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A Comparison of Geometric Algebra and Harmonic Domain for Linear Circuit Analysis

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Abstract

Electric circuit analysis under non-sinusoidal, non-linear situations has been a hot topic for a long time. Many scientific communities hold opposing viewpoints on the additional analysis tool and domain, resulting in a variety of standards and definitions. With the advent of Power Electronic equipment, converters, and Renewable Energy sources, the electric power system has become increasingly sophisticated since its inception. Electronic equipment has transformed the electrical system and provided a slew of advantages to industrial applications. Unfortunately, this comes at the expense of power system distortion (voltage and current). Understanding power flow in non-sinusoidal linear and non-linear circuit circumstances is required for this. As a result, a novel mathematical framework to analyze the circuit in such an environment is always required. Finally, in sinusoidal conditions, a consensus can be reached on norms that comply with well-known, established standards. The work provided here compares the use of harmonic domain and geometric algebra in circuits with disturbances for sinusoidal and non-sinusoidal excitations in order to demonstrate the accuracy of geometrical algebra in power flow calculations.

KEYWORDS:
Geometric algebra, Harmonic Domain, Multivector, power flow

1 | INTRODUCTION

Most of the work done till date by researchers has resulted in the development of power theories for sinusoidal and non-sinusoidal circuits. Sinusoidal power theories are well accepted, but there is still a conflict or rather disagreement for non-sinusoidal, non-linear circuits.The concept of apparent power and active power for these circuits has been a broad research area throughout the century since the Steinmetz experiment on electric arc.[1]This is because non-linear loads exhibit distortion in voltage and current waveform and the power calculation deviates from those of sinusoidal circuit calculation. Traditionally complex numbers have been extensively used for demonstrating the mathematics in electrical circuits. But recently in some research works[2,3,4] it is demonstrated that geometric algebra can be used instead of complex number in the electric circuits to study the power flow, mainly in circuits with linear loads. The study presented here uses geometric algebra as a mathematical tool for circuit analysis and compares the results with another circuit analysis approach i.e. harmonic domain[5] , circuit examples with results have been presented for proving the authenticity of geometric algebra.

Linear circuit analysis is an essential subject for any electrical engineer. Almost every engineer is well-versed in complex algebra in linear circuit analysis and can interpret its result quite easily. However, using geometric algebra instead of complex algebra gives a thorough insight of power flow phenomenon in these circuits.In this paper,an attempt is made to show the power

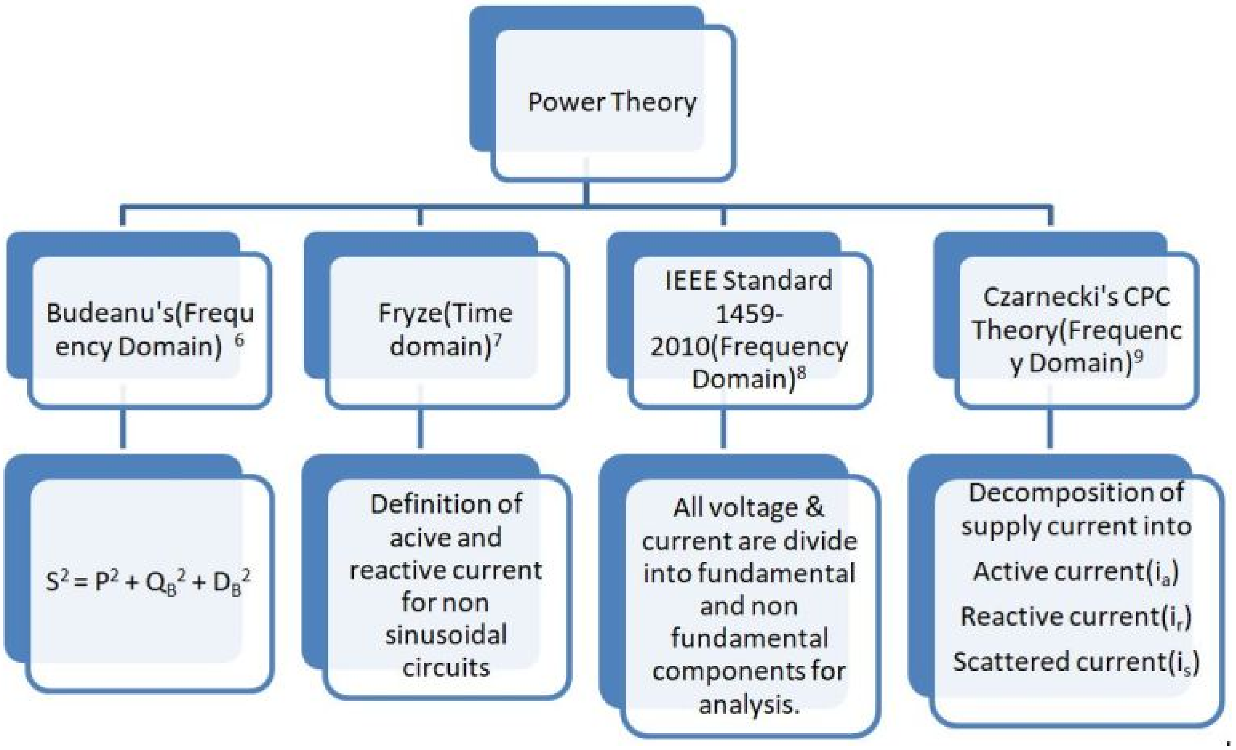

FIGURE 1 Power Theory Chronology

flow and side by side validating it with another established approach for a consensus of agreement.Figure1 gives a chronological idea about the development of power theories[6,7,8] and standards,[9] but it does not say that the work is limited to this picture only.in fact there are notable contribution in reactive power by Akagi,[10] Peng[11] and Dai[12] which constitute further work in power theories.

In this paper, we examine the power quality parameters of linear circuits using both Geometrical Algebra and Harmonic domains, with the goal of demonstrating the benefit of Geometrical Algebra for estimating power quality parameters.

2 | GEOMETRICAL ALGEBRA AND HARMONIC DOMAIN BASICS

In this section our main focus is to discuss the circuit analysis by the above mentioned techniques and therefore only the main aspects related to electrical circuits are concerned,[13] for further detail on Geometrical Algebra[3] and harmonic domain it can be referred to[5]

2.1 | Geometrical algebra

Geometric algebra introduction came in the year 1878, but it was forgotten for a long time. In 1966 David Hestenes again revived the geometrical interpretation, and since then it is considered a powerful mathematical tool for analysis in various engineering domains. The geometric algebra of a vector space is algebra over a field, and is known for its multiplication operator known as the geometric product on a space of elements called multi-vector. Geometric algebra can be defined as the Clifford algebra of a vector space. Clifford described a new product known as a geometric product which is conservative. He combined the work of Hamilton and Grassmann into a single structure and coined it as Clifford Algebra (CA). Scalars and vector representation have the usual interpretation, detailed theory on algebraic operator and its properties can be referred in.[14,15] The bi-vectors provide a picture of pseudo vector quantities, e.g. area, angular rotation, torque etc. By Geometric representation of two vectors it is possible to represent vectors in terms of

- Magnitude.

- Direction.
- Sense.
- Grade.

for a two dimensional geometry with basis e_1 and e_2 the representation is

$$G_2 = 1, e_1, e_2, e_{12} \tag{1}$$

where 1 is a scalar, e_1 and e_2 are vectors and e_{12} represent a pseudo scalar. e_i(here i=1to 2) is the basis vector which generates the multi-vector in G_2 .In general for n-vectors, the geometric sub space A is given as

$$A = \sum_{k=0}^{n} A_k = A_0 + A_1 + ... + A_k + ... + A_n \tag{2}$$

It has been demonstrated through[3,16] that the use of the complex number as a fundamental tool in the frequency domain can be replaced by geometric algebra (GA), as complex numbers are subspace of GA. Also, they do not impart any physical significance to the calculation, it is a mere way of representation of magnitude and direction, however with GA representation we get information about the sense or shape of interacting vector, i.e. a geometric shape. The first attempt of implementing GA in electrical circuit was done by Menti,[2] however Castro Nunez[3] extended this application and compared with previous power theories to point out the flaws in the calculation of power parameters. Montoya et al[4,16] work is the more recent work in this series and he has modified the work of Nunez and has given a modified geometric power definition based on current decomposition and also a means to include sub-harmonics, in-fact their continuous work[16] in this field have opened new avenues of research in electrical circuit analysis by employing geometric algebra. Below equation shows the transformation from the time domain to the geometric domain for circuit analysis adopted from.[3]

$$\begin{aligned}
&x_c 1(t) = X_{rms} cos(\omega t) \mapsto e_1 \\
&x_s 1(t) = X_{rms} sin(\omega t) \mapsto -e_2 \\
&. \\
&. \\
&x_c n(t) = X_{rms} cos(\omega t) \mapsto \wedge_{i=2}^{n+1} e_i \\
&x_s n(t) = X_{rms} sin(\omega t) \mapsto \wedge_{i=1, i\neq 2}^{n+1} e_i
\end{aligned} \tag{3}$$

where n refers to the highest multiple of fundamental frequency in the excitation signal.

The transformation shown above in equation(3) is used for converting time domain signal into geometric domain for analysis and adapt to Fourier series expansion. The power quality parameters of circuit differ from their meaning in complex notation as by using GA they give multivectorial representation, for example the total power calculated by GA is known as net apparent power[17] which is different from apparent power obtained by complex algebra calculation. For calculation of power quality indices for any linear circuit by GA, the relations are adopted from[4] after modification from initial proposal of[3] and are given in Table 1, for convenience of reader.

2.1.1 | Geometric product

The geometric product between two vector entities a and b is defined as

$$M = ab = a \cdot b + a \wedge b \tag{4}$$

basically the wedge product is given as

$$a \wedge b = \| a\| \, \| b\| \sin \Phi e_{1,2} \tag{5}$$

the property revealed by equation(5) is missing in linear algebra and the result of this equation is neither a vector or scalar but a new term known as bi-vector. for power calculation in geometrical algebra these bi-vector has a significant role. also this product is different from the traditional cross product.In Figure 2, (a) the scalar or dot product has the usual meaning while (b) the wedge product gives the algebraic constructions used in geometry to study areas, volumes and higher-order k- vectors. The wedge product is anti-commutative[15] i.e,

$$ab = -ba \tag{6}$$

TABLE 1 Geometric circuit equation

Geometrical circuit equation	Interpretation
$M = VI = P + CN_{r(ps)} + CN_{r(hi)} + CN_d$	Net multivector power
P is Scalar part of M or active power	
$CNr(ps) + CNr(hi) = Q$	Reactive power of power source and harmonic interaction
$\Vert M \Vert = \sqrt{\langle \bar{M} M \rangle_0}$	Norm or Absolute value of a multivector or rms value
$Zh = R + (1/hwC - hwL)e_{12}$	Impedance evaluated at harmonic level h
I= Y·V	I= current, Y= admittance, V=voltage
Y= G + j B	G=conductance, B=susceptance
$i_g = \sum_{i=1}^{n} V \cdot G_i$	i_g= current component due to conductance
$i_b = \sum_{i=1}^{n} V \cdot B_i$	i_b = current component due to conductance

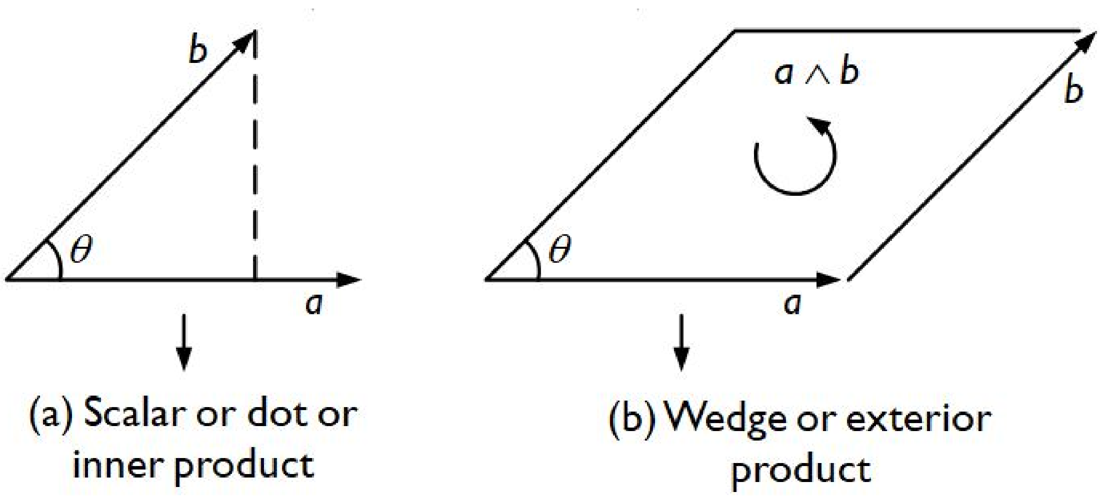

FIGURE 2 geometric product

Also the terms i_g and i_b in Table.1 are correspondingly the parallel and the quadrature component of current, the quadrature component is due to reactance and parallel due to resistance of circuit.

2.2 | Harmonic Domain

Analysis of circuit by harmonic domain(frequency domain) is a widely accepted technique and is also used by budenau.[6] It uses Fourier series expansion for representation of signal in a specific vector arrangement as shown below

$$X = \begin{Bmatrix} X\_h \\ . \\ . \\ X\_1 \\ X_0 \\ X_1 \\ . \\ . \\ X_h \end{Bmatrix} \tag{7}$$

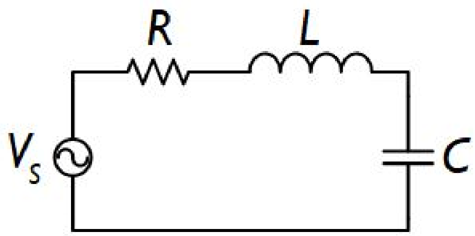

FIGURE 3 Series RLC circuit

Where X represents HD signal, ‘h’ represents highest harmonic number and ‘0’ represents DC component.Euler’s identity converts the time-domain or excitation signal into an HD arrangement.An example is shown for the HD arrangement. e.g.

$$V(t) = \sqrt{2}[100\sin(\omega t) + 25\sin(\omega 3t)] \tag{8}$$

by using Euler’s identity the given signal in equation(8) can be represented as

$$V(t) = 50e^{j\omega t} + 50e^{-j\omega 3t} - 12.5e^{j\omega t} + 12.5e^{-j\omega 3t} \tag{9}$$

Now this can be represented in a HD arrangement equation(10)

$$V = \begin{bmatrix} j12.5 & 0 & 50 & 50 & 0 & -j12.5 \end{bmatrix}^T \tag{10}$$

Further in HD the impedance’s are translated and solved for power quality parameters like rms value, apparent power, pf.for impedance and admittance they are written using complex notation, for solution in harmonic domain mat-lab has been extensively used and the results are shown in the numerical examples.

3 | NUMERICAL EXAMPLES

For the comparative study in the linear circuit different circuits are taken with sinusoidal and non-sinusoidal excitation

3.1 | Sinusoidal Excitation

For the circuit in figure.3 the excitation voltage and circuit parameters are

$$V_s = 50\sqrt{2}\cos\omega t \tag{11}$$

$R = 2\Omega$, L=2H , C=1F and $angular frequency \omega = 1 rad/sec$ Table 2, demonstrates the comparative study for power quality indices by geometric algebra and harmonic domain for circuit in figure 3, the computation of parameters by GA is demonstrated in equation(13) G_2 is used as in the excitation only fundamental frequency is present the voltage signal is converted from time domain to geometric domain by using the transformation given in equation(3) as

$$V_s = 50e_1 \tag{12}$$

the impedance is calculated at fundamental frequency ω =1

TABLE 2 Comparative result for 3.1

Geometric Domain	Harmonic Domain
V_{rms}=50V	V_{rms}=50V
I_{rms}=22.36A	I_{rms}=22.36A
M=1118.033VA	S=1118VA
P=1000W	P=1000W
Q=500VA	Q=500VA
D=0	D=0
pf=0.8944	pf=0.8944

TABLE 3 Comparative result for 3.2

Geometric Domain	Harmonic Domain
V_{rms}=173.2V	V_{rms}=173.17V
I_{rms}=67.01A	I_{rms}=67.006A
M=13.19KVA	S=11.6KVA
P=4.49KW	P=4.48KW
Q=12KVA	Q=3.7KVA
D=2.9KVA	D=10.1KVA
pf=0.34	pf=0.38

$$
\begin{aligned}
&Z1 = R + (\frac{1}{\omega C} - \omega L) \cdot e_{12} = 2 - e_{12} \\
&Y1 = 0.4 + 0.2 \cdot e_{12} \\
&i_g = 20 \cdot e_1 \\
&i_b = -10 \cdot e_2 \\
&i = i_g + i_b = 20 \cdot e_1 - 10 \cdot e_2 \\
&M = sort(ecmul(V_s, i)) = 1000 + 500 \cdot e_{12} \\
&ic = 20 \cdot e_1 - 10 \cdot e_2 \\
&irms = sqrt(evectorpart(ecmul(i, ic), 0)) = 22.3606 \\
&Vrms = sqrt(evectorpart(ecmul(V, Vc), 0)) = 50
\end{aligned}
\tag{13}
$$

in equation(13) M is the net apparent power and its scalar part i.e 1000 is the active power. and from table 2, it is evident that with both geometric algebra and harmonic domain, the results for power quality parameters are same.Also, it can be easily verified by calculating that source power and power in load are equal; hence principle of energy conservation is also observed.

3.2 | Non-Sinusoidal Excitation

for this example the same circuit is taken as in figure 3, but the excitation is non-sinusoidal as

$$V_s = \sqrt{2}[100 \sin \omega t + 100 \sin 3\omega t + 100 \sin \omega t] \tag{14}$$

and R=1 Ω, L=1H , C=0.66 F, and angular frequency ω =1 rad/sec

This circuit has 3 harmonics and table 3, gives our comparative study It can be seen from Table 3. there is no conflict in the calculation of active power. But, there is a difference in the harmonic domain and GA result of reactive power . And generally

[0]**Abbreviations:** ecmul, e-clifford multiplication

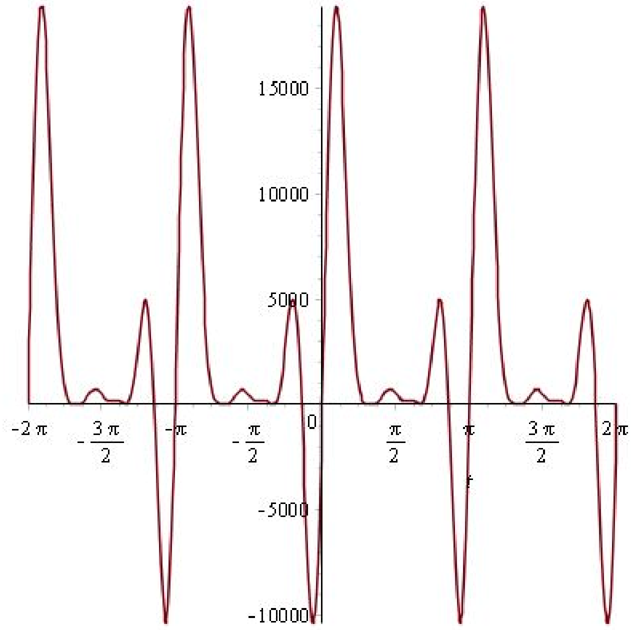

FIGURE 4 Non sinusoidal power by GD for 3.2

in circuit compensation we compensate for this calculated value which can lead to erroneous calculation. figure 4 and 5 gives the non sinusoidal power by both geometrical algebra and harmonic domain.

3.3 | Non sinusoidal Excitation with higher order harmonics

In this example a LTI series system is taken where the excitation voltage and current both are having harmonics. For mathematical simplicity only odd harmonics are considered , since odd harmonics are of greater concern considering the end user equipment safety and system stability where source excitation is

$$V_s = 100 \cdot \sqrt{2}[\sin \omega t + \sin 3\omega t + \sin 5\omega t + \sin 7\omega t + \sin 9\omega t + \sin 11\omega t] \tag{15}$$

and current is given by

$$\begin{aligned} i_s =& 50 \sin \omega t + 50 \sin 3\omega t + 17.12 \sin 5\omega t + 8.47 \sin 7\omega t + 5.05 \sin 9\omega t + 3.35 \sin 11\omega t \\ &+ 50 \cos \omega t - 50 \cos 3\omega t + 37.67 \cos 5\omega t - 27.85 \sin 7\omega t - 21.91 \sin 9\omega t - 18.01 \cos 11\omega t \end{aligned} \tag{16}$$

The above expression of current is obtained by converting back from GA to time domain by utilizing the circuit parameters which are R=1ohm, L=1H and C=2F with angular frequency ω=1rad/sec. The multi vector power in GA for this example is shown as a figure 7 which was executed on GA toolbox. the calculation in equation(13) are also performed using GA toolbox. also the voltage and current signal distortions are given in figure 8.

3.4 | Non Sinusoidal Parallel LC Circuit

In this example a RC parallel circuit is analysed with non sinusoidal supply as in equation(17)

$$V_s = \sqrt{2}[100 \sin \omega t + 100 \sin 3\omega t] \tag{17}$$

and circuit parameter as L=1H and C=0.33F with angular frequency ω=1rad/sec. and the comparative results are shown in table 4.

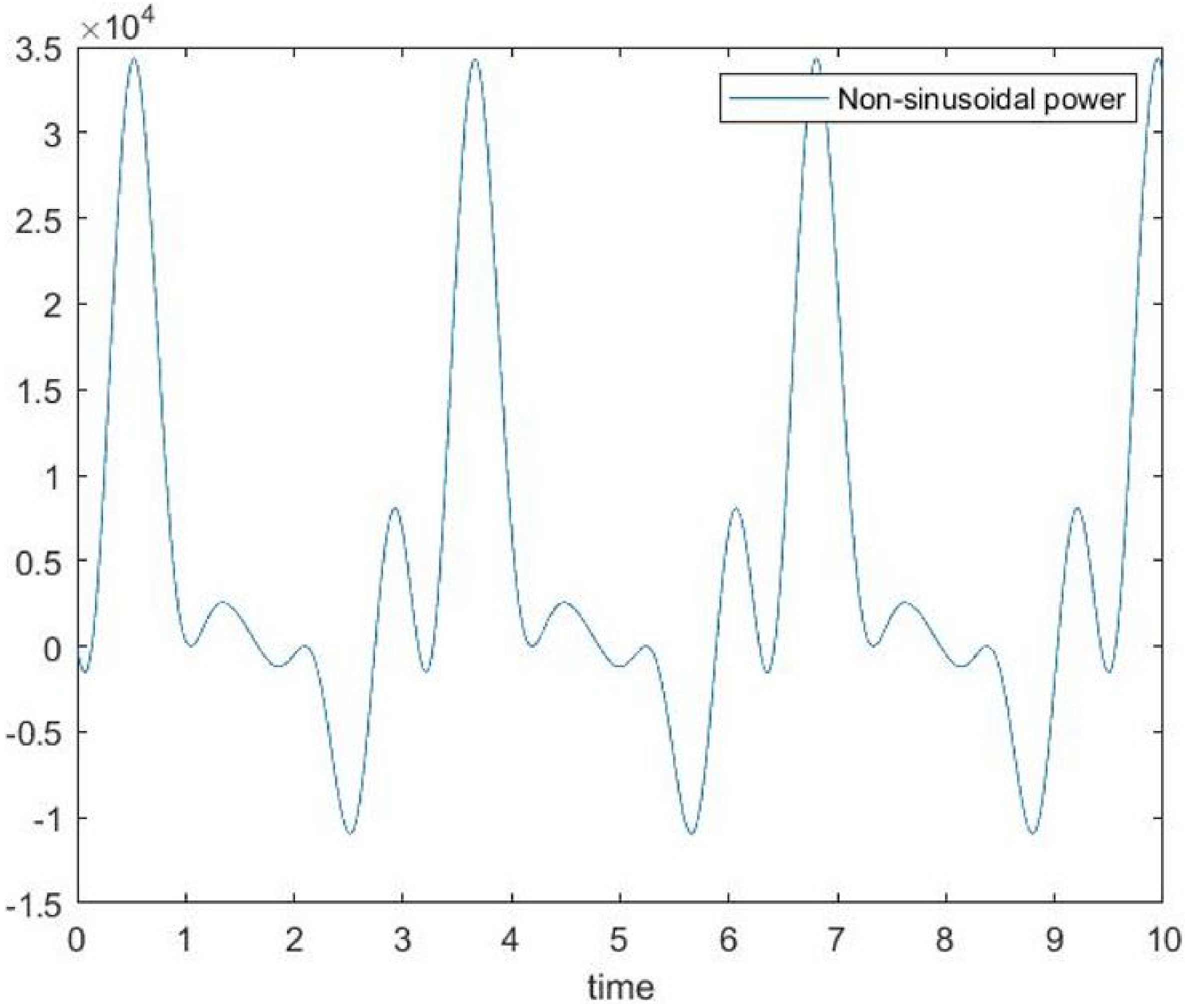

FIGURE 5 Non sinusoidal power by HD for 3.2

TABLE 4 Comparative result for 3.4

Geometric Domain	Harmonic Domain
V_{rms}=141.4V	V_{rms}=141.4V
I_{rms}=94.28A	I_{rms}=94.26A
M=18.85KVA	S=13.32KVA
P=0	P=0KW
Q=18.85KVA	Q=6.67KVA
D=0	D=7.68KVA
pf=0	pf=0

4 | CONCLUSION

In this work, the examples presented in previous theories have been rigorously examined, and some new examples with geometrical algebra have also been presented, and all of these examples are solved in Harmonic domain to demonstrate the circuit

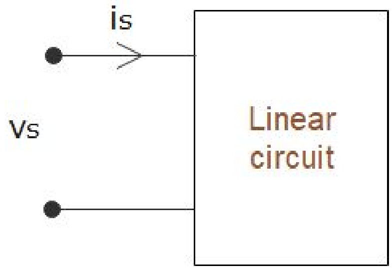

FIGURE 6 Generic series circuit

$$
\begin{aligned}
M := {} & 1034.271393\,\varepsilon + 8629.109395\,e_{1,2} + 10000.00000\,e_{3,4} \\
& - 6712.328767\,e_{5,6} + 2560.079632\,e_{7,8} - 1353.368843\,e_{9,10} \\
& + 841.5424639\,e_{11,12} + 8767.123288\,e_{1,2,5,6} \\
& - 6552.590416\,e_{1,2,7,8} + 4976.478364\,e_{1,2,9,10} \\
& - 3992.788027\,e_{1,2,11,12} + 8767.123288\,e_{3,4,5,6} \\
& - 5847.750865\,e_{5,6,7,8} + 2217.946744\,e_{7,8,9,10} \\
& - 1183.675352\,e_{9,10,11,12} - 3287.671233\,e_{1,2,3,4,5,6} \\
& + 7785.467128\,e_{1,2,5,6,7,8} - 5958.134524\,e_{1,2,7,8,9,10} \\
& + 4587.243919\,e_{1,2,9,10,11,12} + 7785.467128\,e_{3,4,5,6,7,8} \\
& - 5505.617978\,e_{5,6,7,8,9,10} + 2048.253253\,e_{7,8,9,10,11,12} \\
& - 4152.249135\,e_{1,2,3,4,5,6,7,8} \\
& + 7191.011236\,e_{1,2,5,6,7,8,9,10} \\
& - 5568.900079\,e_{1,2,7,8,9,10,11,12} \\
& + 7191.011236\,e_{3,4,5,6,7,8,9,10} \\
& - 5335.924486\,e_{5,6,7,8,9,10,11,12} \\
& - 4494.382022\,e_{1,2,3,4,5,6,7,8,9,10} \\
& + 6801.776791\,e_{1,2,5,6,7,8,9,10,11,12} \\
& + 6801.776791\,e_{3,4,5,6,7,8,9,10,11,12} \\
& - 4664.075514\,e_{1,2,3,4,5,6,7,8,9,10,11,12}
\end{aligned}
$$

FIGURE 7 Multivector power for 3.3

power quality analysis in a comparative and more justified manner, and it is observed that in Geometric domain there is a computational ease of operation. When we move into non-sinusoidal analysis, however, the reactive power calculated by the geometric domain is on the high side, and other power parameters are also different from those calculated by the harmonic domain, implying that there is some ambiguity in terms of capturing the harmonic interaction. Because GA gives the right power output in the sinusoidal example and agrees with the results, We can propose that the geometric algebra framework be used for linear circuits with sinusoidal and non sinusoidal excitation.

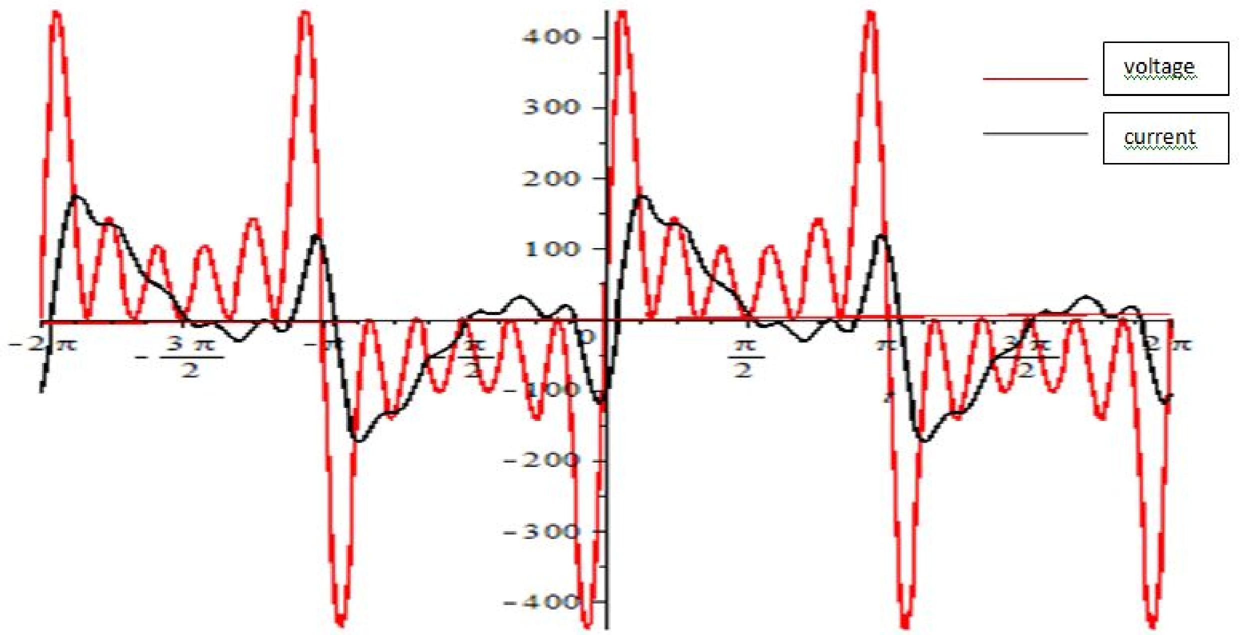

FIGURE 8 Distorted voltage and current for 3.3

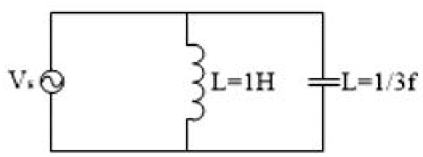

FIGURE 9 Parallel LC circuit

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