The minimal interaction induced by the translation subgroup has a gap in the low-symmetric state.

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Abstract

The paper investigates the low-symmetric state of the compensating field of the distortion tensor and proves that there is a gap in this state. It is shown that the distortion tensor is the compensating field of the minimal interaction induced by a translation subgroup. In this paper, for the first time, an exact wave solution for sound pressure in a continuous medium is obtained from the equations of state for the distortion tensor. It is shown that the sound is described as "massive" wave of the distortion tensor, the spectrum of which has the minimal frequency, which corresponds to a gap. The presence of a gap in the low-symmetric state gives reason to believe that the distortion tensor, as the compensating interaction field, describes a strong fundamental interaction. As it is known, the description of the gap in the strong fundamental interaction has been declared a Millennium problem by the Clay Mathematical Institute.

1. Introduction

In 2000, at the International Mathematical Congress, the Millennium problem was formulated: the description of the gap of the strong fundamental interaction. It is known that there is a gap in the strong fundamental interaction, but there is no description of it. The solution to this problem was assumed within the framework of the Yang-Mills model [1, 2]. However, in the description of the problem itself, it was emphasized that it is impossible to obtain a gap for Yang-Mills fields since the non-Abelian symmetry gauge group leads to nonlinear self-action of Yang-Mills fields in a low-symmetric state and the absence of wave solutions for Yang-Mills fields.

On the other hand, in [3] a model was constructed with a tensor compensating field of minimal interaction and an Abelian gauge group, which was induced by a commutative subgroup of spatial translations. Therefore, there is reason to believe that in such a model a gap in the low-symmetric state will be obtained since in Abelian models there is no self-action in a low-symmetric state.

The minimal interaction in the field theory is understood as the interaction that is given by an extended derivative with a compensating interaction field [4]. The construction of a minimal interaction in the form of an extended derivative with a compensating interaction field was firstly proposed by Pauli for an electromagnetic field [5]. The existence of the extended derivative in electrodynamics was justified by the invariance of the Lagrangian with respect to the local Abelian gauge group of symmetry. The gauge symmetry of the Lagrangian [4,5] was associated with a quantum mechanical description using a complex wave function that has uncertainty in the phase.

On the other hand, in [3] the Lagrangian gauge group was induced by a local irreducible representation of a subgroup of spatial translations. It was proved in [6] that the charge in the extended derivative of the minimal interaction induced by the translation subgroup [3] is the wave vector or quantum momentum, and the compensating field is the distortion tensor A_{ij}.

As it is known, the distortion tensor was first defined in the theory of elasticity [7]. It sets the dislocation density and is defined up to the gradient of the displacement vector. Since in [6] it was shown that the distortion tensor A_{ij} is an interaction field, the distortion exists not only in a solid state, but wherever there is momentum.

Thus, the distortion tensor is not always related to the density of dislocations in a solid state. Rather, on the contrary, under certain conditions it does indeed describe dislocations in a solid state [6], but, in general, it is responsible for the interaction. Quantization of the Burgers vector in a solid state is similar to quantization of the magnetic flux in Abrikosov vortices [8]. The magnetic flux, as it is known, is not always quantized.

The law of conservation of momentum is associated with spatial translational symmetry. Therefore, in this model, the quantum momentum is realized as a coefficient in front of the compensating field: the distortion tensor A_{ij} in the extended derivative [6], as well as an electric charge for the electromagnetic interaction.

In [9] it was shown that the attraction of equal and opposite-directed quantum momenta leads to the pairing of electrons in the superconducting state [10]. In [9] it was proved that when the coherence length of a Cooper pair of electrons is less than a micrometer, the attraction of opposite and equal quantum momenta of electrons becomes greater than their natural electrical repulsion. The existence of Cooper pairs, in fact, is proof of the attraction of quantum momenta as charges.

In [10] this interaction of electrons was called the electron-phonon interaction due to the fact that acoustic phonons were observed during the formation of Cooper pairs. Therefore, in the theory of BCS, the pairing of electrons was associated with phonons, from where the name electron-phonon interaction came from.

Thus, we have the following initial data.

Firstly, the quantum momentum behaves as a charge of minimal interaction [6,9], it sets the minimal interaction in the form of an extended derivative, similar to the electric charge for the electromagnetic fundamental interaction [4, 5].

Secondly, the compensating field of this interaction is the distortion tensor: A_{ij}, which was firstly introduced in the theory of elasticity [7] to describe plastic deformations with dislocations, as a generalization of the strain tensor during the destruction of a continuous elastic medium.

Thirdly, according to the BCS theory [10], sound waves are associated with this interaction.

The paper shows that this interaction has a gap in the low-symmetric state when describing the waves of the distortion tensor. Therefore, most likely, this interaction corresponds to a strong fundamental interaction, which, as is well known from the Clay Mathematical Institute (CMI) Millennium problem, has a gap in the low-symmetric state.

2. The problem statement.

At small distances for small wavelengths of particles, a quantum momentum, as a charge, describes a very strong interaction, since the magnitude of the quantum momentum is inversely proportional to the wavelength. The smaller an elementary particle, the more energy it takes to destroy it. Therefore, large hadron colliders are being built to study elementary particles, which make it possible to achieve very high energies.

Just as it was possible to describe the attraction of electrons in the Cooper pair [9], it is also possible to describe the attraction of two protons with equal and opposite-directed quantum momenta. Therefore, it can be expected that with the help of this formalism it will be possible to describe the attraction of protons and neutrons in the nucleus of an atom as the attraction of their quantum momenta. As is known, initially the strong fundamental interaction was found in the nucleus of an atom. At the same time, the problem of describing the attraction of nucleons in the nucleus of an atom is, in general, three-dimensional, and not one-dimensional, as for paired electrons, with a total quantum momentum equal to zero. However, the solution of this problem: the attraction of nucleons in the nucleus of an atom is beyond the scope of this paper.

The plan to prove the fact that the quantum momentum can be the charge of the strong fundamental interaction will be based on the description of the gap of the strong fundamental interaction, declared as a Millennium problem in 2000.

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As is known, a gap is a characteristic of a low-symmetric state in which the gauge or gradient symmetry of the equations of state for the compensating interaction field is broken. The characteristic feature of the gap is the minimal frequency of the waves of the compensating interaction field in the low-symmetric state. Therefore, it is necessary to prove that the waves of the distortion tensor have a minimal frequency in the low-symmetric state, when the gradient symmetry of the equations of state is broken.

In the future, we will call waves with minimal frequency "massive", by analogy with the solutions of the Klein-Gordon equation for a massive scalar field [4].

It is known that the Yang-Mills fields have self-action in the low-symmetric state. Therefore, they do not have wave solutions and, therefore, they can't describe the gap.

In connection with the Millennium problem, the question arises, which interactions have a gap in the low-symmetric state?

Firstly, it is the electromagnetic interaction. It is known that the gap exists in a low-symmetric state of the electromagnetic field, which occurs when the gauge or gradient symmetry of the Maxwell equations is broken [11].

When the gauge symmetry of the electromagnetic field is broken, the superconducting state is formed, which are described in the London equations $A_i = -\delta'^2 j_i$ [11, 12], here A_i - electromagnetic potential, j_i - current density, δ' - depth of magnetic field penetration in the superconductor.

As you know, London was able to describe the Meissner effect, the ejection of a magnetic field from a superconductor. When the London equations are fulfilled, the gradient symmetry of the Maxwell equations is broken. In this case, electromagnetic waves in the superconducting state satisfy the inhomogeneous d'Alembert equation, which has wave solutions with a minimal frequency.

The minimal frequency is related to the depth of penetration of the magnetic field into the superconductor by the relation: $\omega'_0\delta' = c'$, where c' is the velocity of electromagnetic waves in the superconductor. Thus, the minimal frequency of electromagnetic waves in the superconducting state is an indicator of broken gauge symmetry in the low-symmetric state of the electromagnetic field.

In this example, physical interaction models containing a gap or a minimal frequency in the low-symmetric state of the compensating field end in the field theory.

Note that the concept of the gauge symmetry is usually used in field theory along with gradient symmetry. In field theory, it is assumed that the abstract gauge symmetry of the Lagrangian sets the minimal interaction [2, 4, 5].

Since in [3] it was possible to construct the expanded derivative induced by the usual translation subgroup, there is no reason to use an abstract gauge group of internal symmetries of the Lagrangian. In paragraph 4 of [13] it was shown that the Abelian symmetry gauge group for electromagnetic interaction can also be interpreted as a local irreducible representation of the subgroup of time translations. At the same time, it is not necessary to postulate that the electromagnetic potential changes sign during time inversion, as is done in the field theory [2, 4].

Thus, it is possible to get away from abstract local gauge groups of internal symmetries of the Lagrangian, to construct the interaction in the form of an expanded derivative, and use local representations of the global subgroup of space-time translations. In this case, the equations of state for compensating fields will have gradient symmetry, which is also called gauge symmetry.

In the future, the name gradient symmetry will be used in relation to equations of state, and the name gauge symmetry will be used in relation to abstract models of field theory that are not related to space-time symmetry, for example, for Yang-Mills fields.

As noted above, there is a very important clue in the theory of BCS [10]. In it, the pairing of electrons was associated with the electron-phonon interaction. It was shown in [9] that lattice vibrations have no relation to the attraction of electrons. The attraction of electrons is caused by the interaction of their quantum momenta with each other using the distortion tensor, which is the compensating interaction field in the expanded derivative as well as the electromagnetic potential.

The observed acoustic waves during the formation of Cooper pairs give reason to believe that the waves of the distortion tensor in a continuous elastic medium describe sound. After all, the distortion tensor was originally introduced in the theory of elasticity as a generalization of the strain tensor [7].

It is obvious that sound exists in a continuous medium with density: ρ . Therefore, it is necessary to investigate the equations of state for the compensating field: the distortion tensor, in a continuous medium, and make sure that they describe the sound waves.

In this paper, for the first time, an exact wave solution for sound pressure in a continuous elastic medium is obtained in the form: $P = P_0 \exp(-i\omega(t - x/c))$, where c is the speed of the sound, and P_0 is the amplitude of the pressure in the sound wave. This solution is derived from the equation of state for the distortion tensor when the momentum is proportional to the velocity field: $p_i = \rho \upsilon_i$.

As it is known, sound waves in gases and liquids are described by Euler's equations of hydrodynamics [14]. However, this description does not stand up to criticism.

Firstly, the Euler's equation is nonlinear and has no wave solutions. It is easy to see this if you substitute wave solutions into Euler's equation. Therefore, in order to obtain wave solutions the Euler's equation is linearized. I. e., the nonlinear terms in the Euler equation are neglected. Obviously, this can’t be done, since the nonlinear potential term in Euler's equation is responsible for the kinetic energy of the continuum medium in the Bernoulli equation. Consequently, it is also responsible for the kinetic energy of the mechanical wave.

Secondly, the linearized Euler equations don’t contain the quadratic d'Alembert operator, since the Euler equation is linear in derivatives. Therefore, in order to obtain wave equations with the d'Alembert operator, velocity and pressure are usually defined as derivatives of the scalar potential function φ : $\upsilon_i = \partial\varphi / \partial x_i$, $p = -\rho\partial\varphi / \partial t$ [14]. However, the wave solutions obtained in this way contain a linear dependence of the pressure from the frequency: $p = \omega\rho \operatorname{Re}(i\varphi_0 \exp(-i\omega(t - x/c)))$, see pp. 351-354 [14].

These solutions, in general, do not correspond to sound waves, since in sound waves the frequency does not depend on pressure. This is well known and is used when extracting sound from all musical instruments. For example, "forte" and "piano" have the meaning of "louder" and "quieter". It is known that the frequency of sound does not change when the same keys are pressed on a piano with different pressures.

Since in a continuous elastic medium the velocity field υ_i and the distortion tensor A_{ij} are proportional to the conjugate observed fields: the momentum p_i and the stress tensor σ_{ij} accordingly, then the equations of the state for compensating fields υ_i , A_{ij} [6] in a continuous elastic medium are inhomogeneous d'Alembert equations. Therefore, not ordinary mechanical waves are responsible for sound in a continuous elastic medium, but "massive" waves of the distortion tensor, which have a spectrum with a minimal frequency.

As you know, the Clay Mathematical Institute (CMI) declared the description of the gap of the strong fundamental interaction-the problem of the Millennium. Consequently, there is a reason to believe that the "massive" waves of the distortion tensor in a continuous elastic medium describe the gap of the strong fundamental interaction.

This conclusion is based on the fact that so far the gap has been described only in the low-symmetric state of the electromagnetic field [11], and for a strong fundamental interaction, the gap has not been described so far. At the same time, there are no and there can be no other gaps related to the symmetry of space-time.

Indeed, since the expanded derivative of the minimal interaction [6] is associated with a subgroup of spatial translations, and the expanded derivative of the electromagnetic interaction is associated with temporal translations [13], there can be no other gaps, based on the translational symmetry of space-time.

It is not difficult to make sure that there are no other nonequivalent local irreducible representations, except for the representations of the translation subgroup, suitable for constructing the expanded derivative of the minimal interaction.

The assumptions that fundamental interactions in nature can be induced by abstract local gauge groups of internal symmetries of the Lagrangian, unrelated to the symmetry of space-time, in our opinion, are not substantiated and unlikely.

And now let's give a mathematical justification for what was said above.

3. The equation of state for the compensating field of interaction: υ_i, A_{ij}.

In the paper [6] the minimal interaction was recorded in the form of an expanded derivative:

$$D_j\psi_{\vec{k}} = \left(\frac{\partial}{\partial x_j} - i\sum_p \kappa_p A_{pj}\right)\psi_{\vec{k}}\,. \tag{1}$$

Where the wave vector κ_p is the coefficient in front of the compensating field, A_{pj} is the distortion tensor [6]. Here $\psi_{\vec{k}}$ is the wave function (or order parameter [3]) which is transformed by the local irreducible representation of the translation subgroup: $\hat{a}_q\psi_{\vec{k}} = \exp(i\delta_{pq}k_p a_q)\psi_{\vec{k}}$, where $k_p = k_p(x_j)$, and A_{pj} is transformed by:

$$\hat{a}_q(\kappa_p A_{pj}) = \kappa_p A_{pj} + \delta_{pq}\,\partial(k_p a_q)/\partial x_j\,. \tag{2}$$

Then the extended derivative (1) is the eigenfunction of the translation operator:
$\hat{a}_q(D_j\psi_{\vec{k}}) = \exp(i\delta_{pq}k_p a_q)D_j\psi_{\vec{k}}$.

Similarly, the velocity field υ_i compensates for the time derivative $\psi_{\vec{k}}$:

$$D_0\psi_{\vec{k}} = \left(\frac{\partial}{\partial t} - i\sum_n \kappa_n \upsilon_n\right)\psi_{\vec{k}}\,, \tag{3}$$

$$\hat{a}_q(\kappa_i\upsilon_i) = \kappa_i\upsilon_i + \delta_{iq}\,\partial(k_i a_q)/\partial t\,. \tag{4}$$

The equations of state for the compensating interaction field: υ_i, A_{ij}, or the 4-distortion tensor [15, 16] induced by the quantum momentum as the charge [6,9] have the form:

$$p_i = -\frac{\gamma}{c^2}\frac{\partial \varepsilon_{ij}}{\partial x_j}\,, \tag{5}$$

$$\sigma_{ij} = \gamma\, e_{jkp}\frac{\partial \rho_{ip}}{\partial x_k} - \frac{\gamma}{c^2}\frac{\partial \varepsilon_{ij}}{\partial t}\,, \tag{6}$$

where

$$\varepsilon_{ij} = -\frac{\partial \upsilon_i}{\partial x_j} + \frac{\partial A_{ij}}{\partial t} \tag{7}$$

the centrally symmetric tension of the compensating fields υ_i, A_{ij}, and

$$\rho_{ip} = -e_{pkn}\frac{\partial A_{in}}{\partial x_k}, \tag{8}$$

vortex tension of the distortion tensor A_{ij}. Here γ - the dimensional coefficient, e_{jkn} - the anti-symmetric Levi-Civita tensor.

These equations of state (5, 6) can be obtained by variation of the Lagrangian: $\delta L/\delta\upsilon_i = 0$, $\delta L/\delta A_{ij} = 0$, for the 4-distortion tensor υ_i, A_{ij} [15, 16]:

$$L = p_i\upsilon_i - \sigma_{ij}A_{ij} + \frac{\gamma}{2}\left(\frac{1}{c^2}\varepsilon_{ij}\varepsilon_{ij} - \rho_{ij}\rho_{ij}\right), \tag{9}$$

where $p_i = \partial L/\partial\upsilon_i$, $\sigma_{ij} = -\partial L/\partial A_{ij}$. This conclusion is analogous to the conclusion of Maxwell's equations from the Lagrangian of the electromagnetic field in the field theory [17].

The Lagrangian (9) is a consequence of the Lagrangian $L = L(\psi_{\vec{k}}, \upsilon_i, A_{ij})$, where the momentum p_i and the stress tensor σ_{ij} are the first integrals dependent on the fields $\psi_{\vec{k}}, \upsilon_i, A_{ij}$, according to E. Noether's theorem [9]. However, when the momentum p_i and stress tensor σ_{ij} are external sources for the fields υ_i, A_{ij}, the Lagrangian has the form (9).

From (5,6) by direct differentiation follows the continuity equation, which has the meaning of the law of conservation of momentum, written in differential form:

$$\frac{\partial\sigma_{ij}}{\partial x_j} = \frac{\partial p_i}{\partial t}. \tag{10}$$

To obtain wave equations with the d'Alembert operator, as in the field theory [17], we use the pseudo Lorentz calibration condition [15, 16] for fields υ_i, A_{ij}.

$$\frac{\partial A_{ij}}{\partial x_j} = c^{-2}\frac{\partial\upsilon_i}{\partial t}. \tag{11}$$

Substituting (7, 8) in (5, 6), taking into account the condition (11), we obtain the equations of state in the form:

$$\frac{c^2}{\gamma}p_i = \left(\frac{\partial^2}{\partial x_n\partial x_n} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\upsilon_i, \tag{12}$$

$$\frac{1}{\gamma}\sigma_{ij} = \left(\frac{\partial^2}{\partial x_n\partial x_n} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)A_{ij}. \tag{13}$$

The equations of state (12, 13) for interaction fields: υ_i, A_{ij}, contain the d'Alembert operator. This is a consequence of the gradient invariance of the equations of the state (5-8). It follows from (12, 13) that the source of the interaction fields: υ_i, A_{ij}, are conjugate observable fields: p_i and σ_{ij}. According to the continuity equation (10), the stress tensor σ_{ij} is the momentum flow with a minus sign: $\sigma_{ij} = -p_i v_j$, where v_j is the flow rate. Thus, the equations (12, 13) are similar to Maxwell's equations [17], where charge and current density act as the source of the electromagnetic field.

4. "Massive" waves of the distortion tensor in a continuous elastic medium.

As it is known, in the isotropic continuous medium with density ρ, there is the directly proportional relationship between momentum and velocity: $p_i = \rho\upsilon_i$ (the difference between a quantum momentum and the conventional momentum is shown below, in paragraph 6). Then the equations of state (12, 13) for the fields υ_i, A_{ij} in a continuous medium will have the form of the inhomogeneous wave equation:

$$\frac{\rho c^2}{\gamma}\upsilon_i = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\upsilon_i , \tag{14}$$

$$\frac{\rho c^2}{\gamma}A_{ij} = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})A_{ij} . \tag{15}$$

Indeed, equation (14) is obtained by direct substitution $p_i = \rho\upsilon_i$ in (12). It follows from the known relation between momentum and velocity in a continuous medium with density ρ.

Equation (15) is obtained from the relation $p_i = \rho\upsilon_i$ and two equations: the continuity equation (10) and the gauge condition (11), which also has the form of a continuity equation. It follows from the continuity equation (10) that the stress tensor can be represented as the momentum flow with a minus sign $\sigma_{ij} = -p_i v_j$. Similarly, it follows from equation (11) that the distortion tensor A_{ij} can be represented as the flow of the velocity field with a minus sign and a coefficient $1/c^2$: $A_{ij} = -c^{-2}\upsilon_i v_j$. Since in a continuous medium the velocity field coincides with the flow velocity: $\upsilon_i = v_j$, we obtain: $\sigma_{ij} = -\rho\upsilon_i\upsilon_j$, $A_{ij} = -c^{-2}\upsilon_i\upsilon_j$. Whence it follows that in a continuous medium with density ρ:

$$\sigma_{ij} = \rho c^2 A_{ij} . \tag{16}$$

This means that there is elasticity in a continuous medium, since the distortion tensor A_{ij} was originally introduced into elasticity theory as a generalization of the strain tensor [7]. Equation (15) follows from the relation: $\sigma_{ij} = \rho c^2 A_{ij}$, which has the form of Hooke's law for the distortion tensor. Here the coefficient: $K = \rho c^2$, in fact, characterizes the elasticity of the continuous medium.

Thus, twelve equations of state (12, 13), for the compensating field of interaction: υ_i, A_{ij}, in a continuous isotropic medium turned into the same inhomogeneous d'Aalembert equation with the same spectrum. Let's say at once that this phenomenon is associated with the violation of the gradient symmetry of the equations of state (12, 13) and with the phase transition to the state of a continuous elastic medium, which will be discussed later in paragraph 6.

Let's call the equations (14,15) the equation of a "massive" wave due to the fact that it is similar to the Klein-Gordon equation for a massive scalar field [4], and due to the fact that the spectrum with the minimal frequency in the equations (14,15) is given by the density of the continuous medium ρ not equal to zero.

Indeed, the equations (14, 15) have solutions: $\upsilon_i = \upsilon_{i0}\exp(i\vec{q}\vec{x} - i\omega t)$, $A_{ij} = A_{ij0}\exp(i\vec{q}\vec{x} - i\omega t)$, with spectrum: $\omega = c\sqrt{\vec{q}^2 + c^{-2}\omega_0^2}$ and minimal frequency:

$$\omega_0 = c^2\sqrt{\rho/\gamma} . \tag{17}$$

The frequency ω_0 is also called the zero frequency, below which there can be no wave solutions for the equations of state (14, 15). Knowing the minimal frequency: $\omega_0 = c^2\sqrt{\rho/\gamma}$, the dimensional constant can be calculated from the expression $\gamma = \rho c^4 / \omega_0^2$, from which its physical meaning follows.

We show that the waves of the 4-distortion tensor describe sound waves. In the future, we will omit the 4-distortion tensor, and use the expression distortion tensor to denote a pair of fields: υ_i, A_{ij}.

The equations of state (5-8) by construction have gradient invariance:

$$A_{ij} \to A_{ij} + \partial u_i/\partial x_j \text{ , } \upsilon_i \to \upsilon_i + \partial u_i/\partial t \text{ ,} \tag{18}$$

where u_i is the displacement vector [7]. The displacement vector u_i satisfies the wave equation in a continuous medium. This follows from the pseudo Lorentz condition (11) when substituting a gradient transformation (18):

$$\frac{\partial^2 u_i}{\partial x_n \partial x_n} = \frac{1}{c^2}\frac{\partial^2 u_i}{\partial t^2} \text{ .} \tag{19}$$

Thus, (19) describes mechanical waves in a continuous medium, and equations (14, 15) describe sound in a continuous medium. But sound, or distortion tensor waves, are not mechanical waves, since the spectrum of sound waves has a minimal frequency and is differs from the spectrum of mechanical waves.

Indeed, mechanical waves can occur at any low frequency, and sound waves exist only when their frequency is higher than the minimal frequency. In this case, sound waves in a continuous medium coincide with mechanical waves. Since there can be only one displacement and one velocity in a continuous medium.

This, at first glance, unexpected effect is well known on the example of the low-symmetric state of the electromagnetic field, when the gradient symmetry of Maxwell's equations is broken: $A_i = -\delta^2 j_i$ [11, 12]. In the superconducting state, the charge and electromagnetic field waves propagate together, while the electromagnetic field waves having a gap or minimal frequency. While charge waves can propagate with any arbitrarily small specified frequency.

5. The exact wave solution for the sound pressure in gases and liquids.

Consider the propagation of waves of the distortion tensor in a liquid or gas. In gases and liquids the stress tensor has a symmetric diagonal form and depends on the pressure: $\sigma_{ij} = -\delta_{ij}P$. Then $A_{ij} = \beta\sigma_{ij}$, where $\beta = 1/\rho c^2$ is the compressibility of the continuous medium. Consequently, the distortion tensor in gases and liquids has a symmetric form: $A_{ij} = -\beta\delta_{ij}P$.

Note also that here the density ρ is a constant. In this model ρ is the constant equilibrium density of the continuous medium, and the dependence of the density in the sound wave on the coordinates, according to the construction, is given by the field A_{ij} as an independent variable describing deformation of continuous medium. The expression $A_{ij} = -\beta\delta_{ij}P$ means that the distortion tensor is diagonal A_{ij} and its diagonal elements $A_{ij} = -\delta_{ij}A$ are proportional to the pressure: $\rho c^2 A = P$ or $A = \beta P$. Then the value determines A the deviation of the density from the equilibrium value: $\rho' = \rho A$. As is known, the deviation of pressure and density in the sound wave are related by the ratio $P = c^2\rho'$ [14].

Substitute $A_{ij} = -\beta\delta_{ij}P$ in (15), then get:

$$\frac{\rho c^2}{\gamma}P = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})P. \quad (20)$$

Equation (20) has an exact wave solution for pressure: $P = P_0 \exp(i\vec{k}\vec{x} - i\omega t)$.

Equation (20) has never been taken into account before when describing a continuous elastic medium, since the equations of state for the distortion tensor (5, 6), as an independent variable [6], describing the physical state, were not taken into account before.

Thus, for the first time an exact wave solution for sound pressure in a continuous elastic medium with density ρ (20) is obtained from the equation of state (6,13,15) without any approximations. These solutions are accurate and do not contain the dependence of the pressure amplitude on the frequency, as solutions obtained from the Euler equations of hydrodynamics, see pp. 351-354 [14].

On the other hand, the spectrum of wave solutions of equation (20) contains a minimal frequency below which there are no sound waves. At first glance, this is an unexpected result. To make sure that the wave solutions (20) describe sound waves, we calculate the energy density of the sound waves for the solutions obtained from equation (20).

According to Lagrangian (9), the energy density of sound waves consists of the sum of the mechanical energy density given by the first two terms of the Lagrangian and the energy density of the distortion tensor tensions.

The energy density of mechanical waves is known and equal, see $E = \beta P^2$ [14].

The energy density of the strains of the distortion tensor has the form:

$E = \frac{\gamma}{2}(c^{-2}\varepsilon_{ij}\varepsilon_{ij} + \rho_{ij}\rho_{ij})$ and is equal to: $E = \beta P^2 \frac{\omega^2}{\omega_0^2}$.

Indeed, the centrally symmetric tension ε_{ij} (7) in a continuous medium has the form $\varepsilon_{ij} = -\frac{\partial \upsilon_i}{\partial x_j} - \beta\delta_{ij}\frac{\partial P}{\partial t}$. Substituting in this expression the wave solution for longitudinal waves, and considering that the amplitude of pressure and velocity in a sound wave are related as $P = \rho c \upsilon$ [14], which follows from the continuity equation (10), we see that in the sound wave centrally symmetric tension (7) identically equal to zero: $\varepsilon_{ij} = 0$. Substituting wave solutions into the expression: $E = \frac{\gamma}{2}\rho_{ij}\rho_{ij}$, and given that $\gamma = \rho c^4 / \omega_0^2$, we obtain an expression $E = \beta P^2 \frac{\omega^2}{\omega_0^2}$ for the energy density of the distortion tensor in a continuous elastic medium.

Thus, the energy density of the sound wave is equal to the sum of the mechanical energy and the energy density of the distortion tensor tensions:

$$E = \beta P^2 + \beta P^2 \frac{\omega^2}{\omega_0^2} . \tag{21}$$

The fact that the energy density of the sound wave (21) contains the energy density of mechanical vibrations of a continuous medium is the expected result. The expected result is also the dependence of the energy density of the distortion tensor on the square of the frequency, since it is known that the energy density of electromagnetic waves, for example, depends on the frequency squared. This follows from the fact that the Lagrangian (9) is a quadratic invariant function of the tensions of the distortion tensor (7, 8).

A nontrivial result is the dependence of the energy density of the tensions of the distortion tensor: ε_{ij}, ρ_{ij}, in a continuous medium on pressure. This is equivalent to the dependence of the energy density of electromagnetic waves on the current density, which is observed only in the superconducting state when the gradient symmetry of the equations of state is broken.

The dependence of the energy density of the distortion tensor on the pressure in a continuous elastic medium makes it possible to compare the mechanical energy density and the energy density of the tensions of the distortion tensor (21). Two conclusions follow from expression (21).

Firstly, the minimal frequency in the expression for energy density (21) cannot be zero. Indeed, in a continuous medium with density ρ, the minimal frequency (17) is always more than zero.

Secondly, in the frequency range above the minimal frequency, for example by an order of magnitude, the energy density of mechanical vibrations in the sound wave can be neglected (21). This fact is reflected in the diagram equal volume Fletcher-Manson (F-M) [18], according to which the sound volume is measured in decibels.

Indeed, when the frequency of sound waves increases, for example, by two orders of magnitude from 20 Hz to 2 kHz, the energy density of sound waves increases by four orders of magnitude, and the energy density of mechanical vibrations $E = \beta P^2$ - doesn't change. This explains the increase in sound volume by four orders of magnitude, or 40 dB for the same sound pressure when switching from a frequency of 20 Hz to 2 kHz, according to the F-M diagram. Whence it follows that the wave energy density of the distortion tensor (21) corresponds to the volume of sound in the F-M diagram. This removes the existing contradiction between the description of sound by mechanical waves and psychoacoustics, which is reflected in the F-M diagram. A more detailed study of the F-M diagram is beyond the scope of this paper and is given in the paper [19].

Thus, the energy density of the sound wave is equal to the sum of the energy density of the mechanical wave and the energy density of the distortion tensor (21). The minimal frequency (17) in the spectrum of the sound wave $\omega = c\sqrt{\vec{q}^2 + c^{-2}\omega_0^2}$ indicates that the sound is not mechanical vibrations of a continuous medium. Since the spectrum of mechanical vibrations has the form: $\omega = cq$. Mechanical vibrations are possible with any low frequency, and sound waves occur only when the frequency of the sound source is higher than the minimal frequency (14, 15, 20).

The difference between sound waves and mechanical vibrations in a continuous medium is the same as the difference between electromagnetic waves and charge density waves in a superconductor. The sound waves (distortion tensor waves) and the mechanical waves in a continuous medium, as the electromagnetic waves and the charge density waves in a superconductor, propagate together at the same speed. Sound and mechanical waves can't be separated, just as it is impossible to separate momentum from velocity and the stress tensor from the distortion tensor in a continuous elastic medium.

Note that this description of the sound is free from the disadvantages of describing the sound using Euler's hydrodynamic equations [14].

Firstly, the equation of state (14, 15, 20) contain quadratic d'Alembert operator, unlike the Euler equations. Therefore, the expressions for the amplitude of the sound pressure and the velocity of the continuous medium in the sound wave do not depend on the frequency.

Secondly, in order to obtain an accurate wave solution, it is not necessary to neglect the nonlinear terms in the equation of motion of a continuous medium [20]:

$$\rho \frac{\partial \upsilon_j}{\partial t} = -\frac{\partial P}{\partial x_j} - \rho \upsilon_i \frac{\partial \upsilon_i}{\partial x_j} - \frac{\upsilon_j}{c^2}\frac{\partial P}{\partial t} + \frac{\upsilon_i \upsilon_i}{c^2}\frac{\partial P}{\partial x_j} - \frac{\upsilon_j \upsilon_i}{c^2}\frac{\partial P}{\partial x_i}, \tag{22}$$

since equation (22) for sound wave solutions is identical.

Indeed, the vortex force $f_j^V = \frac{\upsilon_i \upsilon_i}{c^2}\frac{\partial P}{\partial x_j} - \frac{\upsilon_j \upsilon_i}{c^2}\frac{\partial P}{\partial x_i}$ in (22) is identically zero for the longitudinal sound wave.

The centrally symmetric force $f_j^C = -\rho \upsilon_i \frac{\partial \upsilon_i}{\partial x_j} - \frac{\upsilon_j}{c^2}\frac{\partial P}{\partial t}$ is also annulled, since, as shown above, the centrally symmetric tension: $\varepsilon_{ij} = -\frac{\partial \upsilon_i}{\partial x_j} - \beta \delta_{ij}\frac{\partial P}{\partial t}$, is zero.

The continuity equation (10) for the gas or liquid is as follows: $\rho \frac{\partial \upsilon_j}{\partial t} = -\frac{\partial P}{\partial x_j}$. It is performed identically, since in the sound wave the pressure and velocity amplitudes are related by the ratio: $P = \rho c \upsilon$ [14].

Thus, the equations of motion of a continuous elastic medium (22) [20] are fulfilled identically for a sound wave. The same can't be said about Euler's hydrodynamic equations, since in Euler's equations the potential force $-\rho \upsilon_i \, \partial \upsilon_i / \partial x_j$ isn't zero.

The nonlinear terms are not annulled in the Euler equations because there is no force component $-c^{-2}\upsilon_j \, \partial P/\partial t$ (22) in the Euler equations. This force is related to the gradient invariance (18) of the centrally symmetric tension (7) in the highly symmetric state. Being in a low-symmetric state of a continuous elastic medium, where there is no gradient symmetry, it is impossible to justify a centrally symmetric force or tension (7), and, consequently, it is impossible to construct correct equations of motion.

A critique of the derivation of Euler's hydrodynamics equations is given in [21]. Obviously, it is impossible to neglect the potential force $-\rho \upsilon_i \, \partial \upsilon_i / \partial x_j$ in the Euler's equation, which is associated with a change in the kinetic energy of a continuous medium, which is clearly seen, for example, from Bernoulli's equation: $\rho \upsilon^2 / 2 + P = const$.

Note that although in a sound wave the vortex force in (22) is zero, the vortex tension ρ_{ij} is not zero. It is the vortex tension that gives the main contribution to the energy density of the sound wave $E = \frac{\gamma}{2}\rho_{ij}\rho_{ij}$ (21), which is reflected in the F-M diagram.

However, the description of the sound is not the aim of this paper. A more detailed description of the sound and the F-M diagram is given in [19]. The aim of this paper is to describe the gap of minimal interaction induced by the translation subgroup [6, 9].

6. The gap as an indicator of the phase transition to a low-symmetric state.

As it is known, the gap is observed in the low-symmetric state of the compensating interaction field. For the first time, the phase transition to the low-symmetric state of the compensating interaction field was described by Higgs in 1964 [22]. He associated this phase transition with the break of the gauge symmetry of the minimal interaction when the compensating field becomes observable. We show that the gap is a consequence of such a phase transition and that it occurs when the gradient symmetry of the equations of state for the distortion tensor is broken.

The gap or minimal frequency characterizes the low-symmetric state of the compensating interaction field. The appearance of a gap of a strong fundamental interaction is associated with the appearance of a mass. Everyone knows the phrase Higgs Boson, which has already become household name. According to the expression (12) for the minimal frequency of the "massive" wave of the distortion tensor: $\omega_0 = c^2\sqrt{\rho/\gamma}$, this is indeed the case in a continuous elastic medium. Since the minimal frequency or gap exists only in the presence of a non-zero density ρ of the continuous medium. However, in this paper we will not study this very interesting question of the appearance of the mass of a continuous medium in the low-symmetric state of the compensating field of the distortion tensor.

The fact is that the minimal frequency of the compensating field in the low-symmetric state carries very important information. It is an indicator of the phase transition. In our opinion, this is the main function of the gap. We will focus on this function of the gap or the minimal frequency of the waves of the compensating field in the low-symmetric state in more detail.

It is possible that this is not the case for the Higgs phase transition, since during the Higgs phase transition [22] the minimal frequency for compensating Yang-Mills fields in the low-symmetric state was not obtained, due to their self-action. Therefore, the gap of strong fundamental interaction is still being sought, according to the CMI Millennium problem.

In our opinion, it is necessary to separate two concepts: the Yang-Mills field and the gap of the strong fundamental interaction. It is also necessary to determine what is meant by the Higgs phase transition. A specific phase transition described in [22], or a phase transition associated with the break of the gauge symmetry of the interaction field when unobservable interaction fields become observable.

In the future, the Higgs transition will be understood as the phase transition in which the minimal interaction disappears, the gauge or gradient symmetry is broken and the compensating field becomes observable.

At the moment, only one phase transition with a gap in the low-symmetric state has been described. This is the phase transition to a superconducting state with the Meissner effect. In the monograph [11] such a superconducting phase transition is called the Higgs transition. Note that during this phase transition, the compensating fields become not just observable, but proportional to the conjugate observable fields. This is due to the breach of the gauge symmetry of the minimal interaction. For superconductivity, this is due to a break of the gauge symmetry of the Ginzburg-Landau potential [23] or the gradient symmetry of the Maxwell equations.

Indeed, in the London equation, the electromagnetic potential is proportional to the current density: $A_i = -\delta'^2 j_i$. As it is shown above, a similar situation occurs when in a continuous medium the distortion tensor becomes proportional to the conjugate stress tensor: (16). In essence, expression $\sigma_{ij} = \rho c^2 A_{ij}$ (16) is a generalization of Hooke's law for the distortion tensor.

In this connection, the question arises, which interaction describes the quantum momentum as a charge and the distortion tensor as a compensating field of minimal interaction.

In our opinion this is a strong fundamental interaction, because above we managed to describe a gap in a continuous elastic medium with nonzero density ρ (17). After all, in addition to the gap associated with sound (14, 15, 20), and the superconducting gap [11], there are no other gaps due to the symmetry of space-time.

In fact, what to call the minimal interaction induced by the subgroup of spatial translations (1-4) is not important. This interaction can be called the phonon interaction in accordance with the electron-phonon interaction introduced in the BCS theory [10], since the waves of the distortion tensor describe sound in a continuous medium [19]. This interaction can be called quantum interaction, since the quantum momentum is the charge of this interaction [9]. This interaction can be called strong, since at short distances a quantum momentum, as a charge, sets a very strong interaction.

From the practical point of view, the most important is not the name of this interaction and not the description of the gap (17) in the low-symmetric state of the distortion tensor, but the interaction itself and the equations of state (5,6) for the tensions of the distortion tensor (7,8).

After all, the minimal interaction (1, 3) induced by the translation subgroup with the tensor compensating field has not been previously studied in the field theory [2,4]. Since only vector compensating fields have been studied in gauge field theory so far. This is clearly stated in the introduction to the monograph [2], see [6].

After all, the minimal interaction (1, 3) induced by a translation subgroup with a tensor compensating field has not been previously studied in field theory [2,4]. Since only vector compensating fields have been studied in gauge field theory so far. This is clearly stated in the introduction in the monograph [2]: "numerous attempts to link compensating fields with the symmetry of space-time itself have never been successful", see [6].

But if there is the low-symmetric state, then there is also the "normal" high-symmetric state of the distortion tensor. There is the complete analogy with the "normal" state of the electromagnetic field and the superconducting state with the Meissner effect.

In the low-symmetric state twelve equations of state (5, 6) or (12, 13) is degenerate into the same «massive» wave equation (14, 15). In this case, the minimal interaction (1-4) disappears and the momentum becomes proportional to the velocity: $p_i = \rho \upsilon_i$, and the distortion tensor becomes proportional to the stress tensor $\sigma_{ij} = \rho c^2 A_{ij}$ (16), and elasticity appears.

To investigate the high-symmetric state of the distortion tensor, it is necessary to destroy the low-symmetric state of the continuous medium. Then the momentum will "separate" from the velocity and become the quantum momentum, and the distortion tensor will "separate" from the stress tensor and plastic deformations with dislocations will occur. In this case, elasticity disappears and plastic deformations occur, which are described by the distortion tensor [7].

The following paper will be devoted to the description of the phase transition of the destruction of a continuous elastic medium as a low-symmetric state of the distortion tensor. But it is already clear that being in the low-symmetric state – in the continuous elastic medium, it is impossible to understand how the tensions (7, 8) will behave in a high-symmetric state. Because there is no minimal interaction in a continuous elastic medium, since there is no gradient symmetry in a continuous medium.

Moreover, the vortex tension of the distortion tensor ρ_{ij} (8) is pushed out of the continuous medium in the same way as the magnetic field is pushed out of the superconductor.

Indeed, the vortex tension ρ_{ij} in a solid state is the linear defect (8) [7]. The elastic continuous medium does not allow dislocation density (8) to pass into itself, since there is no emptiness in the continuous medium. There are no dislocations in a continuous medium. As it is known, dislocations lead to cracks or linear defects, as a result of which the continuous medium is destroyed.

This situation is well known in the science of the resistance of materials. However, the destruction of materials was not previously described as a phase transition. In the next paper, it will be shown that when the destruction of solid-state as a continuous elastic medium, linear defects are formed in the form of dislocations and cracks, and when the destruction of gas as a continuous elastic medium, an explosion occurs and a high-temperature plasma is formed.

All these phenomena are associated with the manifestation of force tensions ε_{ij}, ρ_{ij} (7,8) the distortion tensor, which are similar to the electric and magnetic fields in electrodynamics [6]. In a continuous elastic medium, these tensions are either absent, for example, $\varepsilon_{ij} = 0$ in a sound wave, or limited ρ_{ij} and manifest themselves in the form of sound. According to (15), the vortex tension ρ_{ij} penetrates into a continuous elastic medium only to the certain penetration depth, just like the magnetic field in the superconductor.

Therefore, the study of the force tensions of the distortion tensor ε_{ij}, ρ_{ij} in the "normal" high-symmetric state is the main task of this theory. Note that the centrally-symmetric tension of the distortion tensor ε_{ij} (7) has never been studied before, and the vortex tension ρ_{ij} (8) has been studied, but only in the solid state [7, 15, 16]. In the solid state the Peach-Kohler force is associated with the vortex tension ρ_{ij} [24], which has the form: $f_j = e_{jnm}\rho_{in}\sigma_{im}$ [15, 16].

The expression "continuous distribution of dislocations" is taken in quotation marks due to the fact that there is no continuous distribution of dislocations since dislocation is a discrete concept. This topic was discussed in detail in the methodological article [6]. Of course ρ_{ij} this is the force characteristic of the minimal interaction (1) - vortex tension (8), similar to the magnetic field in electrodynamics. For example, in [6, 20] it was shown that in a continuous medium the force $f_j = e_{jmn}p_i v_m \rho_{in}$ is proportional to the pressure gradient $f_j = e_{jmn}e_{nki}\upsilon_i v_m c^{-2}\partial P/\partial x_k$.

In the presence of minimal interaction (1), the vortex tension flux ρ_{ij} is quantized and sets the Burgers vector [6], just as the magnetic field flux in the Abrikosov vortices is quantized [8] for the Ginzburg-Landau interaction [23]. But this does not mean that the vortex tension of the compensating interaction field is always quantized, which is well known by the example of the magnetic field.

7. Conclusion

Thus, sound waves in the continuous medium (14, 15, 20) are not associated with the electromagnetic interaction, as previously thought [10], but are associated with the minimal interaction (1, 3) induced by the translation subgroup (2, 4). In this paper, sound waves were obtained as "massive" waves of the distortion tensor and pressure in a continuous elastic medium (14, 20).

In this case, the wave spectrum of the distortion tensor in a continuous medium (14, 15, 20) has a minimal frequency $\omega_0 = c^2\sqrt{\rho/\gamma}$ (17). This means that the distortion tensor, as an interaction field, has the gap in the low-symmetric state.

In addition, the quantum momentum, as a charge [9], describes a very strong interaction at short distances Indeed, the smaller the particle size, the greater of the quantum momentum and the stronger the interaction.

In [9] it was proved that the attraction of oppositely directed quantum momenta leads to the formation of Cooper pairs in the superconducting state. It is known that the shorter the coherence length of paired electrons is the higher the phase transition temperature in HTSC [25]. Therefore, there is a reason to believe that this gap corresponds to the gap of strong fundamental interaction, which is consistent with to the Millennium problem announced by CMI in 2000.

It does not matter how to call the interaction – the strong interaction, as it is formulated in the CMI problem, the phonon interaction, according to the theory of BCS [10], or the quantum interaction, according to the charge - quantum momentum [9]. It is important that the gap is an indicator of the phase transition to the low-symmetric state. In this paper, it is shown that the continuous medium is the low-symmetric state of the distortion tensor. In the continuous medium, the distortion tensor is elastic and describes the sound.

But if there is a low-symmetric state, then there must be a high-symmetric state. The following work is devoted to the description of the phase transition of destruction of a continuous elastic medium as a low-symmetric state of the distortion tensor.

There are only four fundamental interactions in nature. Two of them have a gap: the electromagnetic interaction and the strong interaction. There are also two Abelian models associated with translational space-time symmetry. The first model is related to the Lagrangian invariance with respect to time translations and leads to electromagnetic interaction [13]. The second model is related to the Lagrangian invariance with respect to spatial translations and leads to a minimal interaction (1-4) induced by the quantum momentum as the charge and the tensor compensating interaction field. Most likely, this model describes a strong fundamental interaction.

In the next paper, the tensions of the distortion tensor: ε_{ij}, ρ_{ij} (7,8), similar to the electric and magnetic tensions of the electromagnetic field will be investigated. It will be shown that the critical vortex stress ρ_{ij} (8) destroys the continuous elastic medium, and the centrally symmetric stress ε_{ij} (7) transforms gas into high-temperature plasma.

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