Anytime Capacity of Markov Channels

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***Abstract*—Several new expressions for the anytime capacity of Sahai and Mitter are presented for a time-varying rate-limited channel with noiseless output feedback. These follow from a parametric characterization obtained in the case of Markov channels, and include an explicit formula for the r-bit Markov erasure channel, as well as formulas for memoryless rate processes including Binomial, Poisson, and Geometric distributions. Beside the memoryless erasure channel and the additive white Gaussian noise channel with input power constraint, these are the only cases where explicit anytime capacity formulas are obtained. At the basis of these results is the study of the threshold function for mth moment stabilization of a scalar linear system controlled over a Markov time-varying digital feedback channel that depends on m and on the channel's parameters. This threshold is shown to be a continuous and strictly decreasing function of m and to have as extreme values the Shannon capacity and the zero-error capacity as m tends to zero and infinity, respectively. Its operational interpretation is that of achievable communication rate, subject to a reliability constraint.**

I. Introduction

We consider the problem of moment stabilization of a dynamical system where the estimated state is transmitted for control over a time-varying communication channel. A tutorial review of the problem with extensive references appears in [1].

The notion of Shannon capacity is in general not sufficient to characterize the trade-off between the entropy rate production of the plant, expressed by the growth of the state space spanned in open loop, and the communication rate required for its stabilization. A large Shannon capacity is useless for stabilization if it cannot be used in time for control. For the control signal to be effective, it must be appropriate to the current state of the system. Since decoding the wrong codeword implies applying a wrong signal and driving the system away from stability, applying an effective control signal depends on the history of whether previous codewords were decoded correctly or not. In essence, the stabilization problem is an example of *interactive communication*, where two-way communication occurs through the feedback loop between the plant and the controller. Alternative capacity notions with stronger reliability constraints than having a vanishing probability of error, have been proposed in the context of control, including the zero-error capacity [2], originally introduced by Shannon [3], and the anytime capacity proposed by Sahai and Mitter [4], [5]–[8].

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Within this general framework, we focus on the mth moment stabilization of an unstable scalar system whose state is communicated over a rate-limited channel capable of supporting R_k bits at each time step and evolving randomly in a Markovian fashion, see Fig. 1. The rate process is known casually to both encoder and decoder. Many variations of this "bit-pipe" model have been studied in the literature [9]–[29], including the case of fixed rate channel; the erasure channel where the rate process can assume value zero; and the packet loss channel, where the rate process can oscillate randomly between zero and infinity, allowing a real number of infinite precision to be transported across the channel in one time step. Connections between the rate limited and the packet loss channel have been pointed out in [22], [23], showing that results for the latter model can be recovered by appropriate limiting arguments.

The major contribution of this paper is the introduction of a stability threshold function of the channel's parameters and of the moment stability number m that converges to the Shannon capacity for $m \to 0$, to the zero-error capacity for $m \to \infty$, and it provides a parametric characterization of the anytime capacity for the remaining values of m. This function yields a novel anytime capacity formula in the special case of the r-bit Markov erasure channel, as well as formulas for memoryless rate processes including Binomial, Poisson, and Geometric distributions.

Throughout the paper, the following notation is used. Logarithms are assumed to be in base two; random variables are denoted with uppercase letters, while their realizations with lowercase letters; matrices are also denoted in uppercase letters, using the special typeset A.

II. Moment Stabilization over Markov Channels

We consider the stability of linear dynamical systems when the estimated state is sent to the controller over a digital communication link whose state is described by a Markov process, as depicted in Fig. 1.

A. System model

Consider the scalar dynamical system

$$\begin{aligned} x_{k+1} &= \lambda x_k + u_k + v_k, \\ y_k &= x_k + w_k, \end{aligned} \tag{1}$$

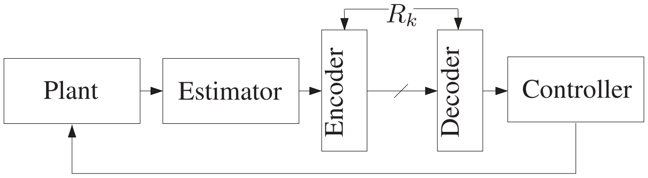

Fig. 1. Feedback loop model. The estimated state is quantized, encoded and sent to a decoder over a digital channel of state R_k that evolves in time according to a Markov process.

where $k \in \mathbb{N}$, and $|\lambda| \geq 1$. The variable x_k represents the state of the system, u_k the control input, v_k is an additive stochastic disturbance, y_k is the sensor measurement, and w_k is the measurement noise. Both disturbance and noise are independent of each other and of the initial condition x_0. They are also independent of the channel state, as defined below.

B. Channel Model

The state observer is connected to the actuator through a noiseless digital communication link that at each time k allows transmission without errors of R_k bits. The rate process $\{R_k\}_{k\geq 0}$ is modeled as a homogeneous positive-recurrent Markov process defined on the finite set

$$\mathcal{R} = \{\bar{r}_1, \ldots, \bar{r}_n\}, \tag{2}$$

for some integer numbers $0 \leq \bar{r}_1 < \cdots < \bar{r}_n$, and with one-step transition probability matrix P having entries

$$p_{ij} = \mathsf{P}\{R_k = \bar{r}_j | R_{k-1} = \bar{r}_i\} \tag{3}$$

for every $i, j \in \{1, \ldots, n\}$. In the sequel, we define $\mathrm{R} \in \mathbb{Z}_+^{n\times n}$ as the diagonal matrix with diagonal entries $\bar{r}_1, \ldots, \bar{r}_n$, i.e.,

$$\mathrm{R} = \mathrm{diag}(\bar{r}_1, \ldots, \bar{r}_n). \tag{4}$$

Encoder and decoder are supposed to have causal knowledge of the rate process.

This noiseless digital link corresponds to a discrete-memoryless channel with Markov state available causally at both the encoder and the decoder. A channel with state is defined by a triple $(\mathcal{X}\times\mathcal{S}, p(y|x,s), \mathcal{Y})$ consisting of an input set $\mathcal{X}$, a state set $\mathcal{S}$, an output set $\mathcal{Y}$, and a transition probability matrix $p(y|x)$ for every $x \in \mathcal{X}$, $s \in \mathcal{S}$, and $y \in \mathcal{Y}$. This channel is memoryless if the output y_k at time k is conditionally independent of everything else given (x_k, s_k). The state sequence is Markov if $S_0, S_1, \ldots$ forms a Markov chain. According to these definitions, our channel model is a discrete-memoryless channel with Markov state $(\mathcal{X}\times\mathcal{S}, p(y|x,s), \mathcal{Y})$ with $\mathcal{X} = \mathcal{Y} = \{1, \ldots, \bar{r}_n\}$, $\mathcal{S} = \{\bar{r}_1, \cdots, \bar{r}_n\}$,

$$p(y|x,s) = \begin{cases} 1 & x = y \text{ and } x \leq s \\ 0. & \text{otherwise} \end{cases} \tag{5}$$

and state transition probabilities

$$p(s_{k+1} = \bar{r}_j | s_k = \bar{r}_i) = p_{ij}. \tag{6}$$

The Shannon capacity of this channel is [30]

$$C = \sum_{i=1}^{n} \pi_i \bar{r}_i, \tag{7}$$

where $(\pi_1, \ldots, \pi_n)$ denotes the unique stationary distribution of P.

The zero-error capacity of this channel is [3]

$$C_0 = \bar{r}_1. \tag{8}$$

The capacities in (7) and (8) are the limiting values of a stability threshold function indicating the channel's rate-reliability constraint required to achieve a given level of stabilization. As $m \to \infty$ and the system is highly stable, then the stability threshold function tends to the zero-error capacity that has a hard reliability constraint of providing no decoding error. Conversely, as $m \to 0$ and the system's stability level decreases, then the stability threshold function tends to the Shannon capacity that has a weak reliability constraint of vanishing probability of error.

C. Stability threshold function

The system (1) is mth moment stable if

$$\sup_k \mathsf{E}[|X_k|^m] < \infty, \tag{9}$$

where the expectation is taken with respect to the random initial condition x_0, the additive disturbance v_k, and the rate process R_k.

The following Theorem establishes the stability threshold for the m-th moment stability of (1) in terms of the unstable mode $|\lambda|$ and the spectral radius $\rho(\cdot)$ of $\mathrm{P}^\mathsf{T} 2^{-m\mathrm{R}}$, where $2^{-m\mathrm{R}}$ denotes the base-2 matrix exponential of $m\mathrm{R}$, i.e.,

$$2^{-m\mathrm{R}} = \mathrm{diag}(2^{-m\bar{r}_1}, \cdots, 2^{-m\bar{r}_n}). \tag{10}$$

Theorem 1. *There exists a control scheme that stabilizes the scalar system* (1) *in mth momemt sense if and only if*

$$\log|\lambda| \lesssim -\frac{1}{m}\log\rho(\mathrm{P}^\mathsf{T} 2^{-m\mathrm{R}}) \triangleq R(m). \tag{11}$$

The usage of the symbol "$\lesssim$" indicates that while the necessary condition holds with a weak inequality, the sufficient condition requires a strong inequality.

We now mention several properties of the threshold function $R(m)$.

Proposition 2. *The following facts hold:*

1) *Monotonicity: $R(m)$ is continuous and strictly decreasing for $m > 0$.*
2) *Convergence to the Shannon capacity:*

$$\lim_{m\to 0} R(m) = \sum_{i=1}^{n} \pi_i \bar{r}_i = C. \tag{12}$$

3) *Convergence to the Zero Error capacity:*

$$R(m) \sim \bar{r}_1 - \frac{1}{m}\log p_{11}, \quad \textit{as } m \to \infty, \tag{13}$$

and hence

$$\lim_{m\to\infty} R(m) = \bar{r}_1 = C_0. \tag{14}$$

4) *Sensitivity with respect to self-loop probabilities:*

$$\frac{dR(m)}{dp_{ii}} = -\frac{2^{-m\bar{r}_{ii}}}{m\rho(\mathrm{P}^\mathsf{T}2^{-m\mathrm{R}})}\,\frac{|\mathrm{D}(1)|}{\sum_{i=1}^n |\mathrm{D}(i)|} < 0, \tag{15}$$

where $\mathrm{D} := \rho(\mathrm{P}^\mathsf{T}2^{-m\mathrm{R}})\mathrm{I} - \mathrm{P}^\mathsf{T}2^{-m\mathrm{R}}$*,* I *denotes the* $n\times n$ *identity matrix, and* $|\mathrm{D}(i)|$ *is the determinant of the matrix obtained by eliminating the* i*th row and the* i*th column from* D*. We also have the asymptotic behavior*

$$\frac{dR(m)}{dp_{11}} \sim -\frac{1}{mp_{11}\ln(2)} \text{ as } m\to\infty. \tag{16}$$

5) *The function* $mR(m)$ *is nonnegative, strictly increasing, and strictly concave. If* $\bar{r}_1 \neq 0$*, then* $mR(m)$ *grows unbounded as* $m\to\infty$*. If instead* $\bar{r}_1 = 0$*, then*

$$\lim_{m\to\infty} mR(m) = -\log p_{11}. \tag{17}$$

III. Anytime Capacity of Markov Channels

We relate the stability threshold function $R(m)$ to the anytime capacity. For the given Markov channel, it provides a parametric representation of the anytime capacity in terms of system's stability level m.

The anytime capacity is defined in the following context [6]. Consider a system for information transmission that allows the decoding time to be infinite, and improves the reliability of the estimated message as time progresses. More precisely, at each step k in the evolution of the plant a new message m_k of r bits is generated that must be sent over the channel. The coder sends a bit over the channel at each k and the decoder upon reception of the new bit updates the estimates for all messages up to time k. It follows that at time k messages

$$m_0, m_1, \dots, m_k$$

are considered for estimation, while estimates

$$\hat{m}_{0|k}, \hat{m}_{1|k}, \dots, \hat{m}_{k|k}$$

are constructed, given all the bits received up to time k. Hence, the processing operation for any message m_i continues indefinitely for all $k \geq i$. A reliability level α is achieved in the given transmission system if for all k the probability that there exists at least a message in the past whose estimate is incorrect decreases α-exponentially with the number of bits received, namely for all $d \leq k$

$$\mathsf{P}\{(\hat{M}_{0|k}, \dots, \hat{M}_{d|k}) \neq (M_0, \dots, M_d)\} = O(2^{-\alpha d}). \tag{18}$$

The described communication system is characterized by a rate-reliability pair (r, α). The work in [6] has shown that for scalar systems the ability to achieve stability depends on the ability to construct such a communication system, in terms of achievable coding and decoding schemes, with a given rate-reliability constraints.

To state this result in the context of our Markov channel, let the α-anytime capacity $C_A(\alpha)$ be the supremum of the rate r that can be achieved with reliability α. The problems of α-reliable communication and mth moment stabilization of a scalar system over a Markov channel are then equivalent in the sense of the following theorem.

Theorem 3 (Sahai, Mitter [6])**.** *The necessary and sufficient condition for* m*th moment stabilization of a scalar system with bounded disturbances and in the presence of channel output feedback over a Markov channel is*

$$\log|\lambda| \lesssim C_A(m\log|\lambda|). \tag{19}$$

The anytime capacity is an intermediate notion between the zero-error capacity and the Shannon capacity. The zero-error capacity requires transmission without error. The Shannon capacity requires the decoding error to tend to zero by increasing the length of the code. In the presence of disturbances, only a critical value of the zero-error capacity can guarantee the almost sure stability of the system [2]. On the other hand, for scalar systems in presence of bounded disturbances, a critical value of the anytime capacity can guarantee the ability to stabilize the system in the weaker mth moment sense.

By combining Theorem 1 and Theorem 3, we obtain the following result.

Theorem 4. *The following holds:*

1) *Parametric characterization of the anytime capacity: For every* $m > 0$*, the anytime capacity* C_A *satisfies*

$$C_A\big(mR(m)\big) = R(m), \tag{20}$$

i.e., for every $\alpha \geq 0$*, there exists a unique* $m(\alpha)$ *such that*

$$m(\alpha)R\big(m(\alpha)\big) = \alpha \tag{21}$$

and

$$C_A(\alpha) = R\big(m(\alpha)\big) = \frac{\alpha}{m(\alpha)}. \tag{22}$$

2) $C_A(\alpha)$ *is a strictly decreasing function function of* $\alpha > 0$.

3) *Convergence to the Shannon capacity:*

$$\lim_{\alpha\to 0} C_A(\alpha) = \sum_{i=1}^n \pi_i r_i = C, \tag{23}$$

4) *Convergence to the Zero Error capacity: If* $\bar{r}_1 = 0$*, then for every* $\alpha \geq \log(1/p_{11})$

$$C_A(\alpha) = 0 = C_0. \tag{24}$$

Conversely, if $\bar{r}_1 \neq 0$*, then* $C_A(\alpha)$ *has unbounded support and*

$$C_A(\alpha) \sim \bar{r}_1\,\frac{\alpha}{\alpha - \log(1/p_{11})}, \quad \text{as } \alpha\to\infty, \tag{25}$$

hence

$$\lim_{\alpha\to\infty} C_A(\alpha) = \bar{r}_1 = C_0. \tag{26}$$

IV. The Markov Erasure Channel

We use the stability threshold function $R(m)$ to compute the anytime capacity of the Markov erasure channel. The Markov erasure channel corresponds to a two-state Markov process where $n = 2$ $\mathcal{R} = \{0, \bar{r}\}$, $p_{12} = q$, and $p_{21} = p$, where $0 < p, q < 1$. In this case,

$$\mathrm{P}^{\mathsf{T}} 2^{-m\mathrm{R}} = \begin{pmatrix} (1-q) & \frac{1}{2^{m\bar{r}}} p \\ q & \frac{1}{2^{m\bar{r}}}(1-p) \end{pmatrix}, \tag{27}$$

and we have the following result.

Theorem 5. *The anytime capacity of the Markov Erasure Channel is*

$$C_A(\alpha) = \frac{\alpha \bar{r}}{\alpha + \log_2\left(\frac{1-p-2^{\alpha}(1-p-q)}{1-(1-q)2^{\alpha}}\right)}, \tag{28}$$

if $0 \leq \alpha < -\log_2(1-q)$*, and 0 otherwise.*

A. Special cases

Several special cases recover previous results in the literature. By (28) it follows that the anytime capacity of the binary erasure channel (BEC) with Markov erasures and with noiseless channel output feedback is

$$C_A(\alpha) = \frac{\alpha}{\alpha + \log_2\left(\frac{1-p-2^{\alpha}(1-p-q)}{1-(1-q)2^{\alpha}}\right)}. \tag{29}$$

By letting $q = 1 - p$, the erasure process becomes i.i.d. and we recover the anytime capacity of the memoryless BEC with erasure probability p derived by Sahai [4, page 129] (in parametric form) and by Xu [8, Theorem 1.3] (in non-parametric form)

$$C_A(\alpha) = \frac{\alpha}{\alpha + \log_2\left(\frac{1-p}{1-p2^{\alpha}}\right)}. \tag{30}$$

By (28), letting $\alpha \to 0$, we have that

$$\lim_{\alpha \to 0} C_A(\alpha) = \frac{q}{p+q}\bar{r} = C, \tag{31}$$

where the expectation is taken with respect to the stationary distribution of P. This recovers the Shannon capacity of an $\bar{r}$-bit erasure channel with Markov erasures and with noiseless channel output feedback.

In the case $n = 2$, $\bar{r}_1 = 0$, $\bar{r}_2 = r$, and an i.i.d. rate process with $\mathsf{P}\{R_k = 0\} = p_1$ and $P\{R_k = r\} = p_2$ for all k's, then the stability condition becomes

$$|\lambda|^m \left(p_1 + p_2 2^{-mr}\right) < 1,$$

which provides a converse to the achievable scheme of Yüksel and Meyn [31, Theorem 3.3].

If we further let $r \to \infty$, then the stability condition $p_1 > 1/|\lambda|^m$ depends only on the erasure rate of the channel. In this case, our condition generalizes the packet loss model result in [24].

V. Memoryless Channels

Consider the special case of an i.i.d. rate process R_k where $R_k \sim R$ has probability mass function $p_i = \mathsf{P}\{R = \bar{r}_i\}$, $\bar{r}_i \in \mathcal{R}$. For t real, Let $M_R(t) = \mathbb{E}(e^{tR})$ denote the moment generating function of R and let $M_R^{-1}(y)$ denotes the inverse of the $M_R(t)$, if it exists, i.e., $M_R^{-1}(y) = t$ if and only if $M_R(t) = y$.

We have the following result.

Theorem 6. *The anytime capacity of a memoryless channel with rate distribution R is*

$$C_A(\alpha) = \frac{\ln 2^{-\alpha}}{M_R^{-1}(2^{-\alpha})} \tag{32}$$

for $\alpha < -\log p_{11}$ *if* $\bar{r}_0 = 0$*, or for any* $\alpha > 0$ *if* $\bar{r}_0 \neq 0$*.*

Theorem 6 shows that in the case where the channel is memoryless, the anytime capacity can be evaluated by computing the inverse of the moment generating function of R, as illustrated in the next three examples.

Example V.1. *Suppose that R is a binomial random variable with parameters k and $1-p$. Then,*

$$M_R(t) = (p + (1-p)e^t)^k \tag{33}$$

and

$$M_R^{-1}(y) = \ln \frac{y^{1/k} - p}{1-p}, \quad y < p^k, \tag{34}$$

and thus by (32)

$$C_A(\alpha) = \frac{\alpha}{\alpha/k + \log_2\left(\frac{1-p}{1-p2^{\alpha/k}}\right)} \tag{35}$$

for $\alpha < -k \log p$*. Notice that* (35) *recovers* (30) *in the special case* $k = 1$ *in which R is Bernoulli with parameter* $1-p$*.*

Example V.2. *Suppose that R is a Poisson random variable with parameter λ. Then,*

$$M_R(t) = e^{\lambda(e^t - 1)} \tag{36}$$

and

$$M_R^{-1}(y) = \ln(1 + 1/\lambda \ln y), \quad y > 0, \tag{37}$$

and thus by (32)

$$C_A(\alpha) = -\frac{\alpha}{\log(1 - \alpha/\lambda \ln 2)} \tag{38}$$

for $\alpha < -\lambda/\ln 2$*.*

Notice in both examples $\bar{r}_1 = 0$, so the anytime capacity has bounded support. In the next example $\bar{r}_1 = 1$ and thus the anytime capacity is defined for all $\alpha > 0$.

Example V.3. *Suppose that R is a geometric random variable with parameter p. Then,*

$$M_R(t) = \frac{pe^t}{1-(1-p)e^t} \tag{39}$$

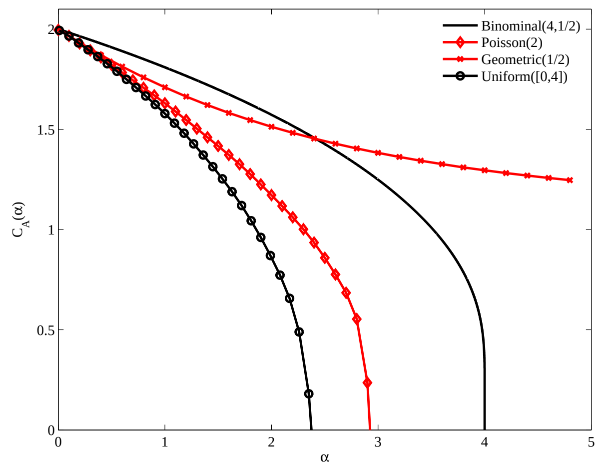

Fig. 2. Comparison of the anytime capacity for different memoryless channels. For the uniform distribution the plot is obtained numerically.

for $t < -\ln(1-t)$ *and*

$$M_R^{-1}(y) = \ln \frac{y}{p + y(1-p)}, \quad y > 0, \tag{40}$$

for $y > 0$*, and thus*

$$C_A(\alpha) = \frac{\alpha}{\log\big((1-p) + p2^{\alpha}\big)}, \quad \alpha > 0. \tag{41}$$

VI. Proofs

All proofs are available on-line in the full preprint of the paper at *http://circuit.ucsd.edu/∼massimo/Papers.html*.

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