Marangoni convection in a hybrid nanofluid filled cylindrical annular enclosure with sinusoidal temperature distribution

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Abstract The current research numerically investigates the Marangoni convection in a cylindrical annulus filled with hybrid nanofluid saturated porous media. The interior and exterior walls are subjected to spatially varying sinusoidal thermal distributions with various amplitude ratios and phase deviations. The limits at the top and bottom are adiabatic. To solve the system of non-dimensional governing equations, the finite difference approach is applied. The major goal of the ongoing study is to investigate the impact of the Marangoni convection, amplitude ratio and phase deviation on the fluid flow, thermal characteristics, local and average Nusselt numbers in the hybrid nanofluid filled vertical cylindrical annulus with magnetic effects. The findings indicate that the sinusoidal temperature promotes multicellular flow in the porous annular region. In the annulus with sinusoidal boundaries, the Marangoni number underperforms while the nanoparticle volume fraction outperforms.

1 Introduction

The fluid flow generated by changes in surface tension caused by temperature gradients is known as Marangoni convection. It plays a prominent role in crystal growth melt, material processing and so on. Sasmal and Hochstein [1] developed a new computational model to study thermocapillary convection in a rectangular cavity with curved and linearly varying surface tension at the free surface, and discovered that increasing capillary number enhances surface deformation, and increasing Marangoni number causes a large distortion in the isotherms. Chen and Xu [2] investigated the Marangoni flow in an enclosure with evenly heated walls using numerical simulation and scaling analysis. Zhang et al. [3] numerically examined the thermocapillary convective flow behaviours in a cavity region with solutal boundary condition and their results show that a steady flow exists for smaller Marangoni number.

Many authors have studied convective heat transfer with non-uniform boundary conditions because it demonstrates higher thermal transmission rates than uniform thermal conditions. Bilgen and Ben [4] numerically examined the convective thermal transport in an enclosure with single vertical wall imposed to sinusoidally varying thermal conditions. Deng and Chang [5] analyzed buoyancy convection in a cavity with sinusoidal thermal distributions applied on both side vertical walls of the cavity region and discovered that heat distribution is greater in an enclosure with sinusoidal thermal distributions applied on both walls than in a single wall with non uniform heating. A numerical analysis on free convection in a cavity filled with cold water has been made by Janagi et al. [6] with sinusoidal temperature distribution at the walls.

A study on fluid flow and heat transfer behaviour in various enclosures with magnetic effects has been met with great interest due to its wide variety of applications in food processing, solar collections, drying technologies and other fields. Hendy and Attar [7] examined the free convection thermal and mass transport in a vertical surface with the effects of magnetic field. Kefayati [8] used the Lattice Boltzmann method to simulate MHD buoyancy driven convection in an enclosure with sinusoidal heat conditions on the walls. Numerous research on convection in cavities with sinusoidal temperature distributions applied to the walls, taking magnetic effects into account, have been found in the literature [9], [10], [11].

Now-a-days nanofluids are the matter of interest in many researches due to its improved thermal profile. Sheremet and Pop [12] explored convection in a porous cavity saturated with nanofluid, employing Buongiorno's proposed model for nanofluid as well as non-uniform thermal conditions at the walls. Bouhalleb and Abbasi [13] conducted a numerical analysis on convection in a nanofluid-filled inclined rectangular cavity under sinusoidal temperature circumstances and discovered that nanoparticle dispersion in water has great thermal performance. Wang et al. [14] analyzed the impact of heat dependent characteristics on buoyancy-driven convection in rectangular cavities filled with power-law nanofluids. Quite recently, using hybrid nanofluids in place of traditional fluids and mono-nanofluids has become more widespread due to their superior thermal performance in a variety of applications. Mashayekhi et al. [15] investigated the flow of a hybrid nanofluid in a silicon-made double-layered microchannel heat sink and discovered that increasing the volume fraction of the nanoparticle results in superior thermal profiles. Biswas et al. [16] evaluated convection in a square porous enclosure saturated with hybrid nanofluid with half-sinusoidal non uniform heating. Shaik et al. [17] exhibited convection in a hybrid nanofluid-filled sinusoidal wavy cavity with magnetic field and heat radiation. Alsabery et al. [18] investigated buoyant convection in a wavy cavity filled with hybrid nanofluid, taking into account the effects of amplitude and thermal source.

While conducting studies on thermocapillary convection in a cylindrical enclosure filled with hybrid nanofluid, the works are limited to the impact of buoyant convection in a sinusoidal boundary cavity, but no such work has been focused on Marangoni convection in a cylindrical enclosure with sinusoidally varying thermal boundary condition. As a result, the primary goal of this research is to investigate Marangoni convection in hybrid nanofluid filled cylindrical porous annular region and are subjected to sinusoidal thermal conditions at the vertical walls with magnetic effects.

2 Mathematical Formulation

Fig. 1 depicts the physical model explored in the current investigation. The fluid flow is examined in the porous annular section MNOP produced by two concentric cylinders. With water as the base fluid, the porous annular region is saturated with silver (Ag) and magnesium oxide (MgO) nanoparticles. The bottom and top surfaces are assumed to be adiabatic. The inner and the outer walls of the annulus are subjected to spatially changing sinusoidal thermal distributions with various phase deviations and amplitude ratios and are given as follows,

$T_1(x)=T_0 + A_l sin(\frac{2\pi x}{D})$ at inner wall

$T_2(x)=T_0 + A_r sin(\frac{2\pi x}{D} + \gamma)$ at outer wall

where, A_l and A_r are the amplitude of the sinusoidal profiles.

The governing equations use the Brinkman extended Darcy model. The Boussinesq approximation is being considered here. For the nanoparticles, the thermal equilibrium condition holds. Table 1 summarises the thermophysical characteristics of nanoparticles. It is assumed that the fluid flow is axisymmetric and laminar. Considering the aforementioned assumptions, the governing equations in dimensional form [19] are mentioned as follows:

$$\frac{\partial}{\partial r}(ur) + \frac{\partial}{\partial x}(vr) = 0 \tag{1}$$

$$\frac{1}{\delta}\frac{\partial u}{\partial \tau} + \frac{1}{\delta^2}\left(u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial x}\right) = -\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial r} + \nu_{hnf}\left(\bigtriangledown^2 u - \frac{u}{r^2}\right) - \frac{\nu_{hnf}}{K}u \tag{2}$$

$$\frac{1}{\delta}\frac{\partial v}{\partial \tau} + \frac{1}{\delta^2}\left(u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial x}\right) = -\frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x} + \nu_{hnf}\left(\bigtriangledown^2 v\right) - \frac{\nu_{hnf}}{K}v$$

$$-\frac{(\rho\beta)_{hnf}}{\rho_{hnf}}g(T - T_c) - \frac{\sigma_{hnf}}{\rho_{hnf}}vB_0^2 \tag{3}$$

$$\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial r} + v\frac{\partial T}{\partial x} = \alpha_{hnf} \bigtriangledown^2 T \tag{4}$$

Here u and v corresponds to the velocity components along r and x directions respectively. K and δ indicates the porous medium's permeability and porosity.

By removing the pressure term from Eqs. (2) and (3), the dimensionless equations are framed as follows.

$$\frac{\partial \theta}{\partial t} + \frac{U}{A}\frac{\partial \theta}{\partial R} + \frac{V}{A}\frac{\partial \theta}{\partial X} = \frac{\alpha_{hnf}}{\alpha_f} \bigtriangledown_1^2 \theta \tag{5}$$

$$\frac{1}{\delta}\frac{\partial \eta}{\partial t} + \frac{1}{\delta^2}\left[\frac{U}{A}\frac{\partial \eta}{\partial R} + \frac{V}{A}\frac{\partial \eta}{\partial X} - \frac{U}{A}\left(\frac{D}{r_i + RD}\right)\eta\right] = Pr\frac{\nu_{hnf}}{\nu_f}\left[\bigtriangledown_1^2\eta - \left(\frac{D}{r_i + RD}\right)^2\right]\eta$$

$$-\frac{Pr}{Da}\frac{\nu_{hnf}}{\nu_f}\eta + Ra\frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_f}\frac{\partial \theta}{\partial R} + Ha^2\frac{\sigma_{hnf}\rho_f}{\rho_{hnf}\sigma_f}\frac{\partial V}{\partial R} \tag{6}$$

$$\eta = \frac{1}{Pr}\left(\frac{r_i}{r_i + RD}\right)\left[\frac{\partial^2 \Psi}{\partial R^2} - \left(\frac{D}{r_i + RD}\right)\frac{\partial \Psi}{\partial R} + \frac{1}{A^2}\frac{\partial^2 \Psi}{\partial X^2}\right] \tag{7}$$

$$U = \frac{r_i}{r_i + RD}\frac{\partial \Psi}{\partial X}; \quad V = -\frac{r_i}{r_i + RD}\frac{\partial \Psi}{\partial R} \tag{8}$$

Here

$$\eta = \frac{1}{Pr}\left[\frac{1}{A^2}\frac{\partial U}{\partial X} - \frac{\partial V}{\partial R}\right]; \quad \bigtriangledown_1^2 = \frac{\partial^2}{\partial R^2} + \left(\frac{D}{r_i + RD}\right)\frac{\partial}{\partial R} + \frac{1}{A^2}\frac{\partial^2}{\partial X^2}$$

In the preceding equations, the non-dimensional variables listed below are used.

$$U = \frac{uD}{\alpha_f}A, \quad V = \frac{vL}{\alpha_f A}, \quad R = \frac{r - r_i}{D}, \quad X = \frac{x}{L}, \quad t = \frac{\tau\alpha_f}{D^2},$$

$$\theta = \frac{(T - T_c)}{\triangle T}, \quad \triangle T = A_l(amplitude), \quad \eta = \frac{\Omega D^2}{\nu_f}, \quad \Psi = \frac{\psi}{r_i\alpha_f}, \quad D = r_0 - r_i$$

The following are the initial and boundary conditions in non-dimensional form.
$t=0: \;\; \Psi=\eta=0; \;\; \theta=0, \quad U=V=0, \;\; 0\leq X\leq 1, 0\leq R\leq 1$

For $t>0: \; \Psi{=}\;\frac{\partial\Psi}{\partial R}{=}0, \; \theta_1(X){=}sin(2\pi XA); \; R=0$

$$\Psi{=}\;\frac{\partial\Psi}{\partial R}{=}0, \; \theta_2(X){=}\epsilon sin(2\pi XA+\gamma); \; R=1$$

$$\Psi{=}\;\frac{\partial\Psi}{\partial X}{=}0, \; \frac{\partial\theta}{\partial X}{=}0; \; X=0$$

$$\Psi{=}\;\frac{\partial U}{\partial X}{=}\frac{\partial^2\Psi}{\partial X^2}{=}0, \; \frac{\partial\theta}{\partial X}{=}0; \; X=1$$

The association between surface tension differences and shear stress determines the boundary condition for the vorticity near the free surface. This causes thermocapillary flow to develop in the annulus. The Taylor series expansion for the stream function is used to derive the boundary condition for vorticity near solid boundaries.

$\eta{=}\left(\frac{r_i}{Pr(RD+r_i)}\right)\frac{\partial^2\Psi}{\partial R^2}; \; 0\leq X\leq 1$ and $R=0; \; R=1$

$\eta{=}\left(\frac{r_i}{A^2Pr(RD+r_i)}\right)\frac{\partial^2\Psi}{\partial X^2}; \; 0\leq R\leq 1$ and $X=0$

$\eta{=}\;\frac{\partial U}{\partial X}{=}MaA\frac{\partial\theta}{\partial R}; \; 0\leq R\leq 1$ and $X=1$

The dimensionless parameters Ra, Ha, Pr, Da, Ma, λ, A, L, ϵ are respectively the Rayleigh number, Hartmann number, Prandtl number, Darcy number, Marangoni number, radii ratio, aspect ratio, amplitude ratio and γ is the phase deviation. These parameters are described as,

$$Ra=\frac{g\beta_f\triangle TD^3}{\nu_f\alpha_f}, \; Ha=B_0D\sqrt{\sigma_e/\rho_f\nu_f}, \; Pr=\frac{\nu_f}{\alpha_f}, \; Da=\frac{K}{D^2},$$

$$Ma=-\frac{\partial\sigma_f}{\partial T}\frac{\triangle TD}{\mu_f\alpha_f}, \; \lambda=\frac{r_0}{r_i}, \; A=\frac{L}{D}, \; \epsilon=\frac{A_r}{A_l}$$

The local thermal transfer rate is specified along the inner and outer walls of the annulus as follows:

$$Nu_l=-\frac{k_{hnf}}{k_f}\frac{\partial\theta}{\partial R} \tag{9}$$

$$Nu_r=-\frac{k_{hnf}}{k_f}\frac{\partial\theta}{\partial R} \tag{10}$$

The average Nusselt number of the annulus is defined as the sum of the average Nusselt number along heating halves of the inner and outer walls and it is given as follows,

$$\overline{Nu} = \frac{1}{A}\int_{heatinghalf} Nu_l dX + \frac{1}{A}\int_{heatinghalf} Nu_r dX \tag{11}$$

A solid structured nanolayer formed by the molecules of liquid near a solid surface is considered in the modified Maxwell model [**?**]. This model is used in this study to define the hybrid nanofluid's effective thermal conductivity and it is provided as follows,

$$k_{hnf} = k_f \frac{2(k_{eq} - k_f)(1+\iota)^3\phi + (k_{eq} + 2k_f)}{(2k_f + k_{eq}) - (1+\iota)^3\phi(k_{eq} - k_f)} \tag{12}$$

Here, $\iota = \frac{h_{nl}}{r_p}$, where h_{nl} denotes the thickness of the nanolayer. r_p denotes the novel radius of the nanoparticles.

The equivalent thermal conductivity of the nanoparticles is represented by k_{eq} and it is provided as

$$\frac{k_{eq}}{k_p} = \xi \frac{2(1-\xi) + (1+\iota)^3(1+2\xi)}{(1+\iota)^3(1+2\xi) - (1-\xi)} \tag{13}$$

where ξ stands for the ratio of heat conductivity of the nanolayer to the nanoparticles heat conductivity $\xi = \frac{k_{nl}}{k_p}$. Here the values for r_p, h_{nl} and k_{nl} are taken as $3nm$,$2nm$, and $100k_f$ respectively. The applied models for the working hybrid nanofluid are given as in [20]

2.1 Numerical solution procedure

The PDEs for energy, vorticity and the elliptic form of stream function equation with the boundary conditions are evaluated using the Finite Difference Method(FDM) (Wilkes [21]). In this study, an Alternating Direction Implicit (ADI) technique is used to discretize the vorticity and energy equations where the energy equation is solved initially along horizontal direction for the first half of the time and then solved along vertical direction for the another half time. After the discretization process, a tri-diagonal system of equations are formed and it is solved by using Thomas Algorithm. The obtained values of the temperature are used in the vorticity equation and the same procedure is repeated to evaluate the vorticity equation. Finally the elliptic stream function equation is resolved using the Successive Over Relaxation(SOR) scheme and the velocity terms are assessed using central difference scheme. To get a converged solution, the following convergence condition must be attained.

$$\frac{|\Phi_{n+1}(i,j) - \Phi_n(i,j)|}{|\Phi_{n+1}(i,j)|} \leq 10^{-5} \tag{14}$$

where Φ represents T, η, Ψ and n denotes the time step.

2.2 Grid sensitivity analysis and numerical validation

An analysis of grid sensitivity has been carried out for numerous grid sizes and the changes in the average Nusselt number is scrutinized in the enclosure. The obtained outputs are displayed in the Table 2. The optimal grid size for the given problem is 81×81, taking into account the accuracy and the processing time.

Before generating simulation results for the ongoing work, the currently created computational code is validated against benchmark results available in the literature. Table 3 compares the numerical results for the square enclosure to the De Vahl Davis G results. [22]. The fluctuation of the average Nusselt number in a cylindrical annular enclosure is associated with the outputs of Sankar et al. in Table 4. [19]. In Fig. 2, the fluid flow and thermal variation for sinusoidally varying thermal profiles are compared with the outputs of Deng and Chang [5]. An excellent degree of agreement is achieved from the above comparison results.

3 Results and discussions

The effect of the thermocapillary convection in a hybrid nanofluid filled cylindrical enclosure with vertical walls having sinusoidally varying thermal distributions is examined in the current study. The numerical simulations are carried out for a wide variety of parameters Marangoni number $Ma : 10^2 - 10^4$, $Ra : 10^3 - 10^6$, Hartmann number $Ha : 10 - 70$, amplitude ratio $\epsilon = 0 - 1$, phase deviation $\gamma : 0 - \pi$, nanoparticle volume fraction $\phi = 0.02, 0.04, 0.08$ and the geometrical parameter radii ratio $\lambda = 1, 2, 5, 10$. Constraints such as Aspect ratio $A = 1$, Prandtl number $Pr = 6.2$, Darcy number $Da = 10^{-2}$ and the porosity of the porous medium $\delta = 0.4$ are held constant. The numerically simulated results are depicted in the form of streamlines and isotherm profiles, the local and global Nusselt number plots along the vertical walls.

Fig. 3 depicts the streamlines and isotherms for various Marangoni number. At the lower Marangoni number ($Ma = 10^2$), the flow structure reveals the formation of four eddies with relatively symmetry about the centre of the enclosure. Since the isotherms show modest convection, it is clear that the conduction dominates the thermal field. With an increase in the Marangoni value ($Ma = 10^3$), two eddies emerge towards the upper boundary, joining previously existing cells. This is due to shear stress created at the free surface boundary as a result of Marangoni flow. As the value of the Marangoni number enhances, the eddies created near the free surface grow larger and become more intense, and so the main stream flow reduces. The isotherms don't vary greatly.

The influence of the amplitude ratio on streamlines and isotherms are demonstrated in Fig. 4. When $\epsilon = 0$, a tri-cellular flow structure is found. The cell generated near the free surface by Marangoni convection disrupts the flow field's horizontal symmetric pattern. The isotherms demonstrate that the thermal transport is highly concentrated near the left wall. As ϵ grows further, the sinusoidal thermal state at the right wall also drives fluid motion, resulting in the formation of two more eddies near the right wall. The isotherm also begins to appear near the right wall. When $\epsilon = 1$, the eddies produced by the sinusoidal temperature near the right wall overpower the cells formed near the left wall. Near the free surface, two more minor eddies can be detected. The isotherm profile remains unchanged.

Fig. 5 illustrates the variation on flow and thermal contours due to the impact of phase deviation. At $\gamma = 0$, four large eddies are seen, with two tiny eddies at the top boundary.The eddies at the top corner of the right wall and the bottom corner of the left wall expand and merge together when the value of phase deviation augments ($\gamma = \pi/4$). The other diagonal cells have shrunk in size. For $\gamma = \pi/2$, a two-cellular structure is found, with additional cell produced towards the upper boundary. Finally, as the phase deviation increases from 0 to π, the four-cellular structure with top two minor eddies is transformed to a two-cellular structure with a single eddy near the top boundary. According to the isotherm pattern, thermal transfer occurs more on the right wall than on the left wall.

The fluctuation of the local Nusselt number along the enclosure's left and rightwalls for varied amplitude ratio (ϵ) is shown in Fig. 6. It is clear from the Fig. 6(a) that the local Nusselt curve of the left side wall is almost same for varying values of amplitude ratio. It demonstrates that the thermal transfer rate at the left side wall does not vary much with increasing amplitude ratio. On the contrary, the local Nusselt curve of the rightwall displays a considerable change, indicating that thermal transfer is significant along the right side wall but has little effect on thermal transport at the right side wall. At $\epsilon = 1$, the amplitude ratio reaches its maximum value as well.

Fig. 7 reports the impact of the phase deviation (γ) on local Nusselt number along the enclosure's left and right side walls. For varied phase deviation values, the local Nusselt curve along the annulus's left wall does not change appreciably, however the thermal transport rate along the annulus's right wall is significantly affected by the phase deviation. The phase deviation has only a little effect on the thermal transport rate along the left wall, but it does affect the thermal transfer rate along the right wall, as shown in the figure.

The effect of the Marangoni number on the amplitude ratio (ϵ) and the phase deviation (γ) is examined in Fig. 8. The average Nusselt curve continuously improves with amplitude ratio, as shown in Fig. 8(a). As a result, assuming a sinusoidal boundary condition on the side walls is more useful than choosing adiabatic or insulated boundary conditions for achieving a high thermal transmission rate. Until $\epsilon = 0.5$, the average Nusselt number augments with the Marangoni number. As the ϵ value further improves, the $\overline{Nu}$ drops with Marangoni number. According to Fig. 8(b), the average Nusselt number value decreases with phase deviation, but there is no noticeable change in the $\overline{Nu}$ curve with respect to Marangoni number except for the greater value of Marangoni number at $\gamma = \pi$.

Fig. 9 represents the impact of the Hartmann number on the amplitude ratio (ϵ) and the phase deviation (γ).The $\overline{Nu}$ curve grows with increasing amplitude ratio, although the Hartmann number marginally reduces the thermal transport rate, as seen in Fig. 9(a). From Fig. 9(b) it is clear that the average Nusselt number values decrease with phase deviation, but there is no discernible change in the $\overline{Nu}$ curve for Hartmann number.

Fig. 10 indicates the impact of nanoparticle volume fraction on amplitude ratio (ϵ) and phase deviation (γ). Figs. 10(a) and 10(b) show that the thermal transmission rate increases with amplitude ratio and nanoparticle volume fraction, but the thermal transport rate falls with phase deviation.

The effect of radii ratio (λ) on the amplitude ratio (ϵ) and phase deviation (γ) is shown in Fig. 11. The augmentation in λ for amplitude ratio and phase deviation results in a high thermal transmission rate. The annular cavity ($\lambda = 2, 5, 10$) clearly outperforms the rectangular cavity ($\lambda = 1$).

4 Conclusions

The numerical simulation of thermocapillary convection in a porous annular region saturated with Ag-MgO/water hybrid nanofluid and with sinusoidally variable temperature conditions at the vertical walls has been performed. The following are the findings of this research.

- In the porous annular enclosure, the sinusoidal thermal condition generates a multicellular flow.
- The Marangoni and Hartmann numbers have little effect on the phase deviation (γ) and amplitude ratio (ϵ).
- The $\overline{Nu}$ improves significantly when the nanoparticle volume fraction (ϕ) and the radii ratio (λ) increase.

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Table 1 The thermophysical characteristics of the base fluid and the nanoparticles [23], [24]are as follows:

Properties	Water	MgO	Ag
$c_p(Jkg^{-1}K^{-1})$	4179	955	235
$\rho(kgm^{-3})$	997.1	3560	10500
$k(Wm^{-1}K^{-1})$	0.613	45	429
$\beta \times 10^{-5}(K^{-1})$	21	1.13	1.89
$\mu(kgm^{-1}s^{-1})$	8.9×10^4	-	-
$\sigma(\Omega^{-1}m^{-1})$	0.05	1.42×10^{-3}	6.30×10^7

Table 2 The grid independent study for the cylindrical annulus for $Ra = 10^4$, $Ma = 10^3$, $\epsilon = 1$, $\gamma = 0$, $Ha = 10$, $\phi = 0.02$, $\lambda = 2$, $A = 1$.

Grid size	Average Nusselt number
41×41	2.4038
61×61	2.5563
81×81	2.6441
101×101	2.7001

Table 3 The comparison of $\overline{Nu}$ on the closed cavity.

	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^6$
De Vahl Davis. G [22]	1.117	2.238	4.509	8.817
Present study	1.137	2.263	4.549	8.862
Error	2.1%	1.1%	0.8%	0.5%

Table 4 The comparison of average nusselt number on annular enclosure.

	$Ra = 10^3$	$Ra = 10^4$	$Ra = 10^5$	$Ra = 10^6$
Sankar et al. [19]	1.48	1.59	3.17	8.39
Present study	1.23	1.33	3.00	8.11