A Novel Perspective for Simulations of the MEW Equation By Trigonometric Cubic B-spline Collocation Method Based on Rubin-Graves Type Linearization

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Abstract

In the present study, the Modified Equal Width (MEW) wave equation is going to be solved numerically by presenting a new technique based on collocation finite element method in which trigonometric cubic B-splines are used as approximate functions. In order to support the present study, three test problems; namely, the motion of a single solitary wave, interaction of two solitary waves and the birth of solitons are studied. The newly obtained results are compared with some of the other published numerical solutions available in the literature. The accuracy of the proposed method is discussed by computing the numerical conserved laws as well as the error norms L_2 and L_∞.

Keywords: Finite element method, collocation method, solitary waves, modified equal width equation, trigonometric B-splines.

AMS classification: 65T40,35Q51, 74J35, 65D07.

1 Introduction

In nature, phenomena which are nonlinear in its characteristics have a deterministic role in various fields of science such as waves, fluid mechanics, plasma physics, optics, solid state physics, kinetics and geogology. Especially, in wave studies, all of the phenomena such as dispersion, dissipation, diffusion, reaction and convection become important [1].

The widely used nonlinear modelling for wave phemona is Korteweg de-Vries (KdV) [2] equation of the following form

$$u_t + 6uu_x + u_{xxx} = 0.$$

Then comes regularised long wave (RLW) equation for describing nonlinear dispersive wave phenomena of the form

$$u_t + u_x + uu_x - u_{xxt} = 0 \tag{1}$$

widely accepted as an alternative to KdV equation. The third equation used for modelling those wave phenomena is known as the equal width (EW) equation and presented in the following form [3]

$$u_t + uu_x - u_{xxt} = 0$$

Finally comes the modified equal width (MEW) equation closely in relation with the RLW (1) is given in the following form under the physical boundary conditions $u \to 0$ if $x \to \pm\infty$

$$u_t + \varepsilon u^2 u_x - \mu u_{xxt} = 0. \tag{2}$$

Here t and x denote time and space coordinates, respectively ε and μ are positive parameters and u is related to the vertical displacement of the water surface. In order to obtain the numerical solutions of MEW (2) for $x \in [a, b]$, the following boundary conditions

$$\begin{aligned} u(a,t) &= u(b,t) = 0, \\ u_x(a,t) &= u_x(b,t) = 0, \\ u_{xxt}(a,t) &= u_{xxt}(b,t) = 0, \end{aligned}$$

and the following initial condition,

$$u(x,0) = f(x), \quad a \leq x \leq b,$$

has been considered, where $f(x)$ is a sufficently smooth function. The MEW (2) equation has the following solitary wave solution

$$u(x,t) = A \sec h \left[k \left(x - x_0 - ct\right)\right] \tag{3}$$

where $k = 1/\sqrt{\mu}$ and $c = A^2/2$. In the literature, solitary waves are defined as traveling waves while retaining their shapes and speeds because of delicate balance between nonlinearity and dispersive effects, whereas, a soliton is a very special type of solitary wave, retaining its shape and speed even after colliding with another wave [4]. Although, those solitary waves can have both positive and negative amplitudes, their speed is positive and proportional to the square of their amplitudes. Morover, as with RLW equation, since all of them have the same number of waves $k = 1/\sqrt{\mu}$, they also have the same width [5]. The conservation constants of the MEW equation for the above boundary conditions are found by Olwer [6] as follows

$$I_1 = \int_{-\infty}^{+\infty} u dx, \quad I_2 = \int_{-\infty}^{+\infty} \left(u^2 + \mu (u_x)^2\right) dx, \quad I_3 = \int_{-\infty}^{+\infty} u^4 dx.$$

In the literature, one can encounter several exact and approximate solutions of the MEW equation given with various initial and boundary conditions.Among others, Hamdi *et al*. [7] have obtained exact solitary wave solutions of the generalized equal width wave equation. Wazwaz [1] have studied the MEW equation and two of its variants with the help of tanh and sine-cosine methods. Esen and Kutluay [8] have utilized a linearized numerical scheme based on finite difference method to find out solitary wave solutions of the one-dimensional MEW equation. Raslan [9] solved generalized EW equation numerically by collocation of cubic B-splines finite element method. Jin [10] has suggested an analytical approach based on the homotopy perturbation method for solving the MEW equation. Lu [11] has introduced variational iteration method for finding the solutions of the MEW equation . Esen [12] has obtained numerical solution of the one-dimensional MEW equation with the help of a lumped Galerkin method using quadratic B-spline finite element method. Çelikkaya [13] has used operator splitting method for numerical solution of modified equal width equation. Essa [14] has applied multigrid method for the numerical solution of the modified equal width wave equation. Zaki [5] has taken the solitary wave interactions for the MEW equation into consideration by collocation method based on quintic B-spline finite elements and he [15] also found out the numerical solution of the EW equation by using least-squares method .Karakoç and Zeybek [16] have obtained the numerical solutions of the generalized equal width (GEW) wave equation by using lumped Galerkin approach with the cubic B-spline functions and they [17] have also used quintic B-spline collocation algorithm with two different linearization techniques. Roshan [18] has sought the solutions for the equation by using the Petrov-Galerkin method. Geyikli and Karakoç [19] obtained numerical solutions of the MEW equation by using collocation method with septic B-spline finite elements with three different linearization techniques and they [20] have also utilized subdomain finite element method with quartic B splines. Saka [21] has obtained numerical solutions for time split the MEW equation and space split the MEW equation using quintic B-spline collocation method. Karakoç and Geyikli [22] have obtained a numerical solution of the MEW equation using sextic B-splines. Geyikli and Karakoç [23] have applied Petrov–Galerkin method with the cubic B-splines for solving the MEW equation. Karakoç and Geyikli [24] have obtained numerical solution of the modified equal width wave equation Karakoç [25] has dealth with applying the cubic B-spline function to develop a numerical method for approximating the analytic solution of the MEW equation. Evans and Raslan [26] have obtained solitary waves for the generalized equal width (GEW) equation. Kaplan and Dereli [27] have solved GEW equation by using moving least squares collocation method. Cheng [28] has applied the improved element-free Galerkin method applied to the MEW equation. Başhan *et al* [29] have used finite difference method combined with differential quadrature method with Rubin and Graves linearizing technique for the numerical solution of the MEW equation. In a recently published article, Başhan *et al* [30] have presented a new perspective for the numerical solution of the MEW equation. One can see some recently published articles about physical pheonema such as given in Refs [31]-[33].The presented method has the advantage of using less computer storage capacity and less running computational time. This resulted in accurate results in short simulation time. More recently, Shallal *et al* [34] have obtained exact solutions of the conformable fractional EW and MEW equations by a new generalized expansion method.

In this article, the error norms L_2 and L_∞ are going to be used to compare the differences between exact and approximate solutions. Those widely used error norms L_2 and L_∞ are computed by the following formulae

$$L_2 = \left\|u^{exact} - U_N\right\|_2 \simeq \sqrt{h\sum_{J=0}^{N}\left|u_j^{exact} - (U_N)_j\right|^2},$$
$$L_\infty = \left\|u^{exact} - U_N\right\|_\infty \simeq \max_j \left|u_j^{exact} - (U_N)_j\right|.$$

In the present article, the MEW equation is going to be handled using finite element trigonometric B-spline cubic collocation method. During the solution process, a Rubin-Graves type linearization technique is going to be utilized to overcome the nonlinear term appearing in the equation. Then the newly obtained results are goning to be compared with some of those available in the literature.

2 Implementation of the method for space discretization

The MEW equation is generally given in the following form

$$u_t + \varepsilon u^2 u_x - \mu u_{xxt} = 0, \qquad a \leq x \leq b$$

together with the physical boundary conditions $U \to 0$ as $x \to \pm\infty$, in which t is time , x is the space coordinate and μ is a positive parameter. For the considered problems, the appropriate boundary conditions will be chosen as

$$\begin{aligned} U(a,t) &= 0, & U(b,t) &= 0, \\ U_x(a,t) &= 0, & U_x(b,t) &= 0. \end{aligned}$$

Let us consider the solution interval $[a, b]$ is divided into N finite elements having equal lengths using the nodal points $x_i,\ i = 0(1)N$ in such a way that $a = x_0 < x_1 \cdots < x_N = b$ and $h = (x_{i+1} - x_i)$. The trigonometric cubic B-splines $T_m^3(x)$, $(m = -1(1)N+1)$, at the knots x_m are defined over the interval $[a, b]$ by [35]

$$T_m^3(x) = \frac{1}{\theta}\begin{cases} \rho^3(x_{m-2}) & , \quad x_{m-2} \le x \le x_{m-1} \\ -\rho^2(x_{m-2})\rho(x_m) & \\ -\rho(x_{m-2})\rho(x_{m+1})\rho(x_{m-1}) & , \quad x_{m-1} \le x \le x_m \\ -\rho(x_{m+2})\rho^2(x_{m-1}) & \\ \rho(x_{m-2})\rho^2(x_{m+1}) & \\ +\rho(x_{m+2})\rho(x_{m-1})\rho(x_{m+1}) & , \quad x_m \le x \le x_{m+1} \\ +\rho^2(x_{m+2})\rho(x_m) & \\ -\rho^3(x_{m+2}) & , \quad x_{m+1} \le x \le x_{m+2} \\ 0 & , \quad \text{otherwise} \end{cases}$$

in which

$$\rho(x_m) = \sin\left(\frac{x - x_m}{2}\right), \ \theta = \sin\left(\frac{h}{2}\right)\sin(h)\sin\left(\frac{3h}{2}\right), \ m = 0(1)N.$$

The set of trigonometric cubic B-splines $\left\{T_{-1}^3(x), T_0^3(x), \ldots, T_{N+1}^3(x)\right\}$ forms a basis for the smooth functions defined over [a,b]. Therefore, an approximation solution $U_N(x,t)$ can be written in terms of the trigonometric cubic B- splines as trial functions:

$$U(x,t) \approx U_N(x,t) = \sum_{i=m-1}^{m+2} T_i^3(x)\delta_i(t) \tag{4}$$

where $\delta_i(t)$'s are unknown, time dependent quantities to be determined from the boundary and trigonometric cubic B-spline collocation conditions. Each trigonometric cubic B-spline covers four elements so that each element $[x_i, x_{i+1}]$ is covered by four trigonometric cubic B-splines. For this problem, the finite elements are identified with the interval $[x_i, x_{i+1}]$. Using the nodal values $U_i, U_i^{'}$ and $U_i^{''}$ are given in terms of the parameter δ_i by:

$$\begin{aligned} U_i &= \alpha_1\delta_{i-1} + \alpha_2\delta_i + \alpha_1\delta_{i+1} \\ U_i^{'} &= \beta_1\delta_{i-1} + \beta_1\delta_{i+1} \\ U_i^{''} &= \gamma_1\delta_{i-1} + \gamma_2\delta_i + \gamma_1\delta_{i+1} \end{aligned}$$

where

$$\begin{aligned} \alpha_1 &= \sin^2\left(\frac{h}{2}\right)\csc(h)\csc\left(\frac{3h}{2}\right), & \alpha_2 &= \frac{2}{(1+2\cos(h))}, \\ \beta_1 &= -\frac{3\csc\left(\frac{3h}{2}\right)}{4}, & \beta_2 &= \frac{3\csc\left(\frac{3h}{2}\right)}{4} \\ \gamma_1 &= \frac{3((1+3\cos(h))\csc^2(\frac{h}{2}))}{16\left(2\cos(\frac{h}{2}) + \cos\left(\frac{3h}{2}\right)\right)}, & \gamma_2 &= -\frac{3\cot^2\left(\frac{h}{2}\right)}{(2+4\cos(h))}. \end{aligned}$$

During the solution process, firstly, for the time discretization forward finite difference scheme and then for the space discritization finite element collocation method based on trigonometric cubic B-spline basis functions are going to be implemented.

3 Implementation of the method for time discretization

Now, the MEW wave equation is discretized as follows

$$u_t + \varepsilon u^2 u_x - \mu u_{xxt} = 0.$$

For this purpose, the Crank-Nicolson type scheme is implemented. Firstly the equation is discretized as,

$$\frac{U^{n+1} - U^n}{\Delta t} + \varepsilon \frac{(U^2 U_x)^{n+1} + (U^2 U_x)^n}{2} - \mu \frac{U_{xx}^{n+1} - U_{xx}^n}{\Delta t} = 0 \tag{5}$$

where Rubin and Graves type linearization technique [36] is used at the left hand side of the Eq. (5) to linearize the nonlinear terms as given below

$$(U^2 U_x)^{n+1} = U^{n+1} U^n U_x^n + U^n U^{n+1} U_x^n + U^n U^n U_x^{n+1} - 2U^n U^n U_x^n,$$

Accordingly, the following iterative scheme is obtained

$$\begin{aligned} &U^{n+1} + \varepsilon \frac{\Delta t}{2} (U^{n+1} U^n U_x^n + U^n U^{n+1} U_x^n + U^n U^n U_x^{n+1} - U^n U^n U_x^n) - \mu U_{xx}^{n+1} \\ &= U^n + \varepsilon \frac{\Delta t}{2} (U^2 U_x)^n - \mu U_{xx}^n \end{aligned}$$

This scheme results in a system of equations consisting of $(N+1)$ equations and $(N+3)$ unknowns. Using the appropriate boundary conditions given with the problem, the unknowns lying outside the solution domain of the problem are eliminated. Thus a solvable system of equations is obtained. Now utilizing this system, the calculations are carried out until the desired time level. But for this, first of all, the initial values of the unknowns at time $t = 0$ are needed. The following section will deal with this step of the solution process.

3.1 Initial state

The initial vector d^0 is determined from the initial and boundary conditions. So the approximation Eq. (4) must be rewritten for the initial condition

$$U_N(x, 0) = \sum_{m=-1}^{N+1} \delta_m^0(t) T_m^3(x)$$

where the δ_m^0's are unknown parameters. The initial numerical approximation $U_N(x,0)$ is required to satisfy the following conditions:

$$U_N(x,0) = U(x_i,0), \qquad i = 0,1,...,N$$
$$(U_N)_x(a,0) = 0, \qquad (U_N)_x(b,0) = 0.$$

Thus, these conditions lead to the matrix equation

$$Wd^0 = b$$

where

$$d^0 = (\delta_0, \delta_1, \delta_2, \ldots, \delta_{N-2}, \delta_{N-1}, \delta_N)^T$$

and

$$b = (U(x_0,0), U(x_1,0), U(x_2,0), \ldots, U(x_{N-2},0), U(x_{N-1},0), U(x_N,0))^T.$$

4 Numerical examples

In this section, three common test problems about the MEW equation are going to be solved and the results will be compared with some of those available in the literature. If the exact solution of the test problem is available, then the accuracy of the numerical method is going to be controlled by using the error norms L_2 and L_∞.

4.1 Problem I: Motion of a single solitary wave

The solitary wave solution of the MEW Eq.(2) is given by

$$u(x,t) = A \sec h(k[x - x_0 - vt])$$

where $k = \sqrt{1/\mu}$, $A = \sqrt{6v/\varepsilon}$. This solution corresponds to motion of a single solitary wave of magnitude A, initially centered at the position x_0 and propagating to the right side with a constant velocity v. The solitary wave type solution (3) of Eq. (2) is not only on a unbounded region, but also at the same time it has a solitary wave solution on the closed interval $a \leq x \leq b$. The three invariants I_1, I_2 and I_3 satisfied by the MEW (2) equation are computed as follows by taking U_j and $U_j^{'}$ as the mesh values calculated from numerical solution

$$I_1 = h\sum_{j=0}^{N} U_j, \quad I_2 = h\sum_{j=0}^{N}\left[U_j^2 + \mu(U_j^{'})^2\right], \quad I_3 = h\sum_{j=0}^{N} U_j^4$$

For this problem the analytical values of the invariants are [5]

$$I_1 = \frac{A\pi}{k}, \quad I_2 = \frac{2A^2}{k} + \frac{2\mu k A^2}{3}, \quad I_3 = \frac{4A^4}{3k}$$

In order to be able to make a comparison with some of the studies in the literature, the parameters as $t = 20$, $\mu = 1$, $x_0 = 30$, $A = 0.25$ and $\Delta t = 0.05$ are used. In Figure 1, the movement of solitary wave has been given for various

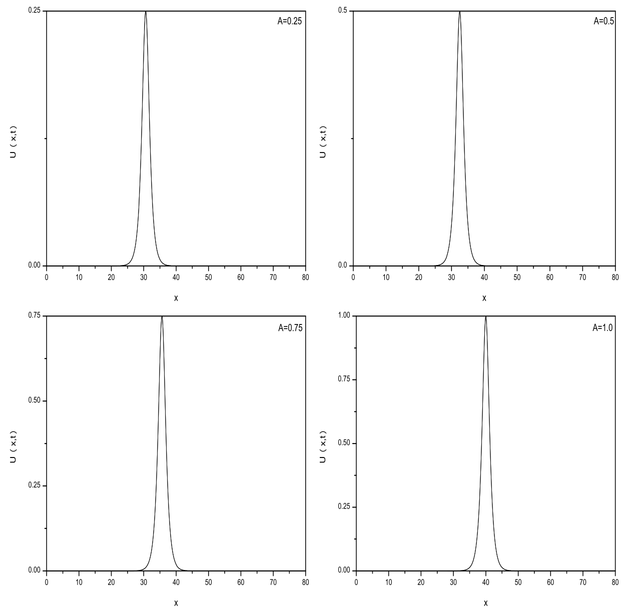


Figure 1: The single solitary wave solutions for values of $A = 0.25, 0.50, 0.75$ and 1.0 at time $t_{final} = 20$.

values of amplitudes $A = 0.25$, 0.5, 0.75 and 1. From the figure, it is seen that the larger wave with large amplitude has traveled a long way because of its faster velocity.

In Table 1, $h = 0.1$, $\Delta t = 0.2$, $A = 0.25$, $x_0 = 30$ are taken over the region $0 \leq x \leq 80$ at times $t_{final} = 5, 10, 15$ and 20. The newly obtained results are compared with some of those available in the literature. From the table it is clearly seen that the present results are better or in good agreement with those given in compared references.

In Table 2, $h = 0.1$, $\Delta t = 0.05$, $A = 0.25$, $x_0 = 30$ are taken over the region $0 \leq x \leq 80$ at times $t_{final} = 20$. The newly obtained results are compared with some of those available in the literature. One can see that the newly obtained results are in good agreement with those given in references.

In Table 3, a comparison of the error norms L_2, L_∞ and invariants I_1, I_2, I_3 of Problem I with those in Ref. [8] for $h = \Delta t = 0.01$, $A = 0.25, 0.50, 0.75, 1.0$ and $x_0 = 30$ on $0 \leq x \leq 80$ at time $t_{final} = 20$. From the table one can see that the present error norms are better than those compared ones.

Table 1: A comparison of the error norms L_2 and L_∞ of Problem I for $h = 0.1$, $\Delta t = 0.2$, $A = 0.25$, $x_0 = 30$ on $0 \le x \le 80$ at times $t_{final} = 5, 10, 15, 20$.

Method	t_{final}	$L_2 \times 10^4$	$L_\infty \times 10^4$	I_1	I_2	I_3
Present	5	1.276445	0.681926	0.7850300	0.1666259	0.0052058
	10	1.319189	0.748057	0.7850300	0.1666259	0.0052058
	15	1.385979	0.828043	0.7850300	0.1666259	0.0052058
	20	1.471099	0.897036	0.7850300	0.1666259	0.0052058
Ref. [8]	5	0.682986	0.610149	0.7853976	0.1664731	0.0052083
	10	1.362867	1.255591	0.7853984	0.1664732	0.0052083
	15	2.036756	1.916829	0.7853976	0.1664733	0.0052083
	20	2.701647	2.576377	0.7853977	0.1664736	0.0052083
Ref. [26]	5	0.473145	0.418872	0.7853712	0.1666095	0.0052078
	10	0.990390	0.840128	0.7853424	0.1665994	0.0052072
	15	1.499677	1.212955	0.7853751	0.1665922	0.0052067
	20	2.021476	1.569539	0.7852864	0.1665818	0.0052061
Ref. [29]	5	0.011570	0.006221	0.7854014	0.1666672	0.0052084
	10	0.010404	0.005784	0.7854008	0.1666668	0.0052084
	15	0.022265	0.014353	0.7854038	0.1666680	0.0052084
	20	0.011493	0.007664	0.7854013	0.1666670	0.0052084

Table 2: A comparison of the error norms L_2, L_∞ and the invariants I_1, I_2, I_3 of Problem I with some of the previous ones for $h = 0.1$, $\Delta t = 0.05$, $A = 0.25$, $x_0 = 30$ on $0 \le x \le 80$ at time $t_{final} = 20$.

Method	$L_2 \times 10^3$	$L_\infty \times 10^3$	I_1	I_2	I_3
Present	0.146806	0.089667	0.7850300	0.1666259	0.0052058
Ref. [8]	0.269281	0.256997	0.7853977	0.1664735	0.0052083
Ref. [12]	0.079694	0.046552	0.7853898	0.1667614	0.0052082
Ref. [13]	0.175081	0.176288	0.7853982	0.1666666	0.0052083
Ref. [14]	0.005208	0.005456	0.7853965	0.1666638	0.0052081
Ref. [20]	0.051873	0.032113	0.7853967	0.1666664	0.0052083
Ref. [22]	0.051774	0.032114	0.7853967	0.1666663	0.0052083
Ref. [23]	0.080146	0.046121	0.7853967	0.1666663	0.0052083
Ref. [24]	0.080098	0.046061	0.7853967	0.1666663	0.0052083
Ref. [25]1	0.175277	0.176465	0.7853966	0.1666662	0.0052083
Ref. [25]2	0.175270	0.176459	0.7853966	0.1666662	0.0052083
Ref. [26]	0.290516	0.249892	0.7849545	0.1664765	0.0051995
Ref. [29]	0.001653	0.001194	0.7853979	0.1666671	0.0052084

Table 3: A comparison of the error norms L_2, L_∞ and the invariants I_1, I_2, I_3 of Problem I with Ref. [8] for $h = \Delta t = 0.01$, $A = 0.25, 0.50, 0.75, 1.0$ and $x_0 = 30$ on $0 \le x \le 80$ at time $t_{final} = 20$.

A		$L_2 \times 10^3$	$L_\infty \times 10^3$	I_1	I_2	I_3
0.25	Present	0.0014686	0.0009014	0.7853945	0.1666663	0.0052083
	Ref. [8]	0.0026985	0.0026867	0.7853963	0.1666644	0.0052083
	Exact			0.7853982	0.1666667	0.0052083
0.50	Present	0.0057187	0.0038677	1.5707889	0.6666650	0.0833329
	Ref. [8]	0.0186465	0.0150972	1.5707920	0.6666588	0.0833333
	Exact			1.5707963	0.6666667	0.0833333
0.75	Present	0.0229900	0.0149503	2.3561834	1.4999963	0.4218729
	Ref. [8]	0.0519345	0.0366739	2.3561860	1.4999790	0.4218745
	Exact			2.3561945	1.5000000	0.4218750
0.10	Present	0.1010366	0.0626081	3.1415779	2.6666660	1.3333267
	Ref. [8]	0.1494558	0.0987068	3.1415790	2.6666350	1.3333310
	Exact			3.1415927	2.6666667	1.3333333

4.2 Problem II: Interaction of two solitary waves

As a second test problem, Eq. (2) together with boundary conditions $U \to 0$ as $x \to \pm\infty$ and the initial condition for all linearization techniques is considered as

$$U(x,0) = \sum_{j=1}^{2} A_j \sec h(k[x - x_j])$$

where $k = \sqrt{1/\mu}$. In order that the collision occurs, the solution domain is taken as $0 \le x \le 80$ for values of $h = 0.1$, $\Delta t = 0.2$, $\mu = 1$, $A_1 = 1$, $A_2 = 0.5$, $x_1 = 15$, $x_2 = 30$. It is seen from Fig. 2 that the larger wave leaves the smaller one its behind. In addition, there was no elastic collision because the waves after the collision left small tail waves behind them. Because of this fact, these two solitary waves are not considered as solitons [5]. Moreover, for values of $A_1 = 1$, $A_2 = 0.5$, $\Delta t = 0.2$, a comparison has been made with those given in Refs. [8] and [26].

In Table 4, a comparison of the invariants I_1, I_2, I_3 of Problem II with Refs. [8] and [26] is made for $h = 0.1$, $\Delta t = 0.2$ on $0 \le x \le 80$ at various times. It is clearly seen that the invariants are well preserved after the initial time untill the end of run-time.

4.3 Problem III: The Maxwellian initial condition

As the last test problem, the initial Maxwellian pulse is considered with the initial condition in solitary waves given by

$$u(x,0) = e^{-x^2} \tag{6}$$

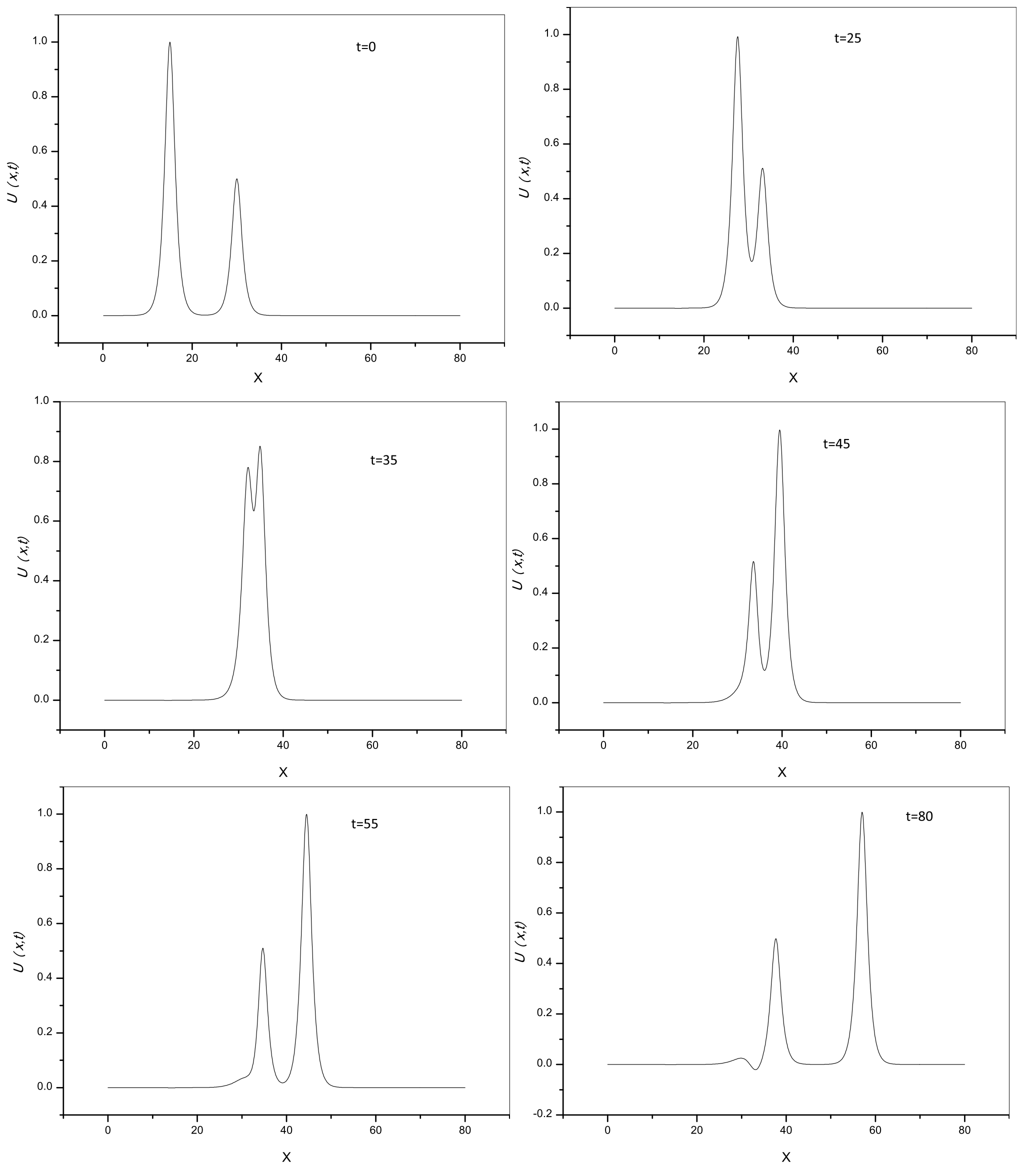


Figure 2: The interaction of two solitary waves at times $t = 0, 25, 35, 45, 55$ and 80.

Table 4: A comparison of Problem II with those from Refs. [8], [26] with $h = 0.1$, $\Delta t = 0.2$ on $0 \leq x \leq 80$.

t	Present method			[8]			[26]		
	I_1	I_2	I_3	I_1	I_2	I_3	I_1	I_2	I_3
0	4.712388	3.333336	1.416669	4.712388	3.329462	1.416669	4.712388	3.332357	1.416670
10	4.710180	3.331961	1.415419	4.712389	3.328927	1.416103	4.712022	3.324678	1.400768
20	4.710180	3.331341	1.414833	4.712387	3.328361	1.415523	4.711697	3.324210	1.401182
30	4.710181	3.329523	1.413184	4.712388	3.327818	1.413882	4.711242	3.346583	1.424847
40	4.710181	3.329690	1.413358	4.712385	3.327112	1.414050	4.711017	3.321250	1.398239
50	4.710180	3.330105	1.413629	4.712388	3.326632	1.414330	4.710804	3.320956	1.398729
55	4.710180	3.329860	1.413359	4.712386	3.326393	1.414062	4.710630	3.323628	1.399068
60	4.710180	3.329600	1.413079	4.712388	3.326228	1.413785			
70	4.710180	3.329056	1.412516	4.712388	3.325891	1.413228			
80	4.710180	3.328490	1.411954	4.712389	3.325434	1.412671			

Table 5: The invariants I_1, I_2, I_3 of Problem III for various values of μ at time $t = 12.5$.

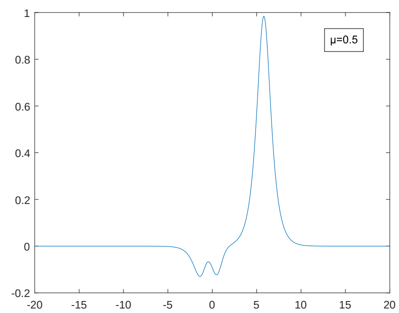
	I_1	I_2	I_3	I_1	I_2	I_3
	$\mu = 0.5$			$\mu = 0.1$		
Present	1.77235	1.87971	0.88597	1.77244	1.37783	0.88619
Ref. [13]	1.77245	1.88008	0.88623	1.77249	1.37774	0.88627
	$\mu = 0.05$			$\mu = 0.02$		
Present	1.77246	1.31444	0.88644	1.77256	1.27424	0.88660
Ref. [13]	1.77254	1.31431	0.88639	1.77275	1.27458	0.88717
	$\mu = 0.005$			$\mu = 0.0025$		
Present	1.77311	1.23603	0.86783	1.76963	1.19626	0.81240
Ref. [13]	1.77465	1.25032	0.89902	1.77868	1.24930	0.92893

with the boundary condition

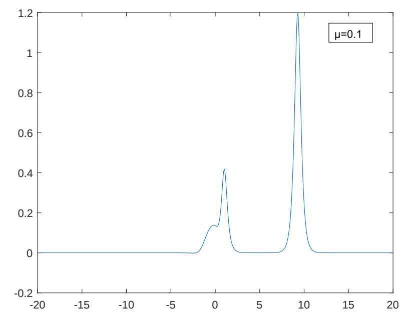
$$u(-20, 0) = u(20, 0) = 0, \quad t > 0.$$

Maxwellian initial condition (6) breaks up into a number of solitary waves depending on values of μ. The calculations are carried out for values of $\mu = 0.5$, 0.1, 0.05, 0.02, 0.005, 0.0025, $h = 0.05$, $\Delta t = 0.01$ and $t = 12.5$. Figure 3 shows Maxwellian initial condition for those parameters on $-20 \leq x \leq 20$ at time $t_{final} = 12.5$.

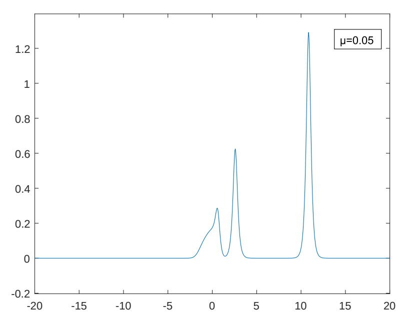
In Table 5, a comparison of the invariants I_1, I_2, I_3 of Problem III with Refs. [13] for various values of $\mu = 0.5$, 0.1, 0.05, 0.02, 0.005, 0.0025 and $h = 0.05$, $\Delta t = 0.01$ and $t = 12.5$ is presented.



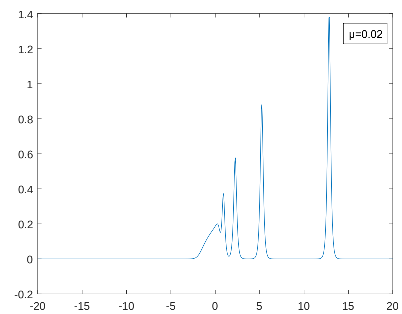
(a)



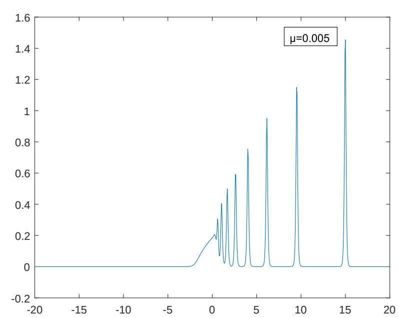
(b)



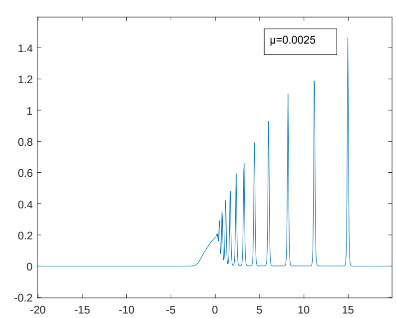
(c)



(d)



(e)



(f)

Figure 3: Maxwellian initial condition for $h = 0.05$, $\Delta t = 0.01$, $\mu = 0.5$, 0.1, 0.05, 0.02, 0.005, 0.0025 on $-20 \leq x \leq 20$ at time $t_{final} = 12.5$.

5 Conclusion

In this paper, numerical solutions of the MEW equation based on the trigonometric cubic B-spline finite element have been presented. Three test problems are worked out to examine the performance of the algorithms. The performance and accuracy of the method is shown by calculating the error norms L_2 and L_∞. For each linearization technique, the error norms are sufficiently small and the invariants are satisfactorily constant in all computer runs. The computed results show that the present method is a remarkably successful numerical technique for solving the MEW equation and advisable for getting numerical solutions of other types of non-linear equations.

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