Intuition and symmetries in electromagnetism: Eigen states of 4 antennas

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Symmetries play an essential role in the field of physics. In this paper, we examine the relationship between the eigen-amplitudes of 4 symmetrical antennas and the symmetry of the amplitudes of their sources (excitations) using mirroring effects. By exploiting the symmetry problem, we can show the advantage of reducing the size of the analysis domain, at least by a factor of two or more (2,4 and 8)(depending on the problem). Several simulation examples have been developed by the MoM-GEC and HFSS to validate this approach.

Problem Formulation:

Setting up of the problem: Let S_1, S_2, S_3 and S_4 be 4 sources of self-amplitudes E_1,E_2,E_3 and E_4 of 4 antennas are given in the figure (1) [1] . Each eigenstate of symmetry (under the condition of the mirror effect) has an amplitude $\tilde{E_1}$,$\tilde{E_2}$,$\tilde{E_3}$ and $\tilde{E_4}$, as we are going to explain [2, 3, 4, 5].

$*$ **Proper states** :

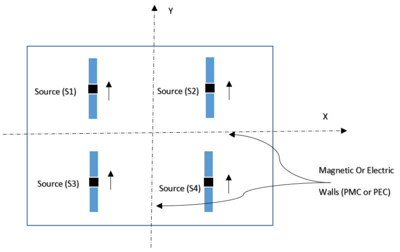

Fig. 1 *Four antenna configuration with a combination of electrical and magnetic symmetries*

Depending on the direction (ox): we can put up magnetic wall or electric wall. Similarly for the direction (oy), we can set up magnetic wall or electric wall, as described in the figure (1) . The combination between the 2 axes (ox) and (oy) allows to establish 4 states of amplitude mirroring, which can be summarized as follows [2, 3, 6, 7]:

States	Walls	Amplitudes of sources
1	magnetic (ox) \ magnetic (oy)	1 1 -1 -1
2	magnetic (ox) \ electric (oy)	1 -1 -1 1
3	electric (ox) \ magnetic (oy)	1 1 1 1
4	electric (ox) \ electric (oy)	1 -1 1 -1

By normalizing the states , we have :

$$u_1 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, u_2 = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, u_3 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \text{ and } u_4 = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

u_1, u_2 , u_3 and u_4 are orthonormal vectors.

Then, using the theorem of superposition, any state can be written [15] :

$$E = \sum_{i=1} E_i V_i = \sum_{i=1} \tilde{E}_i U_i \tag{1}$$

Where,

$$V_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, V_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, V_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } V_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\Rightarrow \text{E} = \underbrace{\begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \\ \hline \end{array}\!\!\!\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{Proper vectors}} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix} = \underbrace{\begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix}}_{\text{Eigen amplitudes of the antennas}} \text{, so,}$$

$$E = \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{pmatrix} = \frac{1}{2} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & u_3 & u_4 \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix} \end{array} \begin{pmatrix} \tilde{E}_1 \\ \tilde{E}_2 \\ \tilde{E}_3 \\ \tilde{E}_4 \end{pmatrix} = P\tilde{E}$$

The passage matrix P is unitary $\Rightarrow (P^{-1} = P^t)$.

$\Rightarrow \tilde{E} = P^{-1} E = P^t E$

$$\Rightarrow \underbrace{E}_{\text{Amplitudes of the antennas}} = P \underbrace{\tilde{E}}_{\text{Amplitudes of the states (sources in symmetries)}}$$

Each antenna self-amplitude in the total configuration of 4 antennas is written as the the superposition of the symmetry amplitudes (states) (all the combination of symmetry between electric and magnetic walls are considered) [8, 9, 10].

Symmetries and phases: In this case, we consider only the case of two sources (a source and its image) having an even or odd symmetry relation (in the presence of magnetic or electric planes) [8]. According to the theorem of image explained in [10, 11, 12], we can simply count the phases established between 2 symmetrical or anti-symmetrical sources (in phase or in phase opposition), as indicated in the figure (2) [13].

In the case of even symmetry, the phase shift established between 2

Fig. 2. Symmetry and antisymmetry between 2 sources

sources was :

$$\Phi_{S12} = 0[2\pi] = 2k\pi \Rightarrow 2 \text{ sources are in phase} \tag{2}$$

Inversely, in the case of odd symmetry the phase shift was :

$$\Phi_{S12} = (2k+1)[\pi] = (2k+1)\pi \Rightarrow 2 \text{ sources are in phase opposition} \tag{3}$$

According to the appendix (See the last section) [14], S can be written as:

$$[S] = \begin{pmatrix} S_{11} e^{-j\Phi 1} & S_{12} e^{-j(\Phi 2 + \Phi 1)} \\ S_{21} e^{-j(\Phi 2 + \Phi 1)} & S_{22} e^{-j\Phi 2} \end{pmatrix} \tag{4}$$

$*$ **Special cases:**

$\diamondsuit$ Case of even symmetry : if we fix $\Phi 1 = 0 \Rightarrow \Phi 2 = 0$.

So,

$$[S] = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \tag{5}$$

$\diamondsuit$ Case of odd symmetry :if we fix $\Phi 1 = 0 \Rightarrow \Phi 2 = \pi$ and vice versa if $\Phi 1 = \pi \Rightarrow \Phi 2 = 0$.

$\Rightarrow$

$$[S] = \begin{pmatrix} S_{11} & S_{12} e^{-j\pi} \\ S_{21} e^{-j\pi} & S_{22} e^{-j\pi} \end{pmatrix} \tag{6}$$

$\diamondsuit$ In the case of an arbitrary phase shift and an odd symmetry: we define any phase $\Phi 1$ will automatically add a phase shift of π to $\Phi 2$:

$\Rightarrow \Phi 2 = \Phi 1 + \pi \Rightarrow \Phi 2 - \Phi 1 = \pi$ (Note that this case is verified under HFSS with the phase commands (e.g. Arg(S_{21})). Finally, the matrix S is written :

$$[S] = \begin{pmatrix} S_{11} e^{-j\Phi 1} & S_{12} e^{-j(2\Phi 1 + \pi)} \\ S_{21} e^{-j(2\Phi 1 + \pi)} & S_{22} e^{-j(\Phi 1 + \pi)} \end{pmatrix} \tag{7}$$

We can generalize these different cases to study the configurations of the previous section (two by two between the 4 antennas). It is now possible

to simulate this problem using commercial software (such as HFSS and CST) or other software.
Finally, we can generalize the case of [S] parameters adapted to 4-antenna structures in each symmetry state (in the presence of all combinations of magnetic and electrical walls), which is written as follows [14]:

$$[S]=\begin{pmatrix} S_{11}e^{-j\Phi 1} & S_{12}e^{-j(\Phi 2-\Phi 1)} & S_{13}e^{-j(\Phi 3-\Phi 1)} & S_{14}e^{-j(\Phi 4-\Phi 1)} \\ S_{21}e^{-j(\Phi 2-\Phi 1)} & S_{22}e^{-j\Phi 2} & S_{23}e^{-j(\Phi 3-\Phi 2)} & S_{24}e^{-j(\Phi 4-\Phi 2)} \\ S_{31}e^{-j(\Phi 3-\Phi 1)} & S_{32}e^{-j(\Phi 3-\Phi 2)} & S_{33}e^{-j(\Phi 3)} & S_{34}e^{-j(\Phi 4-\Phi 3)} \\ S_{41}e^{-j(\Phi 4-\Phi 1)} & S_{42}e^{-j(\Phi 4-\Phi 2)} & S_{43}e^{-j(\Phi 4-\Phi 3)} & S_{44}e^{-j(\Phi 4)} \end{pmatrix} \quad (8)$$

$\Phi 1$, $\Phi 2$, $\Phi 3$ and $\Phi 4$ are parameters that depend on the nature of the symmetry (odd \ even) used at each configuration.
The particular cases of odd-even symmetries of equation(8) will be treated in the same way as the case of 2 sources (see equations (5),(6) , and (7)).

Results: To distinguish the different cases of symmetry, it is necessary to use the phases of the physical quantities J, E, and H and the coupling parameters S, Z, Y, ...etc. In our case, we used the MoM-GEC and HFSS as simulation tools (see figure(3)). In this context, several results were shown to validate this approach. First, we validated the MoM GEC and HFSS on the input impedance of the planar dipole antenna used in the 4 antenna configuration, as indicated in the figure (4)[15]. After that, a validation based on the boundary conditions of 4 antennas, was proven by the surface currents described by the guide modes and test functions, solved with MoM GEC, as given in in the figure (5) [15].

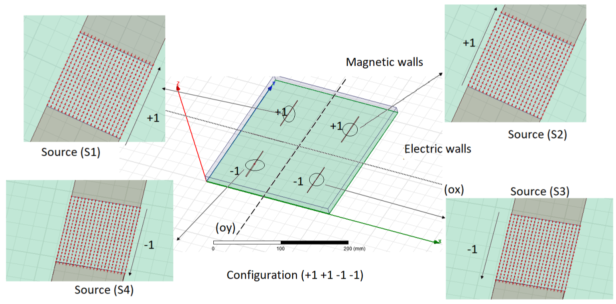

Fig. 3 *Example of a 4 antenna symmetric configuration with amplitudes of 11-1-1 (under HFSS): (ox) electrical walls, (oy) magnetic walls.*

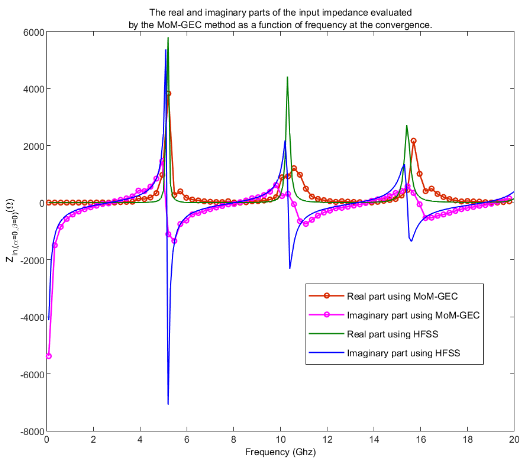

Fig. 4 *Input impedance seen at the source of one of 4 antenna given by the MoM GEC method and the HFSS tool (Validation) :*

A limitation with the MoM-GEC is shown to differentiate the different cases of symmetries. According to the MoM GEC formulation, the coupling parameters Z, Y and S are independent of the phase shifts of the excitation sources (From the coupling expression $Z=\frac{1}{A^t[Z_{i,j}]^{-1}A}$) [15]. These phase shifts will be caused only by the amplitudes of the surface currents J_S and the surface electric field E_S (J_S and E_S are complex numbers). We were able to differentiate these different symmetries with the MoM-GEC using the surface current phase of the 4-antenna structure, as given in the figures (7) and (8). It is an advantage for HFSS to show and distinguish these symmetries with the coupling parameters Z, Y, and S by using the commands Arg(S), Arg(Y), and Arg(Z) (or the HFSS Phase commands).

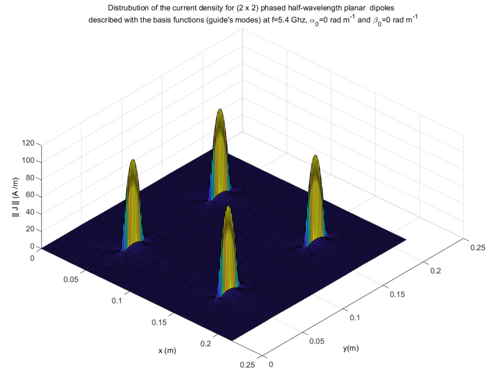

Fig. 5 *Distrubution of the current density for (2 x 2) phased half-wavelength planar dipoles described with the guide's modes functions at f=5.4 Ghz, α_0=0 rad m^{-1}andβ_0=0 rad m^{-1} (MoM-GEC method)*

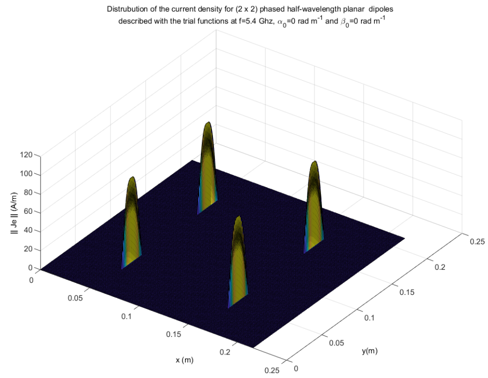

Fig. 6 *Distrubution of the current density for (2 x 2) phased half-wavelength planar dipoles described with the trial functions (test functions) at f=5.4 Ghz, α_0=0 rad m^{-1}andβ_0=0 rad m^{-1} (MoM-GEC method)*

According to the figure (9), we considered Arg(S_{12}) between the interaction of 2 sources of the configuration 1111 (2 symmetrical sources in-phase) and Arg(S_{12}) between the interaction of 2 sources of the configuration 11-1-1 (2 anti-symmetrical sources in phase opposition), we found a phase shift of angle π is established between Arg($S_{12}^{\text{symmetric sources}}$) and Arg($S_{12}^{\text{anti symmetric sources}}$) , at any point of the frequency band [0 20] GHz, as depicted in the figure(9).
This verifies , Arg($S_{12}^{\text{symmetric sources}}$) - Arg($S_{12}^{\text{anti symmetric sources}}$)=$\pi$ (or = $180°$) which seems to the equation: $\Phi_1-\Phi_2=\pi$ and the reasoning which follows the equation (6). Finally, we can distinguish the cases of symmetries by the introduction of phase shift in the matrix of [S] parameters, as explained in the previous section.

Appendix: Technique to count the phase shift between 2 sources (particular cases of odd and even symmetries) : This method explains how to calculate the phase between two sources. By the same reasoning established in [14], we imagine a line tracing placed at the input of a quadripole of known parameter [S] (for example a source and its image), as proposed in the figure(10). This case is considered general to produce the phase shift between 2 sources by the addition of line portions. This line segment provides a phase shift $\Phi 1$ related to the propagation (In our case, related to the source (S_1) and its image (S_2)).
If we first assume that the output is matched, then $a_2=0$ and,
The input reflection coefficient undergoes 2 times the phase shift, so

$$S_{11}^{'}=S_{11}e^{-(2j\Phi 1)} \quad (9)$$

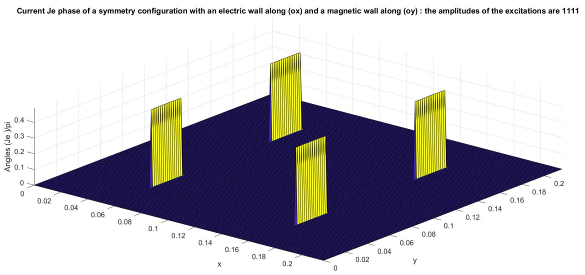

Fig. 7 *Current J_e's phase (or Angle) of a symmetry configuration with an electric wall along (ox) and a magnetic wall along (oy) : the amplitudes of the excitations are 1111 (MoM-GEC method)*

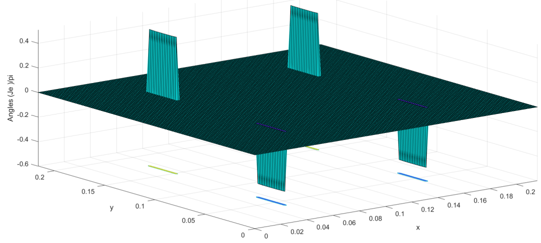

Fig. 8 *Current J_e 's phase (or Angle) of a symmetry configuration with a magnetic wall along (ox) and a magnetic wall along (oy) : the amplitudes of the excitations are 11-1-1 (MoM-GEC method)*

The transmission coefficient from the input to the output undergoes the phase shift once, so

$$S_{21}^{'} = S_{21}e^{-(j\Phi 1)} \tag{10}$$

If we now assume that the input is matched, then $a_1 = 0$, and,
The reflection coefficient seen from the output does not change

$$S_{22}^{'} = S_{22} \tag{11}$$

The reflection coefficient from the output to the input undergoes the phase shift only once, so

$$S_{12}^{'} = S_{12}e^{-(j\Phi 1)} \tag{12}$$

In short, it leads to

$$[S^{'}] = \begin{pmatrix} S_{11}e^{-2j\Phi 1} & S_{12}e^{-j(\Phi 1)} \\ S_{21}e^{-j(\Phi 1)} & S_{22} \end{pmatrix} \tag{13}$$

When the change concerns the two reference planes at the two accesses of a quadrupole, similar reasoning leads to :

$$[S] = \begin{pmatrix} S_{11}e^{-j2\Phi 1} & S_{12}e^{-j(\Phi 2+\Phi 1)} \\ S_{21}e^{-j(\Phi 2+\Phi 1)} & S_{22}e^{-j2\Phi 2} \end{pmatrix} \tag{14}$$

Conclusion: This paper presented the connection between the eigen-amplitudes of the 4 antennas and the symmetry of the amplitude of the associated sources (states) through mirror effects . A phase shift technique has been used to highlight all combinations of symmetry between electric and magnetic walls disposed along (ox) and (oy) axis. To distinguish these different symmetries, a method of calculating the S-parameters is introduced. Note that the main advantage of symmetry is to reduce the domain of analysis .As a perspective, we can apply this symmetry approach to a (largely extended) antenna sub-arrays with different source amplitudes.

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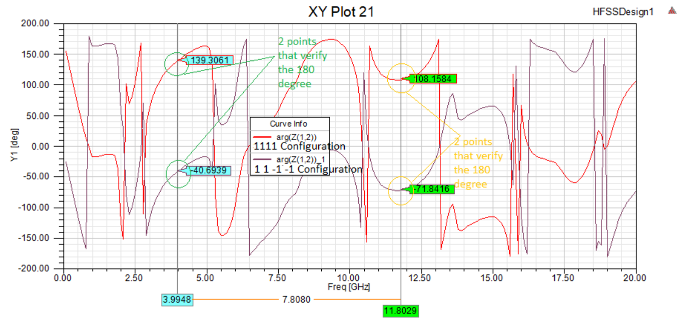

Fig. 9 *Arg (S12) between 2 sources of 4 antennas in two different symmetry configurations 1111 and 11-1-1 : The frequency range from 0 to 20 GHz . (Simulation under HFSS) (It checks $\Phi 2 - \Phi 1 = \pi$ (See after Equation 6))*

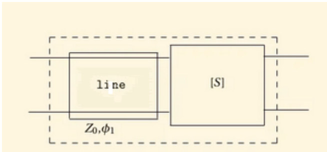

Fig. 10 *A portion of line added at the input of a quadrupole of known matrix [S] to reconstruct the phase established between 2 sources: we can work on the cases of symmetries as particular cases where the sources are in phase or opposition of phase)*

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