Correlations between crack initiation and crack propagation lives of notched specimens under constant and variable amplitude loading

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Abstract

This paper starts with an overview of the application of the three guidelines (GL) of the German Research Association of Mechanical Engineers (FKM). Each of these provides algorithms for calculating fatigue lives of components under constant or variable amplitude loading, however, with underlying different failure criteria, i.e. technical crack initiation life (GL-nonlinear), fatigue crack growth life (GL-fracture mechanics), and total fracture life (GL-linear). This paper introduces the U-Concept which has been evaluated from a large structural durability database. The U-Concept is a small add-on to the Local Strain Approach (LSA) which is the backbone of the GL-nonlinear. It enables 1) to directly calculate the fatigue life to total fracture based on elastic-plastic material behaviour according to the LSA, or 2) to estimate the remaining fatigue life from crack initiation to fracture without a crack growth simulation.

Keywords

Fatigue, Local Strain Approach, Crack initiation, Fracture, FKM guidelines

1. Introduction

There are currently three FKM guidelines for fatigue life calculations of steel components published for the mechanical engineering industry in Germany. The FKM guideline "nonlinear" [1] recommends the Local Strain Approach (LSA) for fatigue life calculations for failure criterion crack initiation, which is able to consider elastic-plastic material behaviour. The FKM guideline "linear" [2] is based on a local stress concept respecting elastic material behaviour for fatigue life calculations for fracture and the third FKM guideline "fracture mechanics" [3] describes the algorithms of crack propagation from crack initiation to fracture. Though the complete range of the fatigue life calculation is covered, the concepts differ clearly in their extent, their complexity and their conditions to the user. The FKM guidelines "linear" and "nonlinear" both use simple concepts based on Woehlercurves for the fatigue life calculation and only require the ultimate tensile strength as material parameter (besides the material class), whereas the fracture mechanics concept of the third FKM guideline uses a fatigue crack growth rate law for a crack propagation calculation requiring experimentally determined crack propagation parameters. A user who calculated the fatigue life for crack initiation with the FKM guideline nonlinear only based on the ultimate tensile strength is considered to calculate the remaining fatigue life with the fracture mechanics guideline. Since fatigue crack growth calculations require 1.) an initial crack length, 2.) information about the crack propagation direction and

3.) crack propagation material parameters, which are not generally at hand. To close this gap, this paper introduces an approximation scheme for the crack propagation life, that is only based on the information of the crack initiation life and which needs no further information. The present paper is an extended version of [4] published in the proceedings of the fourth Conference on Variable Amplitude Loading.

Current publications about fatigue crack propagation can be divided in three areas: a) Modifications and further developments for a better fit of the algorithms to current mechanical problems based on Paris' law, see [5], [6], [7], [8], b) advanced analytical models or further approaches for crack propagation [9, 10], [11], [12], [13], [14] and c) approaches to simplify current crack growth simulation approaches in order to make the algorithms more applicable [15], [16], [17] or independent of material [18] or geometrical influences [14, 19.] Area a) includes researches depending on the stress intensity factor range ΔK and investigating the influence of corrosion [6, 8,] environmental effects like seawater [5] or surface treatments like cold rolling [7] on the material parameters C, m of Paris' law. Area b) contains approaches for the fatigue growth rate by identifying the Crack Tip Opening Displacement (CTOD) [9], mathematically modeling the crack tip stress field (CJP-model) [10], [11], investigations of the crack growth based on micromechanics [12] or approaches with the cohesive zone model to predict mixed-mode crack growth [13]. For a simplification of the crack growth simulation (Area c), Murakami and Beretta [19] introduced the $\sqrt{area}$-model in order to derive the threshold stress intensity factor range ΔK_{th} for different shapes of defects by only taking the square root of the defect area projected onto a plane perpendicular to the applied stress $\sqrt{area}$ and the Vickers hardness *HV* into account. Leonetti et. al. [18] predicted cyclic R-curves and respecting crack opening and closing effects useable for short and long cracks in welded components based only on the Vickers hardness HV using the $\sqrt{area}$-model in combination with approaches by McEvily [20] and Chapetti [21]. Liang et al. [17] combined an effective stress intensity factor range ΔK_{eff} (considering crack opening and closing effects) with the multi-R-ratio model by Bloom [15]. Using Neuber's equation [22] Liang et al. calculated the effective stresses and strains in the notch from the nominal stresses using the plastic load factor K_p and therefore identify ΔK_{eff}. The transformation from elastic-calculated nominal stresses to elastic-plastic stresses and strains in the notch in combination with a damage evaluation using damage parameters is known as the Local Strain Approach (LSA) and is used for fatigue life calculations for failure criterion crack initiation, [23]. Dankert [16] published a first approach for a crack growth calculation in combination with the Local Strain Approach, which where later included in the FKM fracture mechanical guideline [3]. This approach was evolved in [24] and verified in [25]. The paper at hand simplifies the approaches by Dankert in order to develope an approximation method for the fatigue life resulting from the crack growth between crack initiation and fracture for the industrial user based on the FKM guidelines. Therefore, a shifting factor for damage parameter Woehlercurves from failure criterion crack initiation to failure criterion fracture is identified and combined with the Local Strain Approach. Using this concept, a crack growth simulation is no longer necessary to identify the fracture fatigue life of a component under constant or variable amplitude loading.

In section 2 the *FKM guideline linear* for fracture fatigue life calculation is described. Since the approach only considers elastic material behaviour, it is easily applied and widely used in industrial applications. Even so, neglecting the elastic-plastic material behaviour leads to less accuracy in comparison to experimental results, see section 6. Section 3 describes the crack growth simulation of *FKM fracture mechanical guideline* and Section 4 the Local Strain Approach of *FKM guideline nonlinear*. A comparison between section 2 and 4 shows, that both guidelines use Woehler curves approximated by only requiring the ultimate tensile strength, component factors and applications of the Rainflow counting method. Since the applications of both concepts have so much in common, this paper tries to combine the advantages of both guidelines in order to calculate the fatigue life for fracture respecting the elastic-plastic material behaviour. Therefore, Section 5 introduces the U-Concept, a concept which considers a shift of the damage parameter Woehler curve of the Local Strain Approach in order to make it applicable to fracture life estimates. Section 6 shows the results of the U-Concept in comparison to experimental results, a crack growth simulation and calculations with the FKM guideline linear.

2. The FKM guideline linear for fracture

The *FKM guideline linear* [2] consists of a simple concept to calculate the fatigue life for failure criterion fracture based on linear-elastic stresses and strains in the notched area of a component. The concept compares the local stresses in the notch with a component-depending Woehler curve. This Woehler curve can be approximated with

$$N_{\mathrm{FKM,frac}} = 10^6 \cdot \left(\frac{K_t \cdot S_a}{\sigma_{WK} \cdot n_\sigma \cdot K_{AK}}\right)^{-k} \tag{1}$$

with nominal stress amplitude S_a, elastic stress concentration factor K_t, fatigue limit S_D, size effects n_σ and mean stress factor K_{AK}. The Woehlercurve exponent is usually $k = 5$ and the fatigue limit is $\sigma_{WK} = f_{W,\sigma} \cdot R_m$ with factor $f_{W,\sigma}$=0.4-0.45 for steel (except cast steel). Mean stress factor K_{AK} is depending on the nominal stress amplitude S_a, nominal mean stress S_m and ultimate tensile strength R_m. The size effect n_σ considers the influence of the components geometry by including the statistical size effect n_{st}, the fracture mechanical effect by [26] n_{bm}, see [27], and the mechanical size effect n_{vm}.

In [2], the following relation is proposed:

$$n_\sigma = n_{\mathrm{st}} \cdot n_{\mathrm{bm}} \cdot n_{\mathrm{vm}} \tag{2}$$

For surface cracks in steel components the statistical size effect is

$$n_{\mathrm{st}} = \left(\frac{500\ \mathrm{mm}^2}{A_\sigma}\right)^{\frac{1}{k_\sigma}} \tag{3}$$

with the highly stressed area A_σ, depending on the size of the component, and the Weibull-exponent $k_\sigma = 30$ for steel. The strain depending mechanical size effect can be described using the fatigue limit for nominal stresses $\sigma_D = \sigma_D(R_m)$ and plastic strains $\varepsilon_{pl,D} = \varepsilon_{pl,D}(R_m)$, see [2], with cyclic material exponent n' and elastic modulus E (4)

$$n_{\mathrm{vm}} = \sqrt{1 + \frac{E \cdot \varepsilon_{pl,D}}{\sigma_D} \cdot n_{st}^{\frac{1-n'}{n'}}}$$

The fracture mechanical size effect is described by [26] as

$$n_{\mathrm{bm}} = \frac{5 + \sqrt{G\ [\mathrm{mm}]}}{5 \cdot n_{\mathrm{vm}} \cdot n_{\mathrm{st}} + \frac{R_m}{430\ \mathrm{MPa}} \cdot \sqrt{\frac{15 + 2\sqrt{G\ [\mathrm{mm}]}}{5 + \sqrt{G\ [\mathrm{mm}]}}}} \tag{5}$$

With the stress gradient in the notched area G. According to *FKM guideline linear* [2] G can be approximated by the notch radius r and the net diameter d (only for bending)

$$G \approx \frac{2}{r} + \frac{2}{d} \tag{6}$$

So the size effect n_σ is respecting the geometrical and plastic influences of the component.

The *FKM guideline linear* also proposes different other parameters to recognize production- or material-related effects when calculating the fatigue life for fracture, which are not included in the results of this paper. So current *FKM guideline linear* proposes a simple concept based on the ultimate tensile strength R_m for an approximation of the fatigue life of a component under constant amplitude loading based on linear-elastic material behaviour. For variable amplitude loadings, the guideline recommends a Rainflow counting and a consequently or an elementary Miner calculation. To consider elastic-plastic material behaviour and sequence effects, the *FKM guideline linear* proposes an effective Miner sum and is therefore independent of a crack growth simulation.

3. The FKM fracture mechanical guideline for crack growth

In the *FKM fracture mechanical guideline* [3], the crack growth is described by [16], [28]

$$\frac{da}{dN} = C_j \cdot \left(\Delta J_{\mathrm{eff,total}}\right)^{m_j} \tag{7}$$

for short crack growth with the effective cyclic J-Integral $\Delta J_{\mathrm{eff,total}}$. The effective cyclic J-Integral depends on the effective nominal stress amplitude $S_{a,n,\mathrm{eff}}$ described by

$$S_{a,n,\mathrm{eff}} = \frac{\Delta S_{n,\mathrm{eff}}}{2} = \frac{S_{n,\mathrm{max}} - S_{n,\mathrm{cl}}}{2} \tag{8}$$

With the maximum nominal stress of a hysteresis loop $S_{n,\mathrm{max}}$ and the nominal stress when the crack closes $S_{n,\mathrm{cl}}$.

Kumar et al. [29] made the approach for a J-Integral divided in an elastic and a plastic part

$$\Delta J_{\mathrm{eff,total}} = \Delta J_{\mathrm{elastic}} + \Delta J_{\mathrm{plastic}} \tag{9}$$

The elastic J-Integral can be expressed by

$$\Delta J_{\mathrm{elastic}} = \frac{\Delta K_I^2}{E} \tag{10}$$

With elastic modulus E and stress intensity factor of modus I K_I.

The stress intensity factor is defined as

$$K_I = S_{a,n,\mathrm{eff}} \cdot \sqrt{\pi \cdot a} \cdot Y_{\mathrm{el}} \tag{11}$$

With elastic geometry correction function $Y_{\mathrm{el}} = Y_{\mathrm{el}}(a, c, t, w, \rho, \varphi)$ for surface cracks proposed with [16]

$$Y_{\mathrm{el}} = \left[1 + M_2 \cdot \left(\frac{2c}{t}\right)^2 + M_3 \cdot \left(\frac{2c}{t}\right)^4\right] \cdot \xi_1 \cdot F_{\mathrm{notch}} \cdot f_w \cdot \frac{c}{a} \cdot \frac{1}{\sqrt{Q}} \tag{12}$$

with

$$M_2 = \frac{0.05}{0.11 + \left(\frac{c}{a}\right)^{1.5}} \tag{13}$$

$$M_3 = \frac{0.29}{0.23 + \left(\frac{c}{a}\right)^{1.5}} \tag{14}$$

$$\xi_1 = 1 - \frac{\left(\frac{2c}{t}\right)^4 \cdot \sqrt{2.6 - \frac{4c}{t}}}{1 + 4 \cdot \frac{c}{a}} \tag{15}$$

$$F_{\mathrm{notch}} = (1.1215) \cdot \left(1 + \left(\left(K_{t,\infty} - 1\right)^{-2.2} + \left(\sqrt{\frac{C_0}{\lambda} + 1} - 1\right)^{-2.2}\right)^{-5/11}\right) \tag{16}$$

With $C_0 = \bar{a}^2/\bar{b}^2$ and $\lambda = a \cdot \bar{a}/\bar{b}^2$

$$K_{t,\infty} = \left(1 + 2 \cdot \frac{\bar{a}}{\bar{b}}\right) \cdot \left(1 + 0.122 \cdot \left(\frac{1}{1 + \frac{\bar{b}}{\bar{a}}}\right)^{2.5}\right) \tag{17}$$

For symmetric notches on two sides with symmetrically cracks

$$f_w = \frac{1}{1.1215} \cdot \left[1 + 0.1215 \cdot \cos^4\left(\frac{\pi \cdot (\bar{a} + a)}{2 \cdot w}\right)\right] \cdot \sqrt{\left(\frac{2w}{\pi \cdot (\bar{a} + a)}\right) \cdot \tan\left(\frac{\pi \cdot (\bar{a} + a)}{2 \cdot w}\right)} \tag{18}$$

The equations describe a semi-elliptical surface crack. The geometry parameters $a, \bar{a}, \bar{b}, c, t, w$ are shown in Figure 1. The plastic J-Integral is defined by

$$\Delta J_{\mathrm{plastic}} = \Delta J_{\mathrm{elastic}} \cdot Y_{\mathrm{pl}} \tag{19}$$

Leads to

$$\Delta J_{\mathrm{eff,total}} = \Delta J_{\mathrm{elastic}} + \Delta J_{\mathrm{plastic}} = \Delta J_{\mathrm{elastic}} \cdot (1 + Y_{\mathrm{pl}}) \tag{20}$$

The plastic part of the geometry correction function Y_{pl} is proposed according to [16]

$$Y_{\mathrm{pl}} = \frac{0.75}{\sqrt{n'}} \cdot \left(\frac{S_{a,n,\mathrm{eff}}}{K'}\right)^{\frac{1}{n'}} \cdot \Big(\big(1 - K_t(a)\big) \cdot \sin(\varphi) + K_t(a)\Big) \cdot \left(1 + 0.5 \cdot \frac{a}{c}\right) \tag{21}$$

In this paper following conditions were assumed: $\varphi = 0$ and $\frac{a}{c} = 2$ which leads to

$$Y_{\mathrm{pl}} = \frac{1}{\sqrt{n'}} \cdot \left(\frac{S_{a,n,\mathrm{eff}}}{K'}\right)^{\frac{1}{n'}} \cdot K_t(a) \cdot 1.5 \tag{22}$$

4. The FKM guideline nonlinear for crack initiation

The fatigue part of *FKM guideline nonlinear* [1] is based on the Local Strain Approach (LSA), [30, 23]. With the LSA linear elastically calculated stresses and strains in the notched area are transformed to elastic-plastic stresses and strains using flow curves. Based on these elastic-plastic stresses and strains, the stress-strain-curves for a given loading sequence can be simulated and the damage effect for the structure can be predicted using damage parameters and a damage accumulation rule. The basic material parameters used in describing the cyclic stress-strain-behaviour and the damage parameter Woehler curves can be estimated applying simple approximation methods which only require input of the ultimate tensile strength R_m and the material group.

4.1 Cyclic material behaviour

The cyclic stabilized material behaviour is described by Ramberg-Osgood's [31] equation (ROE)

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}} \tag{23}$$

Also Masing's law and the material memory effects has to be considered for simulations of a local stress strain path for a component under cyclic loading, see [23]. For the approximation of the cyclic material parameters the FKM guideline contains an approach by Wächter [32], [33] based on the ulatimate tensile strength.

4.2 Flow curves

In the notched area, flow curves connect the elastically calculated stresses and strains with the elastic-plastically calculated stresses and strains of the ROE. The FKM guideline describes two flow curves approaches: an approach by Neuber and an approach by Seeger and Beste. The approach by Neuber [22] connects the elastic stresses σ_e with the elastic-plastic stresses and strains σ, ε using the plastic factor K_p.

$$\varepsilon = \frac{\sigma_e}{\sigma} \cdot K_p \cdot e^* \tag{24}$$

The approach by Seeger and Beste [34] better fits with FE-results for the high strain area where the strains are mainly plastic. It also connects the elastic stresses σ_e with the elastic-plastic stresses and strains σ, ε using the plastic factor K_p.

$$\varepsilon = \frac{\sigma}{E} \cdot \left[\left(\frac{\sigma_e}{\sigma} \right)^2 \cdot \frac{2}{u^2} \cdot \ln \left(\frac{1}{\cos(u)} \right) - \frac{\sigma_e}{\sigma} + 1 \right] \cdot \left(\frac{e^* \cdot E \cdot K_p}{\sigma_e} \right) \tag{25}$$

With

$$u = \frac{\pi}{2} \cdot \left(\frac{\sigma_e/\sigma - 1}{K_p - 1} \right) \tag{26}$$

The strains e^* can be calculated with

$$e^* = \frac{\sigma_e/K_p}{E} + \left(\frac{\sigma_e/K_p}{K'} \right)^{\frac{1}{n'}} \tag{27}$$

4.3 Simulation of the local stress-strain-path

The stress-strain-path in the notched area has to be simulated for the whole loading sequence to identify the damage effects. The HCM algorithm as described by Clormann [35] is used in the FKM guideline to simulate the elastic-plastic stress-strain-path, identify closed hysteresis loops and calculate the required parameters for the damage accumulation. This algorithm is able to distinguish between the initial and reloading stabilized stress-strain-curve and considers the memory effects.

4.4 Damage Parameter

For the damage evaluation of the closed hysteresis loops, damage parameters are applied. Two different damage parameters P_{RAM} and P_{RAJ} can be used in the guideline for calculating the fatigue life for crack initiation.

4.4.1 Damage parameter P_{RAM}

The damage parameter P_{RAM} is based on Smith, Watson and Topper's [36] approach, modified by a factor to respect the material dependent mean stress sensitivity by Bergmann [37], [38].

$$P_{\mathrm{RAM}} = \sqrt{(\sigma_a + k \cdot \sigma_m) \cdot E \cdot \varepsilon_a} \tag{28}$$

In Bergmann [37] k is material depending but not specified for the material groups, in the FKM guideline the parameter k is described by

$$k = \begin{cases} M_\sigma \cdot (M_\sigma - 2), & \sigma_m \geq 0 \\ \frac{M_\sigma}{3} \cdot \left(\frac{M_\sigma}{3} + 2 \right), & \sigma_m < 0 \end{cases} \tag{29}$$

with mean stress sensitivity M_σ, see [2]. The damage parameter Woehlercurve (P-Woehlercurve) is approximated by the material group and the ultimate tensile strength R_m, see [32], [33], [1]. The grid point $P_{\mathrm{RAM,Z}}$ for the P-Woehlercurve is calculated by

$$P_{\mathrm{RAM,Z}} = \frac{a_{P,Z}}{f_{\mathrm{RAM}}} \cdot \left(\frac{R_m}{\mathrm{MPa}} \right)^{b_{P,Z}} \tag{30}$$

With material group parameters $a_{P,Z}$ and $b_{P,Z}$ and factor f_{RAM} which includes size effects and a safety concept see [1]. According to Wächter [32] the P-Woehlercurve is fapproximated by

$$N_{\mathrm{Calc,CI}} = 10^3 \cdot \begin{cases} \left(\dfrac{P_{\mathrm{RAM}}}{P_{\mathrm{RAM,Z}}}\right)^{\frac{1}{d_1}}, & P_{\mathrm{RAM}} \geq P_{\mathrm{RAM,Z}} \\ \left(\dfrac{P_{\mathrm{RAM}}}{P_{\mathrm{RAM,Z}}}\right)^{\frac{1}{d_2}}, & P_{\mathrm{RAM}} < P_{\mathrm{RAM,Z}} \end{cases} \tag{31}$$

For the damage accumulation the elementary Miner rule is applied

$$D = \sum_i \frac{n_i}{N_{\mathrm{Calc,CI},i}} \tag{32}$$

Since calculations with the LSA respecting elastic-plastic material behaviour, two different runs through the loading sequence have to be calculated, comp. [1]. During the second run only memory 2 effects will occur and additional hysteresis loops could be closed in comparison to the first run, so the damage sums of the first run D_1 and the second run D_2 can differ. Equation *(33)* calculates the remaining runs through the loading sequence before failure occurs.

$$x = \frac{1 - D_1}{D_2} \tag{33}$$

The fatigue life for crack initiation is

$$N_{\mathrm{Calc,CI}} = (1 + x) \cdot H_0 \tag{34}$$

with the scope of the hysteresis collective H_0 from the second run through the loading sequence.

4.4.2 Damage parameter P_{RAJ}

The damage parameter P_{RAJ} has a fracture mechanically background and is able to consider sequence effects as well as crack opening and closing, see [39], [40], [41], [42]. These effects can be considered by using effective stresses and strains, adapting the crack opening strain and consequently decreasing the fatigue limit.

4.4.2.1 Constant amplitude loading

The damage effect of a closed hysteresis loop is defined by effective stresses and strains. The effective parameters are illustrated in where σ_{op} and $\varepsilon_{\mathrm{op}}$ are the crack opening stress and strain and σ_{cl} and $\varepsilon_{\mathrm{cl}}$ the crack closure stress and strain. Damage occurs only when the crack is open. Therefore effective stresses and strains define the area when the microcrack is open and damages the structure.

$$\Delta\sigma_{\mathrm{eff}} = \sigma_{\max} - \sigma_{\mathrm{cl}} \tag{35}$$

$$\Delta\varepsilon_{\mathrm{eff}} = \varepsilon_{\max} - \varepsilon_{\mathrm{cl}} \tag{36}$$

Thus the damage parameter is based on the effective stresses and strains with

$$P_{\mathrm{RAJ}} = 1.24\cdot\frac{(\Delta\sigma_{\mathrm{eff}})^2}{\mathrm{E}}+\frac{1.02}{\sqrt{\mathrm{n}'}}\cdot(\Delta\sigma_{\mathrm{eff}})\cdot\left((\Delta\varepsilon_{\mathrm{eff}})-\frac{(\Delta\sigma_{\mathrm{eff}})}{\mathrm{E}}\right)=\frac{\Delta J_{eff}}{a} \tag{37}$$

with material parameters E, K' and n'. The stress and strain for crack closure can not be defined directly, but since for constant amplitude loading $\varepsilon_{\mathrm{op}} = \varepsilon_{\mathrm{cl}}$, compare Figure 2, the stress and strain for crack closure can be derived by calculating the opening stresses and strains. For the crack opening stress an approach by Newman [43] can be applied with

$$\sigma_{\mathrm{op}} = \sigma_{\max}\begin{cases}\left(A_0+A_1\cdot R+A_2\cdot R^2+A_3\cdot R^3\right), & R\geq 0\\ (A_0+A_1\cdot R), & R<0\end{cases} \tag{38}$$

with constants A_i, see [1]. For constant amplitude loading the crack opening strain can be determined

$$\varepsilon_{\mathrm{op,const}} = \varepsilon_{\mathrm{cl,const}} = \varepsilon_{\min}+\frac{(\sigma_{\mathrm{open}}-\sigma_{\min})}{\mathrm{E}}+2\cdot\left(\frac{\sigma_{\mathrm{open}}-\sigma_{\min}}{2\cdot\mathrm{K}'}\right)^{1/\mathrm{n}'} \tag{39}$$

The damage parameter Woehlercurve (P-Woehlercurve) can be approximated by the material group and the ultimate tensile strength R_m. The reference point $P_{\mathrm{RAJ,Z}}$ for the damage parameter Woehlercurve is

$$P_{RAJ,Z} = \frac{a_{P,\mathrm{Z}}}{f_{\mathrm{RAJ}}}\cdot\left(\frac{R_m}{\mathrm{MPa}}\right)^{b_{P,Z}} \tag{40}$$

The fatigue limit for the mechanical short crack $P_{\mathrm{RAJ,D,0}}$ can be described by

$$P_{\mathrm{RAJ,D,0}} = \frac{a_{P,D}}{f_{\mathrm{RAJ}}}\cdot\left(\frac{R_m}{\mathrm{MPa}}\right)^{b_{P,D}} \tag{41}$$

for $a_{P,Z}$, $b_{P,Z}, a_{P,D}$ and $b_{P,D}$ and factor f_{RAJ} which considers the size effects and a safety concept, see [1]. Taking the actual fatigue limit $P_{\mathrm{RAJ,D}}$ into account, the P-Woehlercurve is described by:

$$N_{Calc,CI} = \begin{cases}\left(\frac{P_{RAJ}}{P_{RAJ,Z}}\right)^{\frac{1}{d}}, & P_{RAJ} \geq P_{RAJ,D}\\ \infty, & P_{RAJ} < P_{RAJ,D}\end{cases} \tag{42}$$

With the fatigue life for crack initiation $N_{\mathrm{Calc,CI}}$. For constant amplitude loading $P_{\mathrm{RAJ,D}} = P_{\mathrm{RAJ,D,0}}$, for variable amplitude loading see *(48)*.

4.4.2.2 Variable amplitude loading

For variable amplitude loading the crack opening strain is affected by the history of the applied loading sequence. Therefore, two different crack opening strains have to be calculated: $\varepsilon_{\mathrm{op,const}}$, for a fictitious constant amplitude loading using *(39)*, $\varepsilon_{\mathrm{op,hist}}$ the crack opening strain derived by former hysteresis loops. Also four different strains need to be taken into account: $\varepsilon_{\mathrm{max,old,hy}}$ and $\varepsilon_{\mathrm{min,old,hy}}$ as history variables for the maximum and minimum strain of all former hysteresis loops and $\varepsilon_{\mathrm{max,old,sq}}$ and $\varepsilon_{\mathrm{min,old,sq}}$ as history variables for the maximum and minimum strain of all former loading points.

To derive the crack opening strain ε_{op}, different cases have to be checked in this order

1.) If $\varepsilon_{\mathrm{max}} \leq \varepsilon_{\mathrm{op,hist}} \Rightarrow \varepsilon_{\mathrm{op}} = \varepsilon_{\mathrm{op,hist}}$ and $P_{\mathrm{RAJ}} = 0$
2.) a) If $\varepsilon_{\mathrm{max,old,hy}} < \varepsilon_{\mathrm{max,old,sq}} \Rightarrow \varepsilon_{\mathrm{op}} = \varepsilon_{\mathrm{op,const}}$
    b) If $\varepsilon_{\mathrm{min,old,hy}} > \varepsilon_{\mathrm{min,old,sq}} \Rightarrow \varepsilon_{\mathrm{op}} = \varepsilon_{\mathrm{op,const}}$
3.) For $\varepsilon_{\mathrm{op,const}} > \varepsilon_{\mathrm{op,hist}} \Rightarrow \varepsilon_{\mathrm{op}} = \varepsilon_{\mathrm{op,hist}}$
4.) For $\varepsilon_{\mathrm{op,const}} \leq \varepsilon_{\mathrm{op,hist}}$
    a) For $\sigma_a \geq 0.4 \cdot \sigma_F$ is $\varepsilon_{op} = \varepsilon_{\mathrm{op,const}}$
    b) For $\sigma_a < 0.4 \cdot \sigma_F$ is $\varepsilon_{op} = \varepsilon_{\mathrm{op,hist}}$

if one of these cases is fulfilled the crack opening strain is determined. For specific case $\varepsilon_{\mathrm{min}} \geq \varepsilon_{\mathrm{op}}$ the crack opening stress is $\sigma_{\mathrm{op}} = \sigma_{\mathrm{min}} = \sigma_{\mathrm{cl}}$. Else for $\varepsilon_{\mathrm{min}} < \varepsilon_{\mathrm{op}}$ the crack opening stress is determined by

$$\varepsilon_{\mathrm{max}} - \varepsilon_{\mathrm{op}} = \frac{\sigma_{\mathrm{max}} - \sigma_{\mathrm{op}}}{E} + 2 \cdot \left(\frac{\sigma_{\mathrm{max}} - \sigma_{\mathrm{op}}}{2K'}\right)^{1/n'} \tag{43}$$

For case 3 with $\varepsilon_{\mathrm{op,const}} > \varepsilon_{\mathrm{op,hist}}$ the crack opening strain history variable $\varepsilon_{\mathrm{op,hist}}$ needs to be updated afterwards by

$$\varepsilon_{\mathrm{op,hist}} = \varepsilon_{\mathrm{op,const}} - \left(\varepsilon_{\mathrm{op,const}} - \varepsilon_{\mathrm{op}}\right) \cdot \exp(15 \cdot N_{\mathrm{CI,const}}^{-1}) \tag{44}$$

with $N_{\mathrm{CI,const}}$ the fatigue life for the current closed hysteresis loop calculated with *(42)*. For any other case, the new history crack opening strain variable $\varepsilon_{\mathrm{op,hist}}$ is set to

$$\varepsilon_{\mathrm{op,hist}} = \varepsilon_{\mathrm{op}} \tag{45}$$

For damage parameter P_{RAJ} a mechanical short crack with crack length a_0 is assumed in the structure before the loading sequence is applied.

$$a_0 = \left(a_{\mathrm{end}}^{1-m_J} - (1-m_J) \cdot C \cdot P_{\mathrm{RAJ,Z}}^{m_J}\right)^{\frac{1}{1-m_J}} \tag{46}$$

This mechanical short crack is propagating when the loading sequence is applied until its length is equal to the failure criterion crack initiation a_{end}. Material parameters of crack propagation C and m can be determined using an approach by [14] with

$$C = 10^{-5}\text{mm} \cdot \left(5 \cdot 10^5 \frac{1}{\text{mm}}\right)^{m_J} \cdot (E)^{-m_J} \tag{47}$$

and $m_J = -\frac{1}{d}$ while d is the exponent of the P-Woehlercurve. The mechanical short crack opens and closes and growths until the crack length a_{end} is achieved. This effect is taken into account in the calculation of the decreasing fatigue limit $P_{\text{RAJ,D}}$ with

$$P_{\text{RAJ,D}} = \Delta J_{\text{eff,th}} \cdot \left(\left(\left(a_{\text{end}}^{1-m_J} - a_0^{1-m_J}\right) \cdot D + a_0^{1-m_J}\right)^{\frac{1}{1-m_J}} + \frac{\Delta J_{\text{eff,th}}}{P_{\text{RAJ,D,0}}} - a_0\right)^{-1} \tag{48}$$

which is equivalent to the consequent Miner rule with actual damage sum D. The threshold for the effective ΔJ can be approximated by

$$\Delta J_{\text{eff,th}} = \frac{E}{\left(5 \cdot 10^6 \frac{1}{\text{mm}}\right)} \tag{49}$$

The damage accumulation rule for variable amplitude loading is equivalent to a consequent Miner rule. First of all, the maximal possible P_{RAJ} and the fatigue limit for crack initiation $P_{\text{RAJ,D,e}}$ has to be determined and 200 classes have to be created, comp. [39].

$$\frac{P_{\text{RAJ,max,class}}}{P_{\text{RAJ,1}}} = \frac{P_{\text{RAJ,1}}}{P_{\text{RAJ,2}}} = \cdots = \frac{P_{\text{RAJ,199}}}{P_{\text{RAJ,D,e}}} \tag{50}$$

Second, the actual fatigue limit $P_{\text{RAJ,D}}$ has to be classified within this 200 classes.

$$P_{\text{RAJ},q-1} \leq P_{\text{RAJ,D}} < P_{\text{RAJ},q} \tag{51}$$

Third, with the class index q the amount of cycles to crack initiation after the second run through the loading sequence N_{-2} can be determined using

$$N_{-2} = H_0 \cdot \sum_{j=q}^{200} \frac{f(j+1) - f(j)}{\sum_{i=1}^{j} \frac{h_i}{N_{\text{Calc,CI},i}}} \tag{52}$$

With

$$f(j) = \frac{a_0^{1-m_J} - \left[\left(\frac{\Delta J_{\text{eff,th}}}{P_{\text{RAJ},j}} + \frac{\Delta J_{\text{eff,th}}}{P_{\text{RAJ,D,0}}}\right) - a_0\right]^{1-m_J}}{a_0^{1-m_J} - a_{\text{end}}^{1-m_J}} \tag{53}$$

So the fatigue life for crack initiation is

$$N_{\text{Calc,CI}} = 2 \cdot H_0 + N_{-2} \tag{54}$$

with the scope of the hysteresis collective H_0 from the second run through the loading sequence.

5. The U-Concept: An Add-On to the Local Strain Approach to consider crack propagation effects for fracture fatigue life calculations

In order to apply the LSA to failure criterion fracture, the influence of crack propagation to the fatigue life needs to be investigated. Therefore, a simplification of the crack growth procedure of the fracture mechanical FKM guideline needs to be derived.

The total J-Integral following *FKM fracture mechanical guideline* [3] is

$$\Delta J_{\mathrm{eff,total}} = \Delta J_{\mathrm{elastic}} + \Delta J_{\mathrm{plastic}} = \Delta J_{\mathrm{elastic}} \cdot \left(1 + Y_{\mathrm{pl}}\right) \qquad (55)$$

With Eq. (21), the plastic part of the geometric correction Y_{pl} depends only on the crack length with decreasing $K_t(a)$. For a first approximation this paper neglects the decreasing of the notch effect and assumes

$$K_t(a) = K_t = \mathrm{const.} \qquad (56)$$

To further make the resulting function independent of the nominal stresses and only depending on the crack length a, the total J-Integral $\Delta J_{\mathrm{eff,total}}$ is divided through the J-Integral for crack initiation $\Delta J_{\mathrm{eff,total}}(a_{CI} = 0.5)$:

$$\frac{\Delta J_{\mathrm{eff,total}}}{\Delta J_{\mathrm{eff,total}}(a_{CI} = 0.5)} = r_J(a) \qquad (57)$$

Therefore, the crack propagation law can be modified to

$$\frac{da}{dN} = C_j \cdot \left(\Delta J_{\mathrm{eff,total}}\right)^{m_j} = C_j \cdot \left(r_J(a) \cdot \Delta J_{\mathrm{eff,total}}(a_{CI} = 0.5)\right)^{m_j} \qquad (58)$$

Respecting furthermore *(37)* and *(42)*, the crack propagation law is simplified to

$$\frac{da}{dN} = C_j \cdot \left(r_J \cdot 0.5 \cdot P_{\mathrm{RAJ,Z}} \cdot N_{\mathrm{Calc,CI}}^{-1/m_j}\right)^{m_j} = C_j \cdot \left(r_J^{m_j} \cdot 0.5^{m_j} \cdot P_{\mathrm{RAJ,Z}}^{m_j} \cdot N_{\mathrm{Calc,CI}}^{-1}\right) \qquad (59)$$

With material constants C_j, m_j, the grid point of the crack initiation Woehlercurve $P_{\mathrm{RAJ,Z}}$ and the calculated fatigue life for crack initiation $N_{\mathrm{Calc,CI}}$. Integrating the crack propagation law leads to

$$N_{\mathrm{fracture}} = N_{\mathrm{Calc,CI}} + \left(C_j \cdot 0.5^{m_j} \cdot \left(\frac{P_{\mathrm{RAJ,Z}}^{m_j}}{N_{\mathrm{Calc,CI}}}\right)\right)^{-1} \cdot \int_{a_{\mathrm{CI}}}^{a_{\mathrm{fracture}}} r_J^{-m_J}\, da \qquad (60)$$

And

$$N_{\mathrm{fracture}} = N_{\mathrm{Calc,CI}} \cdot \left(1 + \left(C_j \cdot 0.5^{m_j} \cdot P_{\mathrm{RAJ,Z}}^{m_j}\right)^{-1} \cdot \int_{a_{CI}}^{a_{\mathrm{fracture}}} r_J^{-m_J}\, da\right) \qquad (61)$$

While $\left(C_j \cdot 0.5^{m_j} \cdot P_{\mathrm{RAJ,Z}}^{m_j}\right)^{-1}$ is independent of the crack length,

$$f_r = \int_{a_{\mathrm{CI}}}^{a_{\mathrm{fracture}}} r_J^{-m_J}\, da \qquad (62)$$

needs to be determined. Considering the behaviour of an *a*-*N*-curve, for the last part of the crack growing before fracture occurs, the crack length a increases extremely while the value of the fatigue life $N(a)$ hardly increases in comparison to its total value. Therefore, this paper defines a technical fracture crack length a_{tf} as failure criterion and neglects the increase of fatigue life between technical and real fracture. Since function $r_J(a)$ is independent of the nominal stress and only depends on the crack length a and with the assumption of the technical fracture crack length a_{tf}, the function f_r is

$$f_r = \int_{a_{\mathrm{CI}}}^{a_{tf}} r_J^{-m_J} \,\mathrm{d}a = \mathrm{const.} \tag{63}$$

Therefore, equation (61) can be simplified to

$$N_{\mathrm{Calc,frac}} = N_{\mathrm{Calc,CI}} \cdot \left(1 + \left(C_j \cdot 0.5^{m_j} \cdot P_{\mathrm{RAJ,Z}}^{m_j} \right)^{-1} \cdot \int_{a_{\mathrm{CI}}}^{a_{\mathrm{tf}}} r_J^{-m_J} \,\mathrm{d}a \right) = f_{\mathrm{const}} \cdot N_{\mathrm{Calc,CI}} \tag{64}$$

So it's possible to approximate a fatigue life for technical fracture based on the fatigue life for crack initiation using a constant shift of the Woehlercurve by factor f_{const}.

Even so, the approximation of Eq. (56) seems rather hard and Eq. (64) also neglects the difference in the slope of Woehlercurves between crack initiation and fracture. To take the influence of the notch to the stress field for a growing crack into account, this means a decreasing of $K_t(a)$, approximation formulas can be used which consider the elastic stress concentration factor K_t, the crack length *a* and the nominal stress *S* when calculating the elastic stress at the crack tip, see [25]. Since the elastic stress concentration factor K_t, the starting crack length a_{CI} and the technical fracture crack length a_{tf} are independet of the stress level, the change in the stress field is here assumed to be independet of the stress level, too. It is therefore already included in Eq. (64).

To consider the different slopes between the Woehler curves for crack initiation and fracture and of course to correct deviations made by the simplifications of this approach, a correction term is introduced which is able to modify the slope of the Woehler curve. This leads to a crack propagation factor

$$\bar{f}_U = f_{\mathrm{const}} \cdot f_U \tag{65}$$

First factor f_{const} considers a constant shift of the damage parameter Woehler curve and the second factor f_U describes a correction of the slope between the Woehler curves from crack initiation to fracture. While a fatigue life calculation for crack initiation with damage parameter P_{RAJ} is able to consider crack opening and closing effects and also crack propagation, the calculation with damage parameter P_{RAM} neglects these effects and only considers the influence of elastic-plastic material behaviour, comp. section 5.1.2. So based on the difference in complexity of both approaches, the complexity of the variations of the U-Concept differ, too. While the U-Concept for P_{RAJ} considers both effects of crack propagation for constant amplitude loading, the U-Concept for P_{RAM} only considers a constant shift of the damage parameter Woehlercurve. The factors were determined by non-artificial intelligence (n-AI) using as training set a large database of experimental results for constant amplitude loadings, see [44], and compared and validated with experimental results from the literature [37, 45].

5.1 Constant amplitude loading

Based on the crack propagation factor the fatigue life for technical fracture is calculated by

$$N_{\text{Calc, fr}} = \bar{f}_U \cdot N_{\text{Calc, CI}} \tag{66}$$

5.1.1 Damage parameter P_{RAJ}

For damage parameter P_{RAJ} the crack propagation factor $\bar{f}_{U,\text{RAJ}}$ can be determined by

$$\bar{f}_{U,RAJ} = \max(1;\ f_{\text{RAJ,const}} \cdot f_{\text{RAJ},U}) \tag{67}$$

With factor $f_{\text{RAJ,const}}$ which considers a constant shift of the damage parameter Woehler curve and factor $f_{\text{RAJ},U}$ which considers the correction of the slope.
The constant shift of the P_{RAJ}-Woehlercurve is described by

$$f_{\text{RAJ,const}} = f(R) \cdot K_p \cdot \max(K_p; 2) \tag{68}$$

With the plastic load factor K_p and the function

$$f(R) = \begin{cases} 0.25 & \text{for } R > 0.5 \\ \dfrac{1-R}{2} & \text{for } 0.5 \geq R \geq -3 \\ 2 & \text{for } R < -3 \end{cases} \tag{69}$$

Which depends only on the nominal stress ratio R of the applied loading sequence.

The slope correction factor $f_{\text{RAJ,U}}$ is depending on the elastic-plastic calculated stresses in the notch and the material parameter Rm.

$$f_{\text{RAJ},U} = \left(10 \cdot \frac{f_{\text{Bending}}}{K_p}\right)^{a_U} \cdot (2 - U_\sigma)^3 \cdot U_\sigma^{d_U} \tag{70}$$

For a closed hysteresis loop with stress amplitude σ_a, stress ratio $R_\sigma = \sigma_{\min}/\sigma_{\max}$ and mean stress σ_m the variable U_σ is

$$U_\sigma = \max\left(\frac{\sigma_a}{R_m}; \frac{\sigma_a + \sigma_m}{R_m}\right) \tag{71}$$

With exponents

$$a_U = \begin{cases} 1 & \text{for } R_\sigma \leq -1 \\ -R_\sigma & \text{for } 0 > R_\sigma > -1 \\ 0 & \text{for } R_\sigma \geq 0 \end{cases} \tag{72}$$

and

$$d_U = \begin{cases} 5 & \text{for } R_\sigma \leq -1 \\ 4 - R_\sigma & \text{for } 0 > R_\sigma > -1 \\ 4 & \text{for } R_\sigma \geq 0 \end{cases} \tag{73}$$

The shape of the function $f_{\text{RAJ},U}$ is similar to experimental results by Saal [45], who plotted the factor $f = N_{\text{fract}}/N_{\text{CI}}$ over the nominal stress amplitude for notched specimens of steel.

To consider the influence of bending to the crack propagation, factor f_{Bending} is proposed and described by

$$f_{\text{Bending}} = K_P/K_t \tag{74}$$

For specimens with circular cross-section according to FKM guideline linear is $f_{\text{Bending}} = 1.7$.

5.1.2 Damage parameter P_{RAM}

The crack propagation factor is

$$\bar{f}_{U,\text{RAM}} = \max\left(1; f_{\text{RAM,const}}/f_{\text{tensile}}\right) \tag{75}$$

A slope correction can be neglected because of the good accordance to experimental results, see section results. The constant shifting factor $f_{\text{RAM,const}}$ is depending on the plastic load factor K_p, the function $f(R)$ see *(69)*, and the ultimate tensile strength.

$$f_{\text{RAM,const}} = f(R) \cdot \begin{cases} 0.5 \cdot K_p^{1.4} \cdot \max(K_p\,;2) & \text{for } R_m \leq 600 \text{ MPa} \\ K_p \cdot \max(K_p\,;2) & \text{for } R_m > 600 \text{ MPa} \end{cases} \tag{76}$$

For constant amplitude loadings with $R > -1$, the shifting factor is adjusted by

$$f_{\text{tensile}} = \begin{cases} 1 & , \quad \text{for } R \leq -1 \\ K_p^{(1+R)\cdot 0.4}, & \text{for } R_m \leq 600 \text{ MPa}, 0 \geq R > -1 \\ K_p^{0.4} & , \quad \text{for } R_m \leq 600 \text{ MPa}, R > 0 \\ K_p^{(1+R)\cdot 0.2}, & \text{for } R_m > 600 \text{ MPa}, 0 \geq R > -1 \\ K_p^{0.2} & , \quad \text{for } R_m > 600 \text{ MPa}, R > 0 \end{cases} \tag{77}$$

5.2 Variable amplitude loading

The factor f_{const} is depending on the stress ratio R of the applied loading sequence. Instead of defining $f(R)$ for every cycle of a variable loading sequence, a mean value is calculated with the nominal stress ratio R_i of every closed hysteresis loop i

$$f_{R,\text{var}}(\bar{R}) = \sum_i \frac{f_{R,\text{const}}(R_i)}{H_0} \tag{78}$$

The function $f_R(\bar{R})$ considers the influence of the nominal stress ratio $\bar{R}$ on the crack growth rate under variable amplitude loading

$$f_R(\bar{R}) = \min\left(f_{R,\text{var}}(\bar{R}); f_{R,\text{var}}(\bar{R} = -1) \cdot f_{R,\text{const}}(R = \bar{R})\right) \tag{79}$$

With $f_{R,\text{const}}(R)$ equivalent to Eq. (69). With the introduction of modified effective Miner sums the accuracy of the calculated technical fatigue lives improved in [46]. Those modified effective Miner sums $D_{\text{eff,NL}}$are multiples of the effective Miner sums $D_{\text{eff,FKM}}$ of FKM guideline linear [2].

$$D_{\text{eff,NL}} = f_{\text{meth}} \cdot D_{\text{eff,FKM}} \tag{80}$$

The method correction factor f_{meth} considers the differences of the applied U-Concept variation. For the KP-approach the method correction factor is defined by

$$f_{\mathrm{meth}} = \begin{cases} 0.5, & \text{for damage parameter } P_{\mathrm{RAM,KP}} \\ 2, & \text{for damage parameter } P_{\mathrm{RAJ,KP}} \\ 3, & \text{for damage parameter } P_{\mathrm{RAJ,U}} \end{cases} \tag{81}$$

5.2.1 Damage accumulation

Also for failure criterion fracture, a damage accumulation with an elementary or consequently Miner rule can be applied. Here, no further changes are necessary. In [46] the influence of the crack length a_{end} to the calculation results under variable amplitude loading was investigated. The given database showed no remarkable influence of changing a_{end} to the fatigue life calculation.

6. Results

This section shows the results of calculations with the proposed U-Concept in comparison to experimental results from the literature and in comparison to calculations based on the local stress concept of the FKM guideline linear for notched specimens of steel under constant or variable amplitude loading. For damage parameter P_{RAJ} results for two different approaches are discussed. Approach $P_{\mathrm{RAJ,U}}$ which considers the slope correction and the constant shift of the Woehler curve and approach $P_{\mathrm{RAJ,KP}}$which considers only a constant shift of the Woehler curve and neglects a slope correction. So calculations with the $P_{\mathrm{RAJ,KP}}$ or $P_{\mathrm{RAM,KP}}$ approach predicts the same slopes for crack initiation and fracture. All calculation results are calculated with estimated Woehler curves with failure probability $P_f = 50\%$. The diagrams also show calculation results with the algorithms of the FKM guideline linear (FKM_{linear}) for failure probability $P_f = 50\%$ based on linear-elastic material behavior. All calculations depend only on one material parameter: the ultimate tensile strength.

6.1 Constant amplitude loading

In [25] crack growth simulations were performed for experiments by [45] in order to determine the fatigue lives for fracture for specimens under constant amplitude loading with different stress ratios. The results of this crack growth calculations (straight lines) in comparison with experiments and in compared with calculation results with the U-Concept (dotted lines) are presented in Figure 3. The calculations with the U-Concept fit the experimental data as well as the results of the crack growth simulation for both notched geometries for stress ratios $= 0, -1, -3$. Figure 4 shows experimental results from a structural durability database of sources [44], [45], [47] in comparison to calculated results with the U-Concept for damage parameter P_{RAM}. The database contains of more than 1500 experiments for notched steel specimens $(1.01 \leq K_p \leq 22.2)$ under constant amplitude loading with a fracture fatigue life less than 1 million cycles. Figure 4 shows the good accordance ($T_N < 5$) of the calculated results by the U-Concept in comparison with the experimental results of the DaBef.

6.2 Variable Amplitude loading

For more than 750 experiments under variable amplitude loading from the DaBef [44] the fatigue lives for fracture were approximated by the U-Concept. Table 1 gives an overview about the different loadings sequences, Figure 5 shows the results of calculations with damage parameter P_{RAJ} and the U-Concept. Comparing the diagrams on top and bottom show a smaller scatter for U-Concept variante $P_{\mathrm{RAJ},KP}$. Figure 6 and Figure 7 show the results for two different loading sequences: Gaussian distribution and MiniTwist. The S-N-diagramms show clearly the better fit of the simpler concept $P_{\mathrm{RAJ},KP}$. There are two different reasons for this behaviour. First of all, the database for fitting of the slope correction function mostly based on experimental results for stress ratios R=-1 or R=0. The slope correction function may therefore not yet have been sufficiently investigated and adapted for the other stress ratios, which lead to the differences in the prediction. The second reason is that the slope correction function describes the full difference between the Woehlercurves crack initiation and fracture which occurs during crack propagation. During a variable amplitude loading, different stress amplitudes are applied to the component, so it seems clear that the difference in the slope for a stress horizon may not fully appear in variable amplitude loading. So for loading sequences with many different stress horizons the slope correction doesn't need to be applied.

Figure 8 shows the comparison of the different approaches of the U-Concept and the FKM guideline linear for damage sum $D = D_{\mathrm{eff,NL}}$. Through the application of an effective miner sum, the scatter decreases, compare Figure 5 with Figure 8, and the mean value is set to 1.0 for the consequent approach. Although the scatter $T_{N,\mathrm{consequent}} > T_{N,\mathrm{elementary}}$ for both approaches of P_{RAJ}, the consequent approaches show a better fit to the course of the experimental results in S-N-diagrams in the HCF area, see Figure 7.

7. Conclusions

This paper proposed the U-Concept, a simple approach to calculate the fatigue life for fracture with the Local Strain Approach. The U-Concept only requires the ultimate tensile strength as material parameter and is useable for unwelded mechanical components of steel under constant or variable amplitude loading. It proposes a crack propagation factor between crack initiation and fracture which depends on the ultimate tensile strength and modifies the damage parameter Woehlercurves. For a better fit with experiments, an effective miner sum can be determined for variable amplitude loading. The presented results showed a good accordance between experimental and calculated fatigue lives for constant and variable amplitude loading even in comparison to crack growth simulations.

The U-Concept can only be used in conjunction with the FKM guideline nonlinear. The fatigue life calculations for technical crack initiation according to the guideline nonlinear are based on approximated damage parameter Wöhler curves and approximated cyclic material behavior using tensile strength. The U-Concept is not validated in combination with experimental Wöhler curves and material parameters, the approximation quality of fatigue life for fracture can therefore be different. The influences of size effects and surface roughness are considered within the FKM guideline nonlinear. If components are to be evaluated that differ greatly in size or surface condition, or both, from those in the database used here, the U concept cannot be applied without further verification. The database typically contains results for machine components with dimensions in the centimeter and decimeter range. Surface roughnesses range from polished to roughness depths of $R_z \approx 30\,\mu m$. Other effects or special cases influencing the fatigue life of components like residual stresses due to surface treatment, overloading, welding, etc. are not taken into account and form the base for further research. Other effects or special cases affecting the fatigue life of components, such as residual stresses due to surface treatment, overloading, welding, etc. are not considered and form the basis for further investigations. If fatigue life calculations with the U-Concept lead to significantly longer fatigue lives compared to fatigue lives calculated with the FKM guideline linear, but are within the validity range of the guideline, the fatigue lives for fracture should be verified experimentally.

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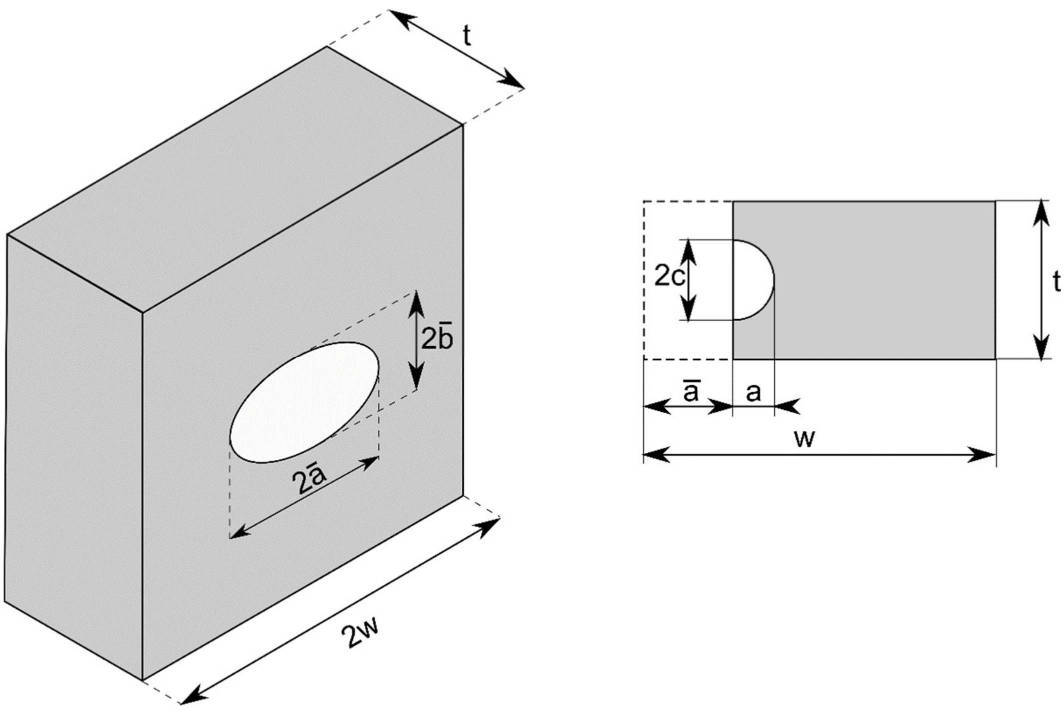

Figure 1: Geometry for the crack growth simulation

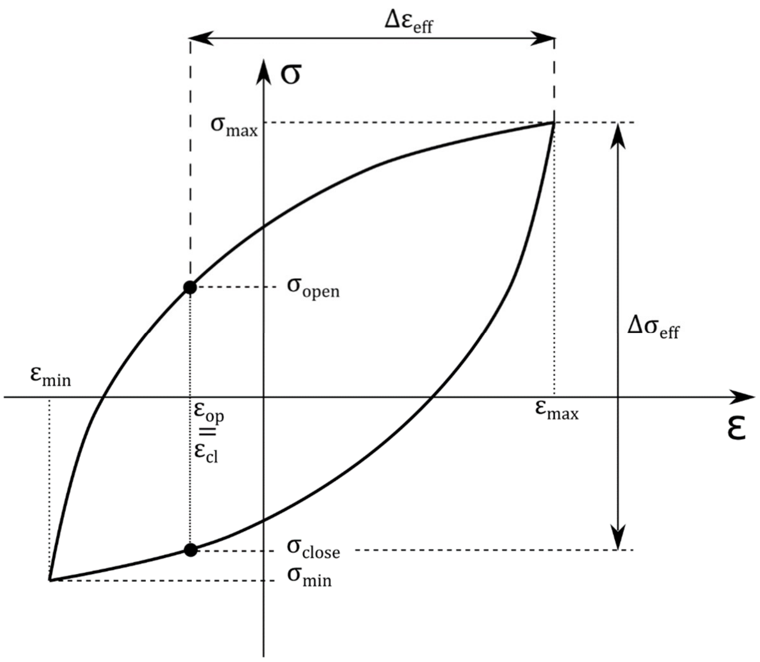

Figure 2: Effective stresses and strains for damage parameter P_{RAJ}

Table 1: Variable amplitude loading sequences from DaBef [44]

Sequence form	$\bar{R}$ **in [-]**	K_t **in [-]**	**No. of Experiments**
Normally distributed	-1 and 0	1.01 – 3.48	229
Linear distributed	-1 and 0	2.08 – 3.31	133
MiniTwist	-0.23	2.53	16
Unsymmetrical	-1	2.19, 2.53	16
Alternating	-0.5	2.19, 2.53	47
Carlos	0 and -1	1.49 – 3.48	156
Increasing	-1	2.19	18
More than normally distributed	-1	2.19	42

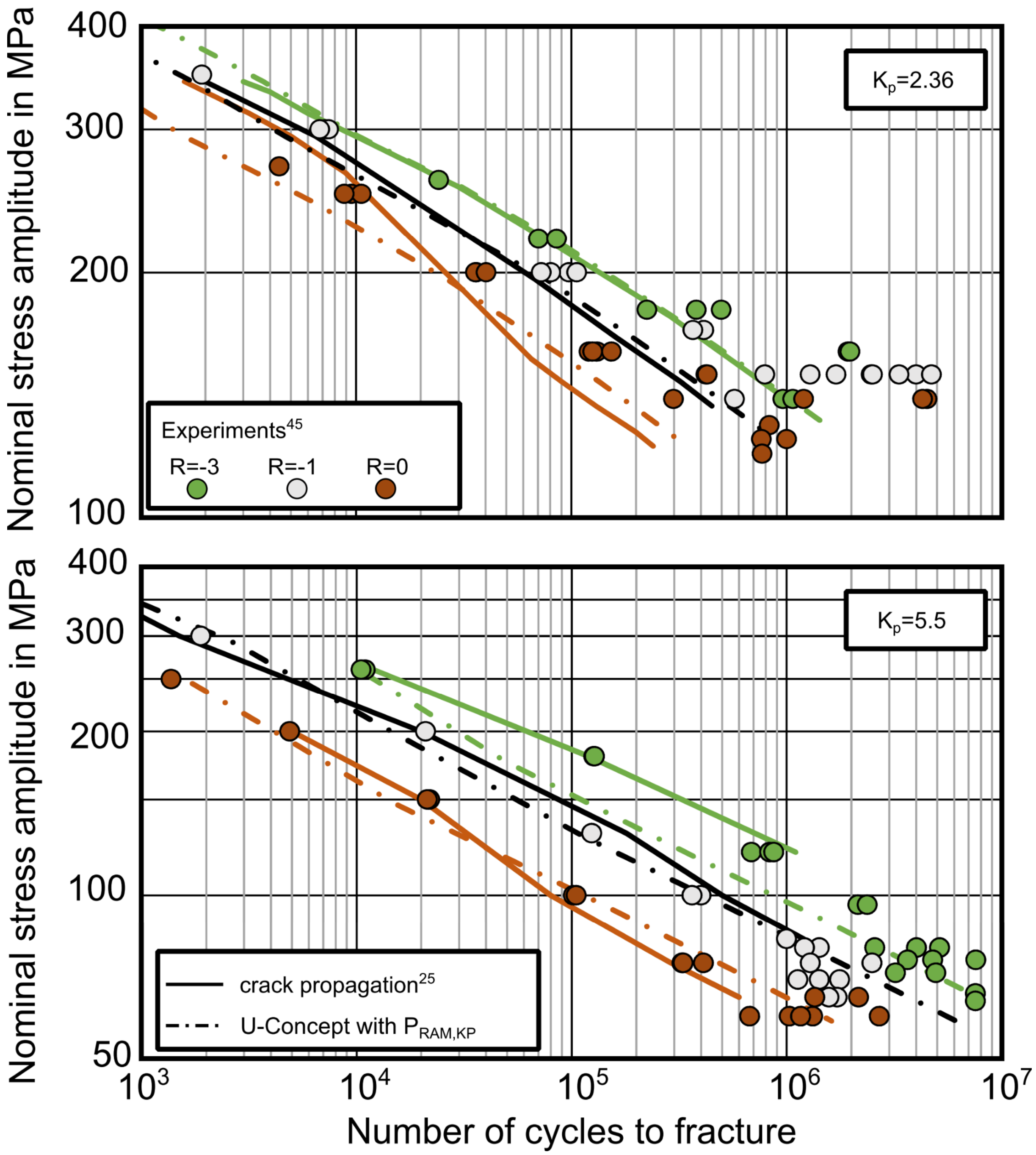

Figure 3: Comparison of experimental results under constant amplitude loading [45] to calculation results with the U-Concept version $P_{\mathrm{RAM},KP}$ and with calculation results by Savaidis [25] from a crack propagation calculation. Top: For $K_p = 2.36$, Bottom: For sharp notched $K_p = 5.5$

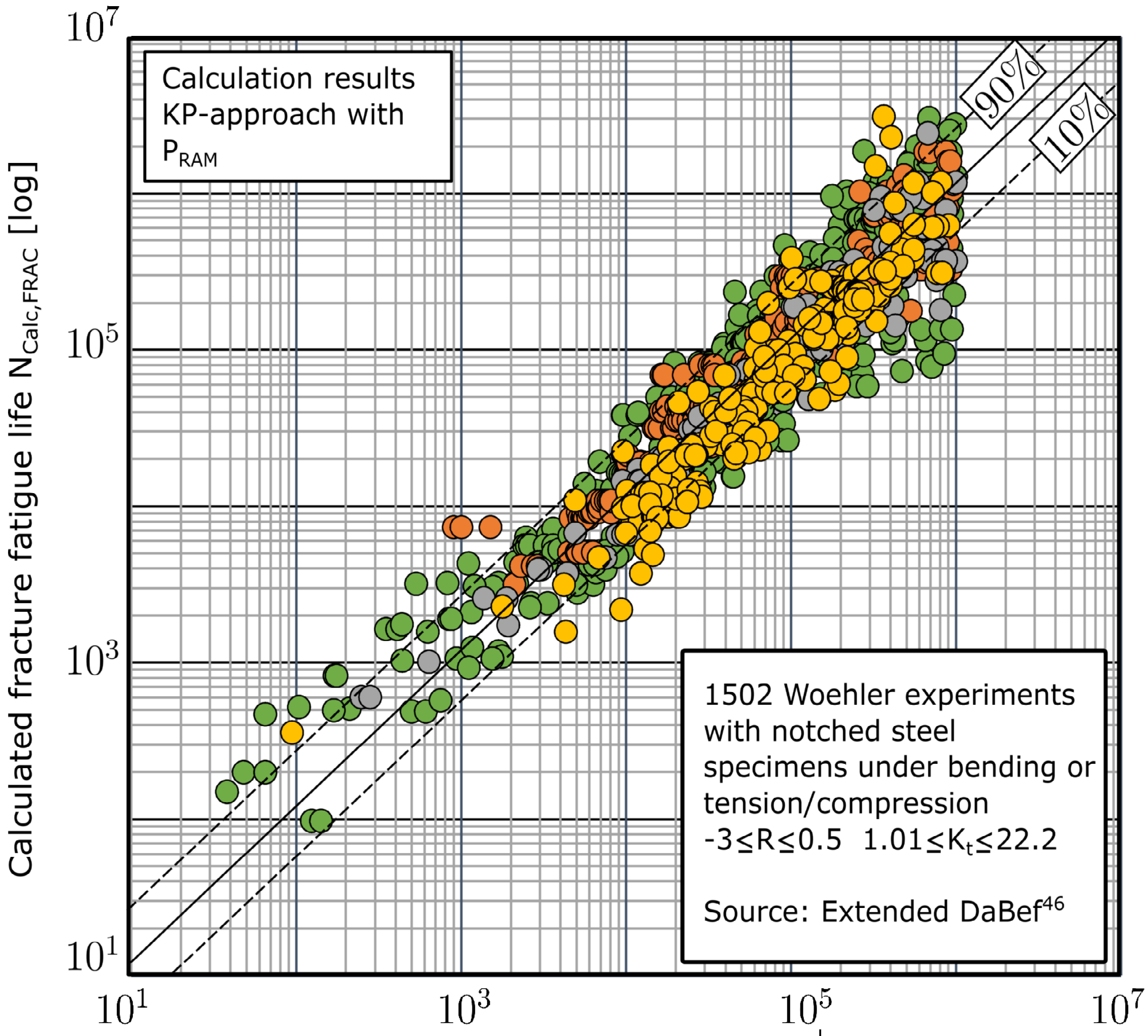

Figure 4: Comparison of experiments with calculation results for notched specimens of steel under constant amplitude loading (bending and tension/compression), [46]

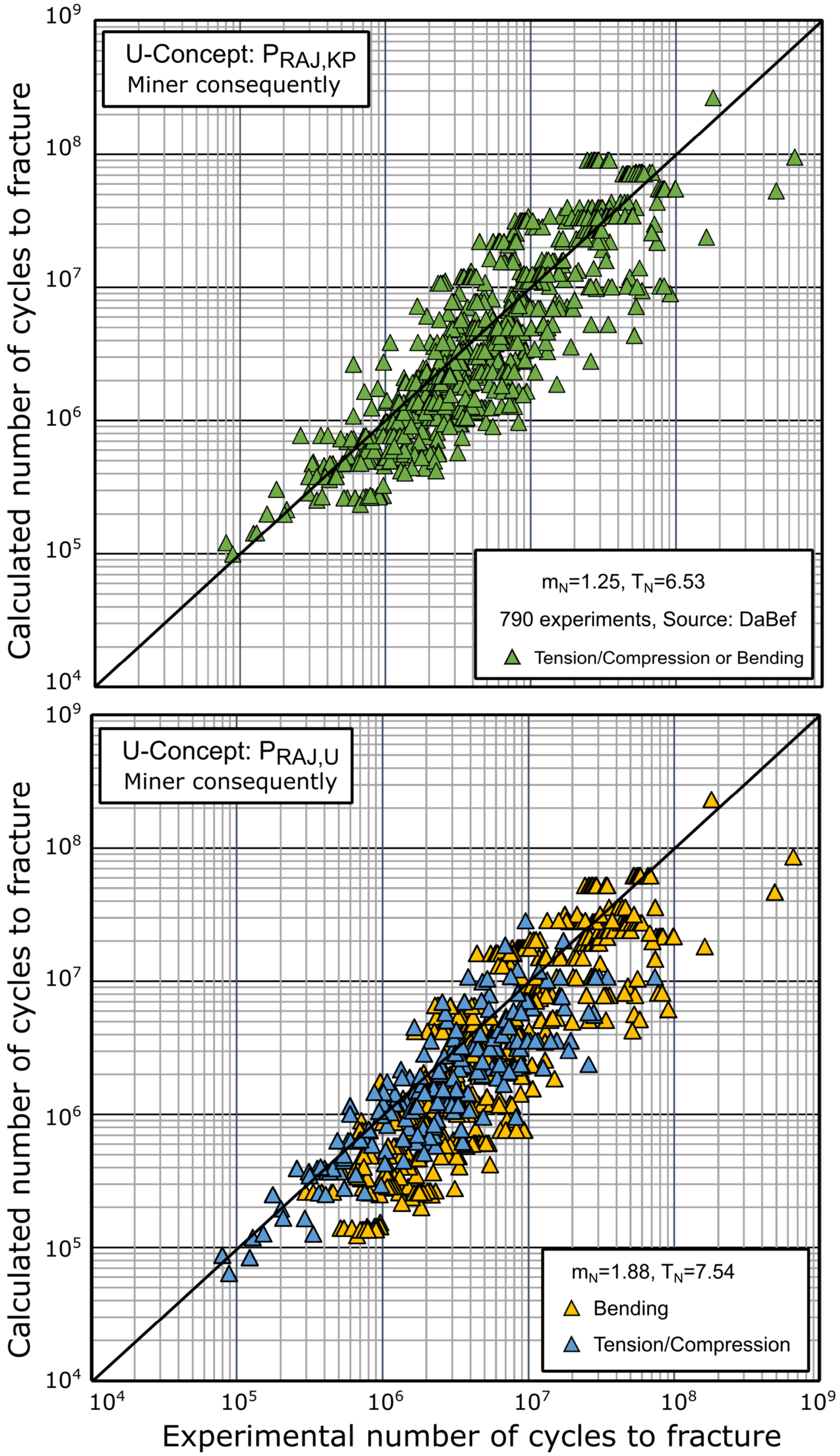

Figure 5: Comparison of experimental results under variable amplitude loading from DaBef [44] with calculations with the U-Concept with $P_{\mathrm{RAJ},KP}$ (top) and $P_{\mathrm{RAJ},U}$ (bottom) for 750 experiments.

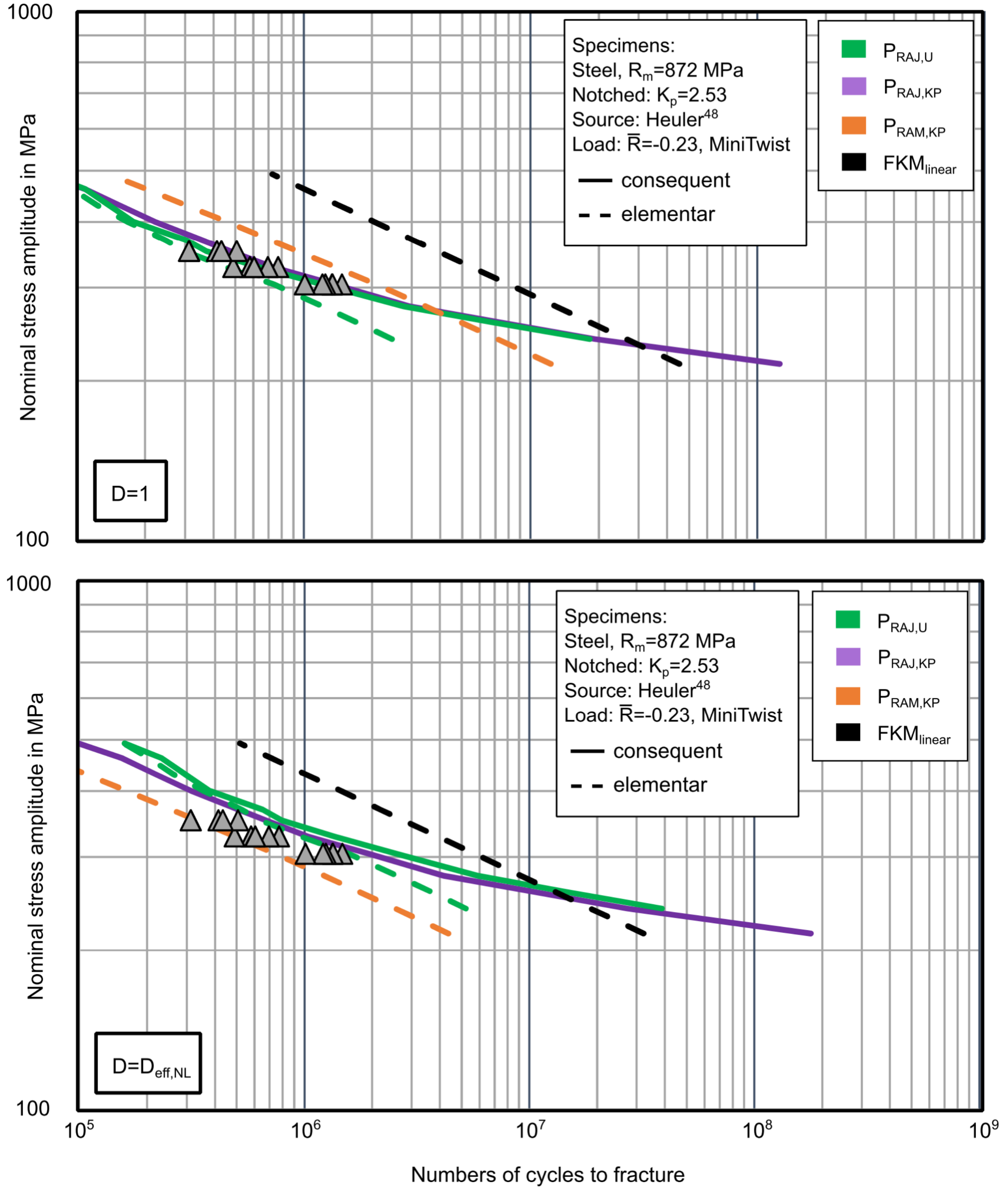

Figure 6: Comparison of experimental results under variable amplitude loading from Heuler [48] to calculation results with the U-Concept and the FKM guideline linear. Top: For Miner sum $D = 1$*, Bottom: For effective Miner sum* $D = D_{eff,NL}$

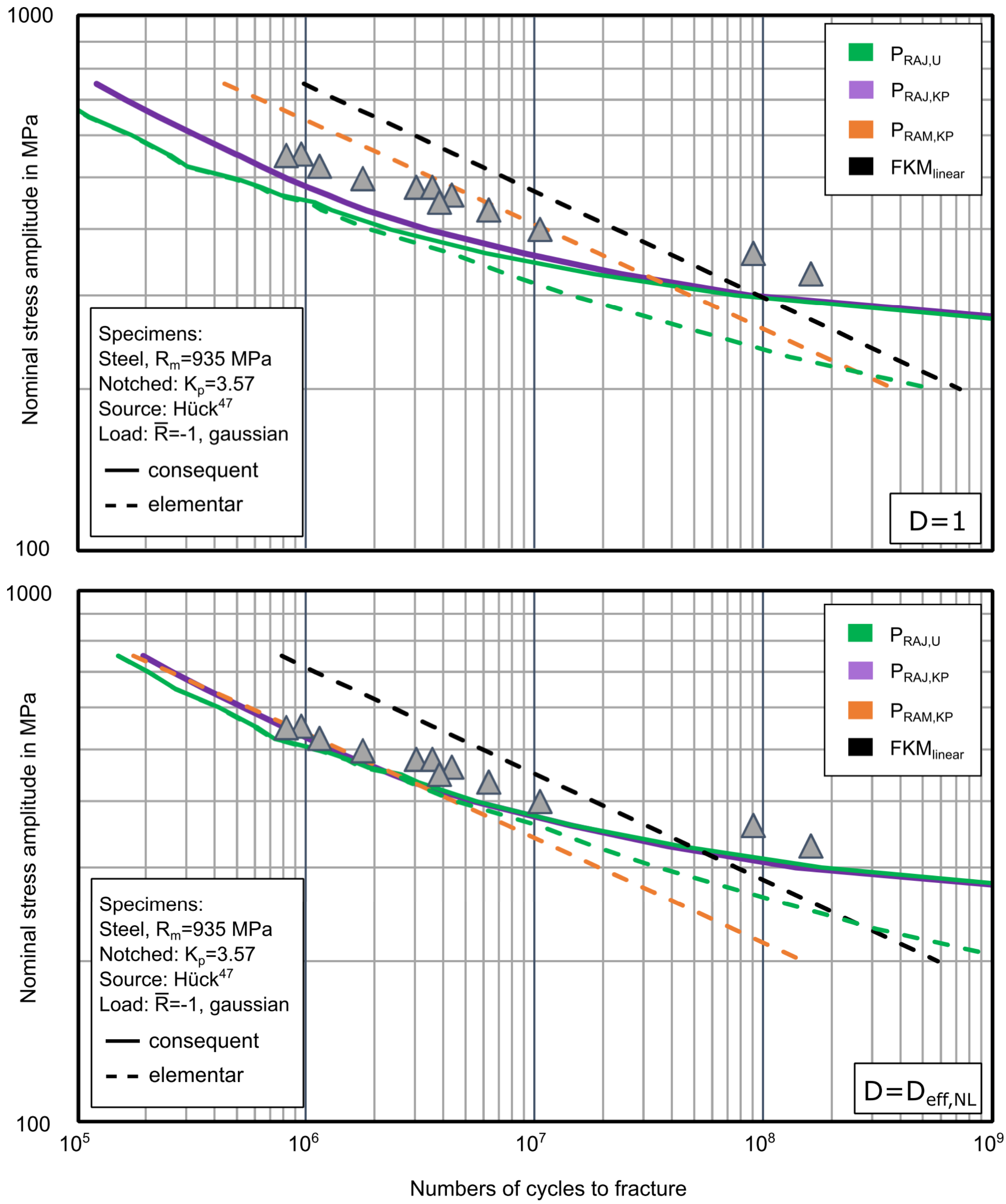

Figure 7: Comparison of experimental results under variable amplitude loading from Hück and Bergmann [47] to calculation results with the U-Concept and the FKM guideline linear. Top: For Miner sum $D = 1$, Bottom: For effective Miner sum $D = D_{\mathrm{eff,NL}}$

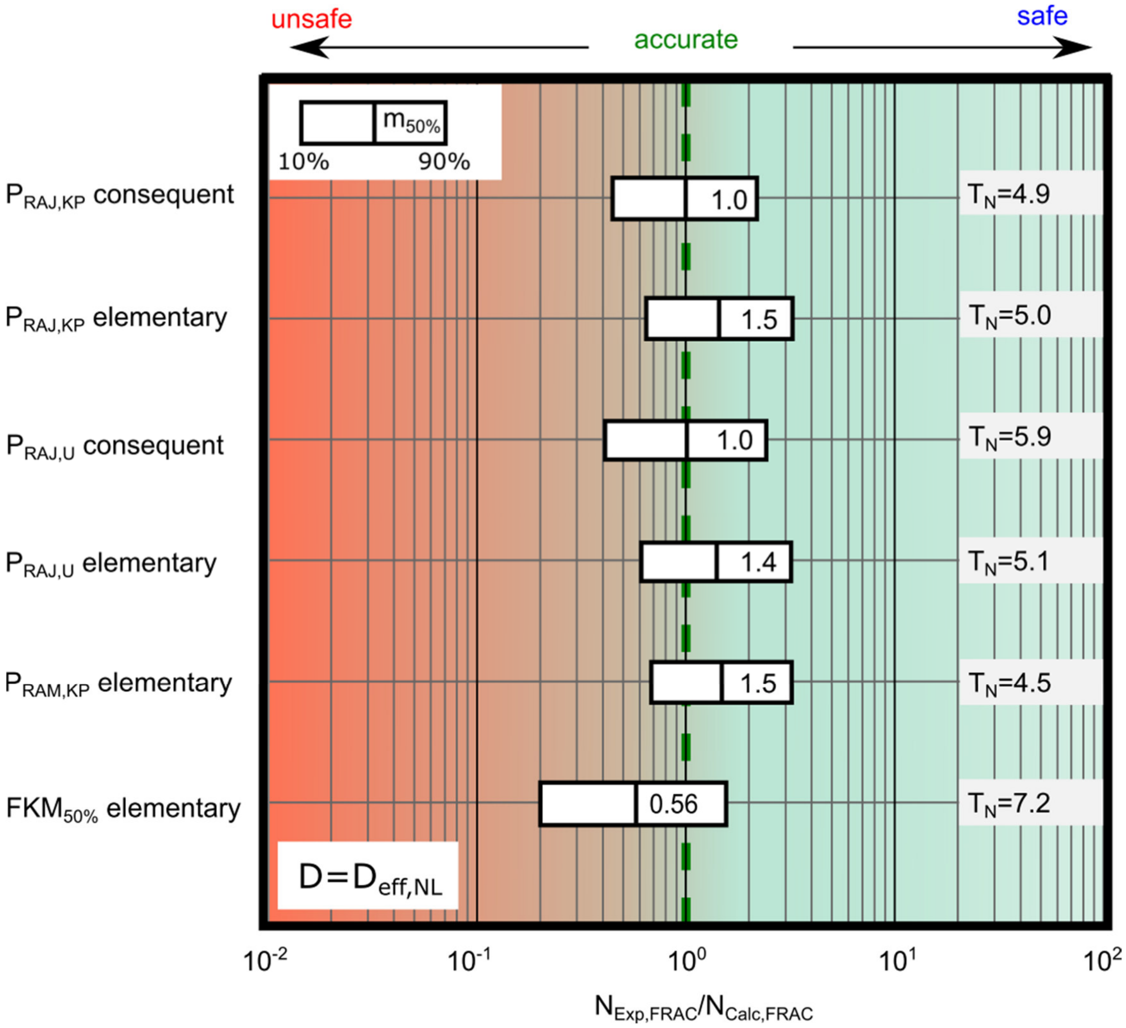

Figure 8: Statistical analysis of a database of n=790 experimental results for variable amplitude loading including loadings sequence types Gaussian distribution, linear distribution, MiniTwist, CARLOS and more for a probability of failure of $P_f = 50\%$ of the Woehlercurves, calculated with modified effective damage sums $D = D_{\mathrm{eff,NL}}$, [46].