Covariance matrix design of transmit waveform for MIMO dual-function radar-communication system

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The multiple-input multiple-output (MIMO) dual-function radar-communication system can adjust the sidelobe level (SLL) by transmit beamforming to realize the communication function in the line-of-sight channel. However, a high SLL of the transmit beampattern can affect target detection performance. We propose a low-sidelobe covariance matrix design of transmit waveform method to address this issue. Minimizing the integrated sidelobe level is taken as the objective function with the constraint of the mainlobe lower and upper bounds. The optimal global solution can be obtained by the interior point algorithm since the objective function is semi-definite programming. Finally, the simulation results verify the performance in transmit beampattern and bit error rate.

Introduction: With the development of 4G and 5G communication, multiple-input multiple-output (MIMO) technology is utilized in various civil systems. Recently, MIMO radar has been widely concerned in the academic community. Similar to MIMO communication, MIMO radar can also obtain higher diversity gain and design freedom by multiple antennas, dramatically improving signal processing capability. The similarity between the two technologies offers the possibility of combining MIMO radar with MIMO communication. One approach to achieving this combination is to design a dual-function radar-communication (DFRC) waveform in the spatial domain [1]. In this case, communication is considered to be the secondary objective. Without compromising the primary radar operation, the information symbols should be embedded in radar transmit waveforms in a particular dimension.

The MIMO DFRC system can adjust the sidelobe level (SLL) by transmit beamforming to realize the communication function in the line-of-sight channel [2]. The technique employs two weight vectors and orthogonal waveforms to control the SLL to represent the communication symbols [3]. This problem of transmit beamforming is a convex optimization with low complexity. However, the synthesized beampattern has a wide mainlobe and a high SLL, which is not conducive to detecting the weak target and suppressing the interference entering from the SLL. It has a severe impact on the performance of radar target detection.

The power distribution of the transmit beampattern can be controlled by designing the covariance matrix of the MIMO radar transmit waveforms [4]. In this letter, we propose a low-sidelobe covariance matrix design method for the MIMO DFRC system. The objective function is minimizing the integrated sidelobe level (ISL) for SLLs reduction. The mainlobe lower and upper bounds are introduced to restrict the radar mainlobe power. The sidelobe control of the transmit beamforming towards the communication direction is employed for information embedding. Besides, the constraints also include that the covariance matrix is semi-positive and the total transmit power is constant. The simulation results verify that the proposed method can achieve a lower SLL and narrower mainlobe without the communication bit error rate (BER) loss.

Traditional method: We consider a DFRC system having M_T transmit antennas with the inter-element spacing of half wavelength. The communication bits are embedded by controlling the SLLs while ensuring the radar detection function in the mainlobe. During each radar pulse, the transmitter is assumed to embed L_B communication bits, which is denoted as the binary sequence $b_l,\ l=1,\dots,L_B$. In phased array mode, the baseband signal model is [2]

$$\boldsymbol{S}(t;\tau)=\sqrt{\frac{M_T}{L_B}}\sum_{l=1}^{L_B}\left(b_l(\tau)\boldsymbol{u}_{\mathrm{L}}^*+\left(b_l(\tau)-1\right)\boldsymbol{u}_{\mathrm{H}}^*\right)\psi_l(t),\tag{1}$$

where $(\cdot)^*$ denotes the conjugate operation, τ is the pulse index, $\sqrt{M_T/L_B}$ is the power normalization factor, $\psi_l(t),\ l=1,\dots,L_B$ are the orthogonal waveforms satisfying that $\int_T\psi_l(t)\psi_{l'}^*(t)dt=0,\ l\neq l'$ with T being the pulse width, $\boldsymbol{u}_{\mathrm{H}}$ and $\boldsymbol{u}_{\mathrm{L}}$ are the $M_T\times 1$ weight vectors of transmit beamforming used to realize different SLLs Δ_{H} and Δ_{L} wherein $\Delta_{\mathrm{H}}>\Delta_{\mathrm{L}}$. For convenience, we suppose the weight vector corresponding to the lth orthogonal waveform $\psi_l(t)$ is $\boldsymbol{u}_l$. The optimization model of transmit beamforming is [2]

$$\min_{\boldsymbol{u}_l}\max_{\theta_i}\left|e^{\mathrm{j}\varphi(\theta_i)}-\boldsymbol{u}_l^{\mathrm{H}}\boldsymbol{a}(\theta_i)\right|,\quad \theta_i\in\Theta,\tag{2a}$$

$$\text{subject to }\left|\boldsymbol{u}_l^{\mathrm{H}}\boldsymbol{a}(\theta_p)\right|\leq\varepsilon,\quad \theta_p\in\bar{\Theta},\tag{2b}$$

$$\boldsymbol{u}_l^{\mathrm{H}}\boldsymbol{a}(\theta_j)=\Delta_l,\quad j=1,\dots,J,\tag{2c}$$

where Θ and $\bar{\Theta}$ are the set of angles for the radar operation and sidelobe region, $\boldsymbol{a}(\theta)$ is the $M_T\times 1$ transmit steering vector, $\varphi(\theta)$ is the phase profile of user's choice, ε is a positive value to control the SLLs, J is the number of communication receiver, and $(\cdot)^{\mathrm{H}}$ denotes conjugate transpose operation, and Δ_l is the SLL radiated towards the communication directions over the lth transmitting beam. The communication receiver at corresponding direction can obtain the original symbols by detecting the received power amounts, which is equivalent to amplitude modulation.

Eq. (2) minimizes the mainlobe fluctuation while controlling the SLLs for beamforming through the Chebyshev criterion. The objective function is employed to conformal the radar mainlobe. For the constraints, the sidelobe level needs to be low enough to meet the target detection performance on the premise of realizing the communication function. However, when the parameter ε takes a low value to get low SLLs, the controlling range for communication is also deficient, which may cause information loss. Moreover, the SLLs simultaneously depend on the transition region nearby the mainlobe. A broad transition region increases the mainlobe width. As a result, the overall transmit beampattern of the traditional method has a wide mainlobe and a high SLL. It has a low resolution and a poor performance in detecting the weak target and suppressing the interference.

Proposed method: In MIMO mode, the baseband signal model can be rewritten as

$$\boldsymbol{S}(t;\tau)=\sqrt{\frac{M_T}{L_B}}\sum_{l=1}^{L_B}\left(b_l(\tau)\boldsymbol{x}_{l,\mathrm{L}}(t)+\left(1-b_l(\tau)\right)\boldsymbol{x}_{l,\mathrm{H}}(t)\right),\tag{3}$$

where $\boldsymbol{x}_{l,\mathrm{L}}$ and $\boldsymbol{x}_{l,\mathrm{H}}$ are the lth orthogonal waveform in frequency domain corresponding to $\boldsymbol{R}_{l,\mathrm{L}}$ and $\boldsymbol{R}_{l,\mathrm{H}}$ for different SLLs Δ_{L} and Δ_{H}, respectively. The overall transmit beampattern at angle θ can be written as

$$P(\theta)=\frac{1}{L_B}\sum_{l=1}^{L_B}\boldsymbol{a}^{\mathrm{H}}(\theta)\boldsymbol{R}_l\boldsymbol{a}(\theta),\tag{4}$$

where $\boldsymbol{R}_l=\boldsymbol{X}_l\boldsymbol{X}_l^{\mathrm{H}}/N,\ l=1,\dots,L_B$ are the $M_T\times M_T$ covariance matrix of transmitting signal $(\boldsymbol{X}_l)_{M_T\times N}$ with N being the sample number.

Assume that P_N discrete points are in the sidelobe region $\bar{\Theta}$, and the mainlobe lower and upper bounds are γ and β, respectively. The optimization model of transmit beamforming can be expressed as

$$\min_{\boldsymbol{R}_l}\sum_{p=1}^{P_N}\left(\boldsymbol{a}^{\mathrm{H}}(\theta_p)\boldsymbol{R}_l\boldsymbol{a}(\theta_p)\right),\quad \theta_p\in\bar{\Theta},\tag{5a}$$

$$\text{subject to }\gamma\leq\boldsymbol{a}^{\mathrm{H}}(\theta_i)\boldsymbol{R}_l\boldsymbol{a}(\theta_i)\leq\beta,\quad \theta_i\in\Theta,\tag{5b}$$

$$\boldsymbol{a}^{\mathrm{H}}(\theta_j)\boldsymbol{R}_l\boldsymbol{a}(\theta_j)=\Delta_l,\quad j=1,\dots,J,\tag{5c}$$

$$\mathrm{tr}(\boldsymbol{R}_l)=1,\ \boldsymbol{R}_l\succ 0,\tag{5d}$$

where $\mathrm{tr}(\cdot)$ denotes the trace operation, and $\boldsymbol{R}_l\succ 0$ represents the semi-positive definite constraint of the covariance matrix. Eq. (5) is a semi-definite programming (SDP), which can be efficiently solved by the interior point method [5].

Simulation results: We consider a MIMO DFRC system composed of a half-wavelength uniform linear array with $M_T = 10$. The mainlobe region is focused towards a single angle $\theta_{\text{radar}} = 0^{\circ}$, the sidelobe region is located at $[-90^{\circ}, -10^{\circ}] \cup [10^{\circ}, 90^{\circ}]$, and the mainlobe lower and upper bounds are set to be $\gamma = 1$ and $\beta = 10$. Assume three communication receivers located at $\theta_1 = -50^{\circ}$, $\theta_2 = -30^{\circ}$, and $\theta_3 = 40^{\circ}$. The communication SLLs associated with four covariance matrixes $\boldsymbol{R}_l,\ l = 1,\ldots,4$ are $\Delta_1^2 = -20$ dB, $\Delta_2^2 = -21.76$ dB, $\Delta_3^2 = -24.77$ dB, and $\Delta_4^2 = -40$ dB with respect to the radar mainlobe.

Fig.1 (a) shows the respective beampatterns formed by each covariance matrix. All covariance matrixes can provide almost the same mainlobe. It indicates that the radar mainlobe cannot be affected by the constraint on the SLL. Besides, the SLLs towards the communication directions are separated in different transmit beampatterns. Fig. 1 (b) shows the overall transmit beampattern synthesized by Eq. (4). The peak sidelobe level (PSL) of the proposed method is evidently lower than that of the method in Ref. [2]. Simultaneously, the mainlobe width is also narrower than the method in Ref. [2]. It illustrates that the proposed method is prior to the traditional one in target detection performance.

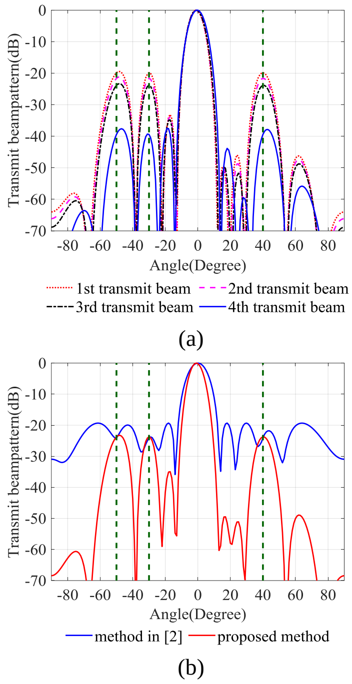

Fig. 1 *Comparison of the transmit beampattern (a) The transmit beampattern of proposed method (b) the overall beampatterns*

Assume that each communication receiver has $M_R = 10$ antennas. A total of 10^6 pulses are transmitted for texting. 1 bit, 4 bits, 16 bits, and 64 bits are embedded during each pulse. Figure 2 shows the BER curves of the method in Ref. [2] and the proposed method. The anti-noise performance of both methods is the same. Since they adjust the SLL by transmitting beamforming to realize the communication function, which is equivalent to amplitude modulation.

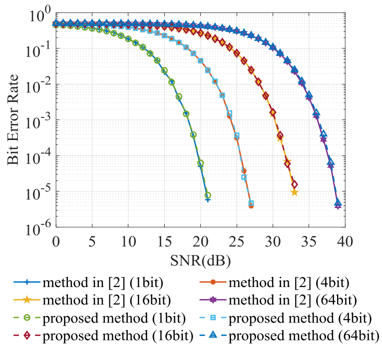

Fig. 2 *BER versus SNR*

Conclusions: This letter has proposed a low-sidelobe covariance matrix design method for the MIMO DFRC system. It has laid the foundation for designing the transmit waveform in the next step. The proposed optimization is an SDP problem with low complexity. By constraining the mainlobe power and minimizing the ISL, the PSL of the overall transmit beampattern is successfully reduced. Moreover, when the lower bound of the mainlobe is constant, the SLL can be further reduced by increasing the mainlobe upper bound. The simulation results demonstrate that the proposed method can improve the target detection performance of radar on the premise of realizing the communication function within the line-of-sight range.

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