Shaking Hands Puzzle

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Alice and Bob go to dinner with n other couples. Everyone shakes hands with everyone they don't know

Alice asks everyone else how many hands they shook, and they all gave a different answer.

How many hands did Bob shake?

The maximum number of hands anyone can shake is $2n$, since there are $2n + 2$ people at the party, and you cannot shake your own hand, or your partner's hand.

There are $2n+1$ unique shake-counts bound between 0 and $2n$, which means all integers in $[0, 2n]$ must be present in the total.

We can deduce that Bob cannot shake hands with $2n$ people. If he did, then he shook hands with all $2n$ people, since he could not have shaken hands with Alice, leaving only the $2n$ from the other couples. This implies there are no people with 0 hand-shakes, which is impossible.

Thus, there must be some other person from another couple who shook $2n$ hands. Call them Charlie

This implies that Charlie's partner must have shaken zero hands, since all other people were shaken by Charlie, and there must be someone with zero handshakes.

At this stage, Charlie and Charlie's partner have $2n$ and 0 shakes, respectively, and can no longer participate in handshakes.

Besides Charlie's couple, all other participants have exactly one handshake.

Thus, we can equivalently reformulate the current state as a game between Alice, Bob and the $n-1$ couples and count the handshakes between themselves, removing the 1 handshake received from Charlie from everybody's count.

This formulation is identical in every manner to the initial setup, but only decreasing n by one. Thus, we can inductively apply the same outcome from the first round until there is only Alice and Bob remaining.

It will take n rounds to reach this state, at which point Bob would have received a handshake from exactly 1 person from each couple at each round, giving Bob a total of n handshakes by the end.