Table 1: Reduced fractional differential Transformations [7,21]

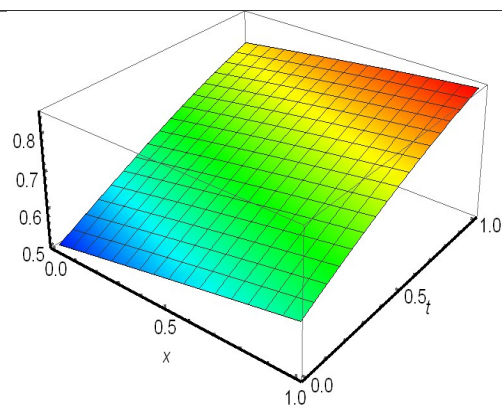| Original function	Transformed function
$u(x,t)$	$U_h(x) = \dfrac{1}{\Gamma(1+h\alpha)} \left[\dfrac{\partial^{ha}}{\partial t^{ha}} u(x,t) \right]_{t=0}$
$u(x,t) = l_1 w(x,t) \pm l_2 v(x,t)$	$U_k(x) = l_1 W_h(x) \pm l_2 V_h(x) \, l_1, l_2 \in R$
$u(x,t) = cw(x,t)(c \in R)$	$U_h(x) = c \, W_h(x)(c \in R)$
$u(x,t) = \dfrac{\partial^r}{\partial x^r} w(x,t)$	$U_h(x) = \dfrac{\partial^r}{\partial x^r} W_h(x)$
$u(x,t) = \dfrac{\partial^{N\alpha}}{\partial t^{N\alpha}} w(x,t)$	$U_h(x) = \dfrac{\Gamma(h\alpha + N\alpha + 1)}{\Gamma(h\alpha + 1)} W_{h+N}(x)$
$u(x,t) = w(x,t) v(x,t)$	$U_h(x) = \displaystyle\sum_{s=0}^{h} V_s(x) W_{h-s}(x)$
$\psi(x,t) = u(x,t) w(x,t) v(x,t)$	$\Psi_k = \displaystyle\sum_{r=0}^{k} \sum_{i=0}^{r} U_i(x) V_{r-i}(x) W_{k-r}(x)$

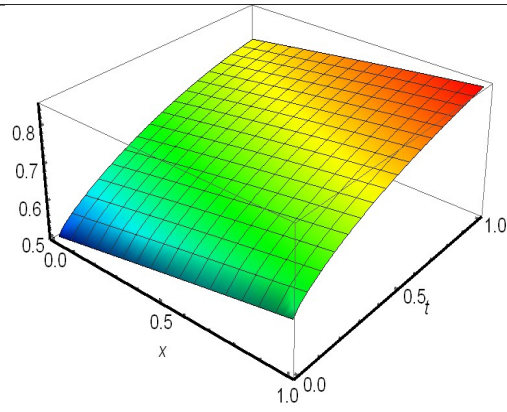**Fig. 4.a** Murray equation graph for $\alpha = 1$

Fig. 4.b Murray equation graph for $\alpha = 0,75$
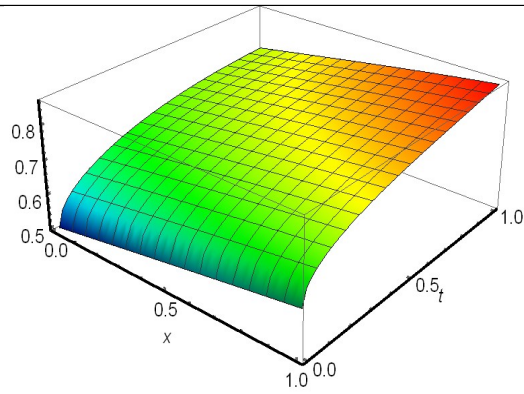

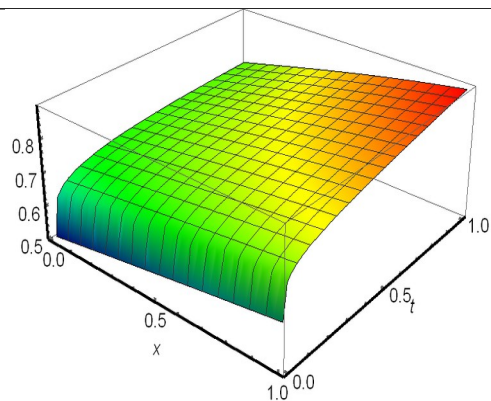**Fig. 4.c** Murray equation graph for $\alpha = 0,50$

Fig. 4.d Murray equation graph for $\alpha = 0,25$

Table 2 When $\alpha = 1$ and $t = 0,125$, the $u(x,t)$ numerical solution of time-fractional differential equation ([4.4](#))

x value	t value	Numerical solution	Analytical solution	Absolute error
0,125	0,125	0,5544702743	0,5544704649	$1,9 \times 10^{-7}$
0,225	0,125	0,5667858200	0,5667860060	$1,8 \times 10^{-7}$
0,325	0,125	0,5790194078	0,5790195874	$1,7 \times 10^{-7}$
0,425	0,125	0,5911567286	0,5911568998	$1,7 \times 10^{-7}$
0,525	0,125	0,6031839359	0,6031840971	$1,6 \times 10^{-7}$
0,625	0,125	0,6150877057	0,6150878555	$1,4 \times 10^{-7}$
0,725	0,125	0,6268552909	0,6268554280	$1,3 \times 10^{-7}$
0,825	0,125	0,6384745708	0,6384746941	$1,2 \times 10^{-7}$
0,925	0,125	0,6499340935	0,6499342022	$1,0 \times 10^{-7}$

Table 3 When $t = 0,125$ and $\alpha = 1$, $\alpha = 0,75$, $\alpha = 0,50$, $\alpha = 0,25$, the $u(x,t)$ numerical solution of time-fractional differential equation ([4.4](#))

x	t	$\alpha = 1$	$\alpha = 0,75$	$\alpha = 0,50$	$\alpha = 0,25$
0,125	0,125	0,5544702743	0,5859103286	0,6334478288	0,6894731167
0,225	0,125	0,5667858200	0,5979239246	0,6446632874	0,7000300566
0,325	0,125	0,5790194078	0,6098216121	0,6557299695	0,7104476486
0,425	0,125	0,5911567286	0,6215906289	0,6666378629	0,7207052979
0,525	0,125	0,6031839359	0,6332187959	0,6773775486	0,7307833264
0,625	0,125	0,6150877057	0,6446945618	0,6879402342	0,7406632610
0,725	0,125	0,6268552909	0,6560070412	0,6983177834	0,7503280899
0,825	0,125	0,6384745708	0,6671460477	0,7085022739	0,7597624789
0,925	0,125	0,6499340935	0,6781021211	0,7184883430	0,7689529449

the function value

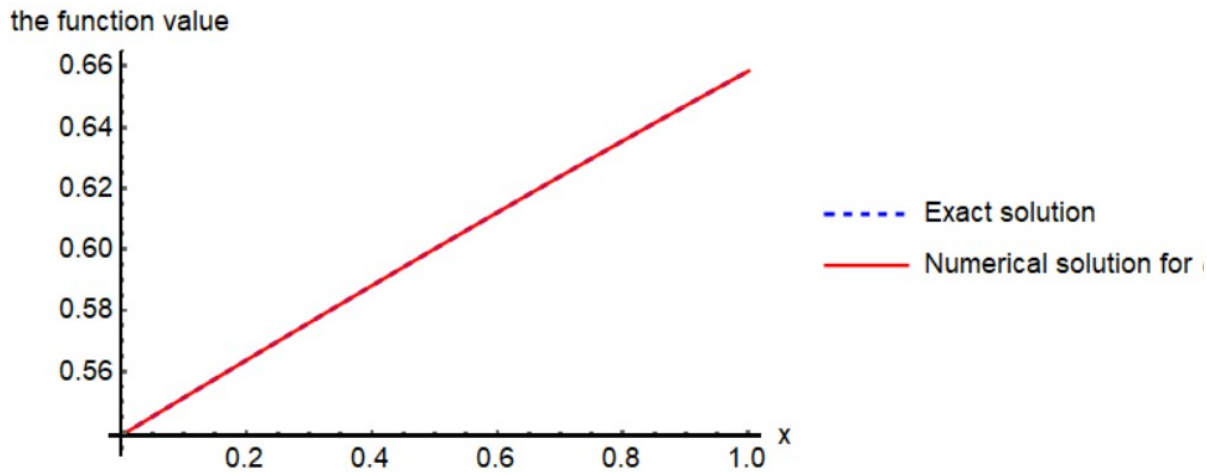**Fig. 5.a** 2D graphic of the exact and numerical solution of $u(x, 0.125)$ of the (4.4) time-fractional differential equation for $\alpha = 1$.
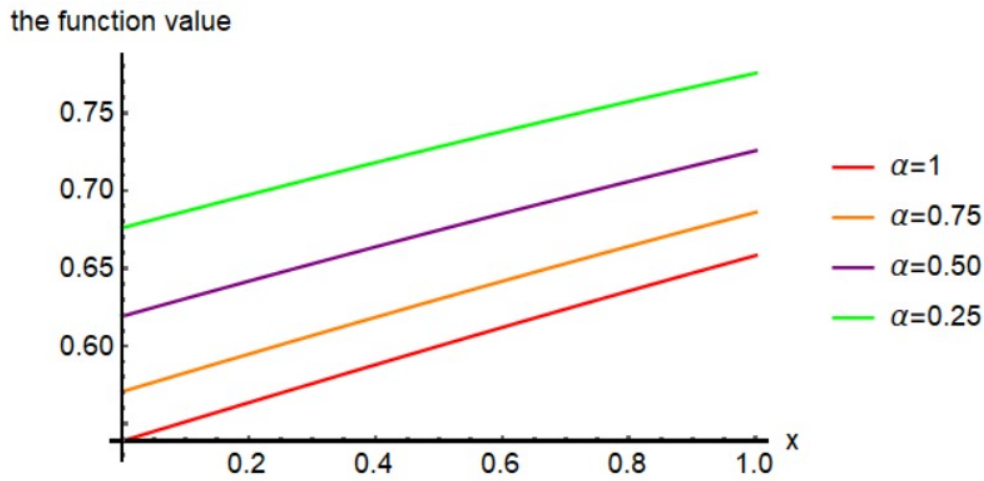
the function value

Fig. 5.b 2D graphic of the numerical solution of $u(x, 0.125)$ of the (4.4) time-fractional differential equation for $\alpha = 1, \alpha = 0,75$, $\alpha = 0,50$, $\alpha = 0,25$ of the (4.4) time fractional differential equation.

Table 4 When $\alpha = 1$ and $t = 0,325$, the $u(x, t)$ numerical solution of time-fractional differential equation (4.4)

x value	t value	Numerical solution	Analytical solution	Absolute error
0,125	0,325	0,6150657708	0,6150878555	0,0000220847
0,225	0,325	0,6268340714	0,6268554281	0,0000213566
0,325	0,325	0,6384542799	0,6384746942	0,0000204143
0,425	0,325	0,6499149279	0,6499342022	0,0000192743
0,525	0,325	0,6612052504	0,6612232068	0,0000179564
0,625	0,325	0,6723152161	0,6723316992	0,0000164831

0,725	0,325	0,6832355529	0,6832504316	0,0000148788
0,825	0,325	0,6939577659	0,6939709354	0,0000131695
0,925	0,325	0,7044741505	0,7044855324	0,0000113819

Table 5 When $t=0,325$ and $\alpha=1$, $\alpha=0,75$, $\alpha=0,50$, $\alpha=0,25$, the $u(x,t)$ numerical solution of time-fractional differential equation ([4.4](#))

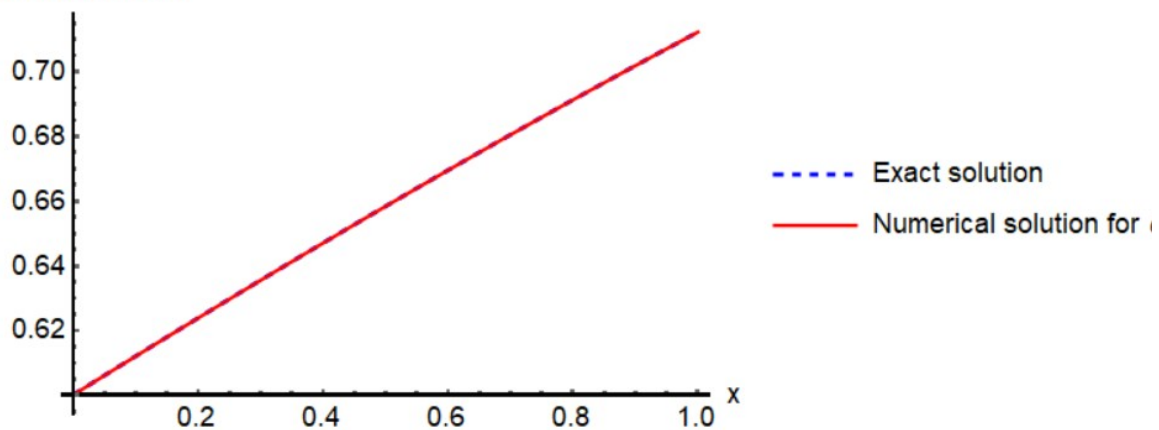| x	t	$\alpha=1$	$\alpha=0,75$	$\alpha=0,50$	$\alpha=0,25$
0,125	0,325	0,6150657708	0,6541561282	0,6919781039	0,7192267946
0,225	0,325	0,6268340714	0,6651658729	0,7021446159	0,7307671602
0,325	0,325	0,6384542799	0,6760042568	0,7121570774	0,7421752989
0,425	0,325	0,6499149279	0,6866625171	0,7220040752	0,7534040021
0,525	0,325	0,6612052504	0,6971325	0,7316746799	0,7644082702
0,625	0,325	0,6723152161	0,7074069155	0,7411585762	0,7751460837
0,725	0,325	0,6832355529	0,7174788827	0,7504461818	0,7855790777
0,825	0,325	0,6939577659	0,7273424109	0,7595287530	0,7956731036
0,925	0,325	0,7044741505	0,7369921707	0,7683984747	0,8053986682

Fig. 6.a 2D graphic of the exact and numerical solution of $u(x,0.325)$ of the ([4.4](#)) time-fractional differential equation for $\alpha=1$.
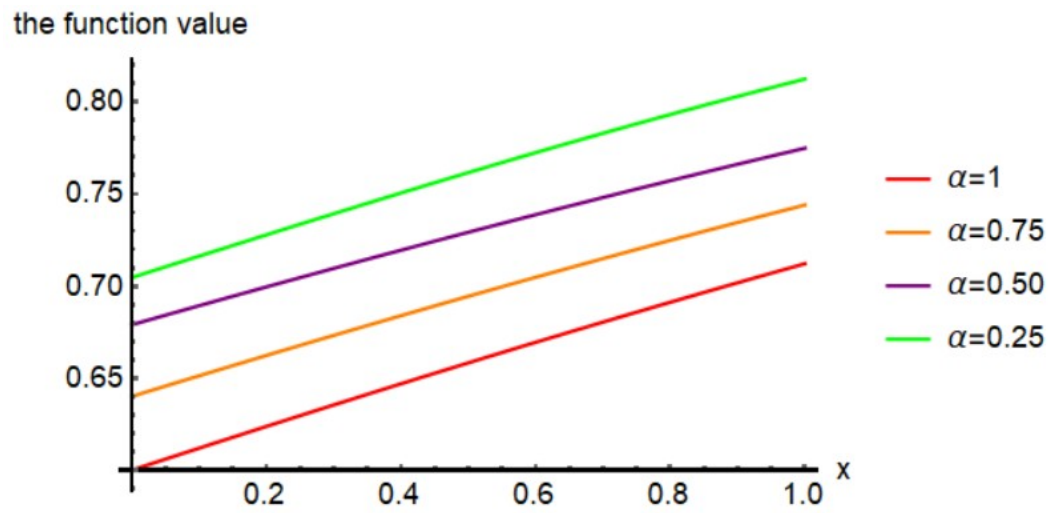
Fig. 6.b 2D graphic of the numerical solution of $u(x, 0.325)$ of the (4.4) time-fractional differential equation for $\alpha=1$, $\alpha=0,75$, $\alpha=0,50$, $\alpha=0,25$ of the (4.4) time fractional differential equation.