{\mu}\psi(\bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}))]d\bar{\varsigma}.
\end{aligned}
$$

As ψ is increasing, thus we have

$$
(4.7) \quad
\begin{aligned}
\psi(\bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma})) &\leq \psi(\mid \bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}) \mid) \\
\psi(\bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma})) &\leq \psi(\mid \bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}) \mid)
\end{aligned}
$$

Thus by (4.6), as $F_2(\bar{\sigma}, \bar{\varsigma}) \leq \bar{0}$, we get

(4.8)

$$| \alpha(\bar{\zeta}, \bar{\eta})(\bar{\sigma}) - \alpha(\bar{\gamma}, \bar{\delta})(\bar{\sigma}) | \leq \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[\bar{\lambda}\psi \mid \bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}) |) + \bar{\mu}\psi(| \bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}) |)]d\bar{\varsigma}$$

$$- \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[\bar{\lambda}\psi(| \bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}) |) + \bar{\mu}\psi(| \bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}) |)]d\bar{\varsigma}$$

$$\leq \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma})[max\{\bar{\lambda}, \bar{\mu}\}\psi(| \bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}) |) + max\{\bar{\lambda}, \bar{\mu}\}\psi(| \bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}) |)]d\bar{\varsigma}$$

$$- \int_{\bar{r}}^{\bar{s}} F_2(\bar{\sigma}, \bar{\varsigma})[max\{\bar{\lambda}, \bar{\mu}\}\psi(| \bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}) |) + max\{\bar{\lambda}, \bar{\mu}\}\psi(| \bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}) |)]d\bar{\varsigma}.$$

Using (4.4), we have

(4.9)

$$| \alpha(\bar{\zeta}, \bar{\eta})(\bar{\sigma}) - \alpha(\bar{\sigma}, \bar{\delta})(\bar{\sigma}) | \leq max\{\bar{\lambda}, \bar{\mu}\} \int_{\bar{r}}^{\bar{s}} F_1(\bar{\sigma}, \bar{\varsigma}) - F_2(\bar{\sigma}, \bar{\varsigma})d\bar{\varsigma}[\psi(| \bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}) |) + \psi(| \bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}) |)]$$

$$\leq max\{\bar{\lambda}, \bar{\mu}\}sup_{\bar{\sigma} \in K} \int_{\bar{r}}^{\bar{s}} [F_1(\bar{\sigma}, \bar{\varsigma}) - F_2(\bar{\sigma}, \bar{\varsigma})]d\bar{\varsigma}[\psi(| \bar{\zeta}(\bar{\varsigma}) - \bar{\gamma}(\bar{\varsigma}) |) + \psi(| \bar{\delta}(\bar{\varsigma}) - \bar{\eta}(\bar{\varsigma}) |)]$$

$$\leq \frac{\psi(| \bar{\zeta} - \bar{\gamma} |) + \psi(| \bar{\delta} - \bar{\eta})}{8}$$

Thus

(4.10) $$2 \mid \alpha(\bar{\zeta}, \bar{\eta})(\bar{\sigma}) - \alpha(\bar{\gamma}, \bar{\delta})(\bar{\sigma}) \mid \leq \frac{\psi(| \bar{\zeta} - \bar{\gamma} |) + \psi(| \bar{\delta} - \bar{\eta})}{4}$$

As ψ is increasing, we have

(4.11)
$$\psi(| \bar{\zeta} - \bar{\gamma} |) \leq \psi(| \bar{\zeta} - \bar{\gamma} | + | \bar{\eta} - \bar{\delta} |),$$
$$\psi(| \bar{\eta} - \bar{\delta} |) \leq \psi(| \bar{\zeta} - \bar{\gamma} | + | \bar{\eta} - \bar{\delta} |).$$

Thus, we get

(4.12) $$\frac{\psi(| \bar{\zeta} - \bar{\gamma} |) + \psi(| \bar{\eta} - \bar{\delta} |)}{2} \leq \psi(| \bar{\zeta} - \bar{\gamma} | + | \bar{\eta} - \bar{\delta} |).$$

Hence, by (4.10) and (4.12)

(4.13) $$2 \mid \alpha(\bar{\zeta}, \bar{\eta})(\bar{\sigma}) - \alpha(\bar{\gamma}, \bar{\delta})(\bar{\sigma}) \mid \leq max\{| \bar{\zeta} - \bar{\gamma} |, | \bar{\eta} - \bar{\delta} |\}.$$

By (3.45) and (4.13), we get

(4.14)

$$\varpi_{M,N}(\alpha(\bar{\zeta}, \bar{\eta}), \alpha(\bar{\gamma}, \bar{\delta}), \frac{\varrho}{2}) = (\frac{\frac{\varrho}{2}}{\frac{\varrho}{2} + | \alpha(\bar{\zeta}, \bar{\eta}) - \alpha(\bar{\gamma}, \bar{\delta}) |}, \frac{| \alpha(\bar{\zeta}, \bar{\eta}) - \alpha(\bar{\gamma}, \bar{\delta}) |}{\frac{\varrho}{2} + | \alpha(\bar{\zeta}, \bar{\eta}) - \alpha(\bar{\gamma}, \bar{\delta}) |})$$

$$= (\frac{\varrho}{\varrho + 2 | \alpha(\bar{\zeta}, \bar{\eta}) - \alpha(\bar{\gamma}, \bar{\delta}) |}, \frac{2 | \alpha(\bar{\zeta}, \bar{\eta}) - \alpha(\bar{\gamma}, \bar{\delta}) |}{\varrho + 2 | \alpha(\bar{\zeta}, \bar{\eta}) - \alpha(\bar{\gamma}, \bar{\delta}) |})$$

$$\geq_{L^*} (\frac{\varrho}{\varrho + max\{| \bar{\zeta} - \bar{\gamma} |, | \bar{\eta} - \bar{\delta} |\}}, \frac{max\{| \bar{\zeta} - \bar{\gamma} |, | \bar{\eta} - \bar{\delta} |\}}{\varrho + max\{| \bar{\zeta} - \bar{\gamma} |, | \bar{\eta} - \bar{\delta} |\}})$$

$$\geq_{L^*} (min\{\frac{\varrho}{\varrho + | \bar{\zeta} - \bar{\gamma} |}, \frac{\varrho}{\varrho + | \bar{\eta} - \bar{\delta} |}\}, max\{\frac{| \bar{\zeta} - \bar{\gamma} |}{\varrho + | \bar{\zeta} - \bar{\gamma} |}, \frac{| \bar{\eta} - \bar{\delta} |}{\varrho + | \bar{\eta} - \bar{v} |}\})$$

$$\geq_{L^*} \Theta(\varpi_{M,N}(\bar{\zeta}, \bar{\gamma}, \varrho), \varpi_{M,N}(\bar{\eta}, \bar{\delta}, \varrho)),$$

that is (3.40). Thus all conditions of Corollary 3.1 are satisfied. Hence α possess a unique fixed point $\bar{z} \in \bar{X}$, so that $\bar{z} = \alpha(\bar{z}, \bar{z})$, thus integral equation (4.1) possess $\bar{z} \in C(K, R)$ as its unique solution. $\qquad\square$

5. Conclusion

In our work we have defined common coupled fixed point theorem for maps in MIFSMS that are weakly compatible. Moreover to prove our result we have given an example and have applied our result to find the solution of nonlinear Fredholm integral equation.

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