Solutions of Kerr Black Holes subject to Naked Singularity and Wormholes

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Abstract: The existence of the "Naked Singularity" has been shown taking the advantage of the Ring Singularity of the Kerr Black Hole and thereby making the way to manipulate the mathematics by taking the larger root of Δ as zero and thereby vanishing the ergosphere and event horizon making the way for the naked ring singularity which can be easily connected via a cylindrical wormhole and as 'a wormhole is a black hole without an event horizon' therefore, this cylindrical connection paved the way for the Einstein-Rosen Bridge allowing particles or null rays to travel from one universe to another ending up in a future directed Cauchy horizon while changing constantly from spatial to temporal and again spatial paving the entrance to another Kerr Black hole (which would act as a white hole) in the other universes.

Key Words: Axially-Symmetric Black Hole - Quasipotential Event Horizon - Cosmic Censorship Hypothesis, Naked Singularity – Hoop - ANEC

Introduction: This paper is totally based on the mathematical physics of the Black holes. In Einstein's theory of "General Relativity", Schwarzschild solution is the vacuum solutions of the Einstein Field Equations that describes the gravity potential from outside the body of a spherically symmetric object having zero charge, zero mass and zero cosmological constant[1]. It was discovered by Karl Schwarzschild in 1916, a little more than a month after the publication of the famous GR and the singularity is a point singularity which can be best described as a coordinate singularity rather than a real singularity, however, the drawback of this theory is that it fails to take into account the real life scenario of black holes with charge and spin angular momentum. The black hole is based on event horizon and Schwarzschild radius. However, Physicists were trying to develop a metric for the real life scenario of a black hole with a spin angular momentum and ultimately the exact solution of a charged rotating black hole had been discovered by Roy Kerr in 1965 as the Kerr-Newman metric[2][3]. The Kerr metric is one of the toughest metric in physics and is the extensional generalization to a rotating body of the Schwarzschild metric. The metric describes the vacuum geometry of space-time around a rotating axially-symmetric black hole with a quasipotential event horizon. In Kerr metric there are two event horizons (inner and outer), two ergospheres and an ergosurface. The most important effect of the Kerr metric is the frame dragging (also known as Lense-Thirring Precession) is a distinctive prediction of General relativity. The first direct observation of the collision of two Kerr Black Holes has been discovered by LIGO in 2016 hence setting up a milestone of General Relativity in the history of Physics. Here, the Kerr metric has been introduced in the Boyer-Lindquist forms and it is derived from the Schwarzschild metric using the Spin-Coefficient formalism. According to the "Cosmic Censorship Hypothesis", a naked singularity cannot exist in nature as nature always hides the singularity via an event horizon. I will not go in detail about the contradiction of 'Chronology Protection Conjecture" [4] whether the Stress-Energy-Momentum Tensor can violate the ANEC (Average Null Energy Conditions) or not with the values of less than zero or greater than, equal to zero, instead I will focus definitely on the creation of the mathematical formulation of a wormhole from a Naked Ring Kerr Singularity of a Kerr Black Hole without any event horizon or ergosphere. Another important thing to mention in this paper is that I have taken the time to be imaginary[5] as because, a singularity being an eternal point of time can only be smoothen out if the time is imaginary rather than real which will allow the particle or null rays inside a wormhole to cross the singularity and making entrance to the other universe. The final conclusion would be to determine the mass-energy equivalence principle as spin angular momentum increases with a decrease in BH mass due to the vanishing event horizon and ergosphere thereby maintaining the equivalence via apparent and absolute masses in relation to spin J along the orthogonal *Z axis. A 'NAKED SINGULARITY'* alters every parameters of a BH and to include those parameters along with affine spin coefficient, it has been proved that without any spin angular momentum the generation of wormhole and vanishing of event horizon and singularity is not possible.

Methodology and Interpretation: A sphere should have a critical radius beyond which it will become so densely packed, considering the sphere is not hollow and having some mass, the sphere tends to collapse gravitationally and becomes a black hole. This critical radius that the sphere is getting transformed into is called 'hoop radius' when substituting the *'r'* of the 'surface area' of the sphere with the 'Schwarzschild radius', a gravitationally collapsed BH is formed. However, while considering this 'hoop' it is not needed to specify whether the black hole has charge or spin rather its always been spherically symmetric. The 'hoop operations' are performed below.

$$R_{(spherical\ hoop)} = \frac{2MG}{rc^2} \tag{1}$$

Substituting $R_{(spherical\ hoop)}$ into $R_s{}^2$ we get the below equations,

$$R_{(hoop)} = 4\pi R_s{}^2 = 4\pi\left(\frac{2MG}{rc^2}\right)^2 = 16\pi\left(\sqrt[2]{\frac{MG}{rc^2}}\right)^4 \tag{2}$$

Note: The Hoop Conjecture was proposed by KIP Throne in the form of a circle.

Here from, to make the metric less messy and for brevity we will group certain things into a symbol and that symbolic representations would be used to manipulate the final result. Here, the foremost thing necessary is to include an 'i' term with 'cdt' to make the time imaginary as this will smoothen out the singularity paving the way of eternal time to an imaginary time reducing the infinite time dilation problem while reaching the singularity.

$$-i^2c^2dt^2 = \tau \tag{3}$$

The black hole has a vertical axis of spin that is 'z' and 'a' is an affine parameter of the spin along 'z' axis with the spin term 'J' [6]mass 'M' and speed of light as 'c'.

$$a = \frac{J}{Mc} \tag{4}$$

Here we are using the Boyer-Lindquist coordinates[7][8] and the standard spherical coordinates used here are r, θ, φ and we will use Σ as the frame dragging equation although later in this paper will see 'a' has a specific value when the over extreme solution of the metric is taken making the larger root of another grouping parameter Δ as zero. However, it's necessary to introduce Σ here as follows,

$$\Sigma = r^2 + a^2Cos^2\theta \tag{5}$$

Now, there are two radiuses, the '$hoop\ radius\ R$' and the $Schwarzschild\ radius$ 'r' and grouping them to 'ω' will lead us to the below equation.

$$\omega = R_{(hoop)}r \tag{6}$$

Here comes the most important parameter Δ whose limit when taken to zero will completely vanish the event horizon and ergospheres, I mean the two, internal and external event horizons and the ergospheres thereby making a way to proceed in developing a *'NAKED SINGULARITY'* which is essential for any particle travels through the singularity as there is no event horizon to break the laws of physics. So, two important things could be achieved up to now,

1. Making time imaginary to remove the eternal time while reaching singularity.
2. Removing the event horizon so that a particle can easily travel through the singularity without the fear of getting trapped inside event horizon.

Taking into account the delta factor in the below equation.

$$\Delta = r^2 - R_{(hoop)}r + a^2 \tag{7}$$

Another important parameter is the' Ξ' which combines the '$affine\ spin\ parameter\ a$' with the square of $Sin\,\theta$ as follows,

$$\Xi = aSin^2\theta \tag{8}$$

A wormhole is always cylindrical and to take the cylindrical polar coordinates vectors $\hat{\rho}, \hat{\varphi}, \hat{z}$ we have grouped these terms into Ω yielding the equations as follows,

$$\Omega = (d\hat{\boldsymbol{\rho}} + \rho d\hat{\boldsymbol{\varphi}} + d\hat{\boldsymbol{z}})^2 \tag{9}$$

Now, we have almost all the tools to construct the '*EXTREMAL KERR METRIC WITH WORMHOLES*' including an additional factor of $\oint_{C(H^+)}(T_{ab}l^al^b)\,\Omega$ where $C(H^+)$ is the *Cauchy-Horizon* where future directed null lines would go and if there are particles then this Stress-Energy-Momentum tensor $(T_{ab}l^al^b)$ would be integrated via a closed integral over the wormhole coordinate Ω.

$$g = \tau = \left(1 - \frac{\omega}{\Sigma}\right)\tau + \frac{\Sigma/\Delta}{\lim_{\Delta\to 0}}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{\omega a(\Xi)}{\Sigma}\right)\frac{\Xi}{a}d\varphi^2 - \frac{2\omega(\Xi)}{\Sigma}\frac{\sqrt[2]{\tau}}{i^2}d\phi^2 + \oint_{C(H^+)}(T_{ab}l^al^b)\,\Omega \tag{10}$$

Coming back to the equation grouping Σ, if we consider r^2 as f^2 and $a^2Cos^2\theta$ as g^2, then with no harm, if we assume Σ as $(S_0)^2$ then the singularity equation satisfies a ring singularity which is a circle.

$$\Sigma = r^2 + a^2Cos^2\theta \equiv (S_0)^2 = f^2 + g^2|_{circle} \tag{11}$$

Again further analyzing the ring singularity, we get to know the value of 'r' as zero if we take the value of θ as $\frac{\pi}{2}$. Here two things are satisfied as follows,

Firstly, at the beginning of the paper a *'wick Rotation'* has been performed to make c^2dt^2 as $-i^2c^2dt^2$ and now regarding the equation of the circle we can see that the value of θ as $\frac{\pi}{2}$ making a way of the *'Wick Rotation'* of c^2dt^2 as $-i^2c^2dt^2$ through a $\frac{\pi}{2}$ Rotational Plane also paving the way for the '$singularity$' 'r' to be zero.

The below equation satisfies the singularity relation as follows[8],

$$\Sigma = r^2 + a^2Cos^2\theta \approx r^2 + a^2Cos^2\theta = 0 \approx r^2 + a^2Cos^2\left(\frac{\pi}{2}\right) => r \cong 0|_{singularity} \tag{12}$$

Manipulating with the *'hoop radius'* and taking $G = 1, c = 1$ we can establish a very important conclusion that '*event horizon*' if exists (if we don't take the value of Δ as zero) then we would arrive the relation that '*event horizon*' is a sphere which is vanishing in case of the *'NAKED SINGULARITY'*. [9]

$$R_{(hoop)} = 4\pi {R_s}^2 = 4\pi\left(\frac{2MG}{rc^2}\right)^2 \equiv r = 2M|_{G=1,c=1} \tag{13}$$

$$r = 2M = x^2 + y^2 + z^2 \cong r = 2\beta|_{M=\beta} = x^2 + y^2 + z^2 \tag{14}$$

$$x^2 + y^2 + z^2 = 2\beta \ldots (2) \tag{15}$$

To manipulate equation (15), as 'a' being an affine spin parameter along the 'z' axis of motion and amplifying one side of 'z' axis, the axis that points outwards from our universe we can make an equation with LHS as 'z' and RHS as $-x^2 + y^2$, what we get is an *'extended Cauchy horizon'* as follows,

$$z^2 = -x^2 + y^2|_{Extended\ Cauchy\ Horizon}C(H^+) \tag{16}$$

The *spacelike timeline* will escape from the BH through *Wormhole* and enters into a Cauchy horizon $C(H^+)$ which is future directed and thereby enters into another Kerr BH in another universe to make an exit through another future Cauchy Horizon. [10][11]

Now, if the *spin angular momentum* '$J = 0$' then 5th part of the RHS of equation (10) would be 0 as it is a cross term between $\phi\ \&\ \tau$ coordinate which beautifully implies that without *SPIN* equation (10) is not valid. This suggests that only Kerr BH can have this property if limit of Δ goes to zero shown as follows.

$$\left(a = \frac{J}{Mc}\ \&\ \Xi = aSin^2\theta\right)_{J=0} \equiv \frac{2\omega(\Xi)}{\Sigma}\frac{\sqrt[2]{\tau}}{i^2}d\phi^2 = 0 \xrightarrow{yields} \frac{\sqrt[2]{\tau}}{i^2}d\phi^2 = 0 \tag{17}$$

Here if Δ approaches to complete zero then the affine parameter 'a' of the spin takes the values as computed by the below equation,

$$\Delta = r^2 - R_{(hoop)}r + a^2 \equiv 0 = r^2 - Rr + a^2 => a^2 = Rr - r^2 = r(R-r) => a = \sqrt[2]{r(R-r)} \quad (18)$$

Therefore, as soon as the *singularity got naked*, the spin of the BH changes as follows,

$$a = \frac{J}{Mc} \quad (19)$$

Substituting the value of '*a*' with the new value $\sqrt[2]{r(R-r)}$, the equation is as below with a new parameter of spin expressed in terms of '*HOOP and SCHWARSZCHILD Radius',*

$$J^* = \sqrt[2]{r(R-r)}\, Mc \quad (20)$$

The mass of the BH will also change as follows,

$$M^* = \frac{J}{\sqrt[2]{r(R-r)}c} \quad (21)$$

Now, here as $\lim_{\Delta \to 0}$. Has been fully satisfied, a wormhole begins to form and the event horizon along with ergosphere starts to vanish thereby its mass is reduced and its spin is increased satisfying the two important relations.

$$M^* \ll M \quad J^* \gg J$$

Substituting the value of $r = 2|_{M=\beta}$ in equation (14) with the value of $16\pi\left(\sqrt[2]{\frac{MG}{rc^2}}\right)^4$ in equation (2) we get $R_{(hoop)} = 16\pi\left(\sqrt[2]{\frac{\beta}{2}}\right)^4_{|c=1,M=\beta,G=1} = 4\pi\beta^2$ & assuming $R_{(hoop)} \approx r \approx M^*$ we can conclude that M^* can only be less than M by a limiting factor of $4\pi\beta^2$.

Mass-energy equivalence: This is an important factor that needs to be taken into account while deducing the parameters for any systems and this being starts with the most famous yet simple equations of special relativity $E=mc^2$. Here, in this paper, the mass M^* gets reduced by a factor of $4\pi\beta^2$ and the spin J^* gets increased which means the rotational energy E_{rot} gets increased thereby adding mass to the BH. Then how does this reduced mass M^*gets compensated by the increased mass due to the kinetic energy of rotation. To do so, we will split the original mass M into two components as follows,

$$M = \begin{cases} M_{apparant}, & M^* < M \\ M_{absolute}, & M^* > M \end{cases}$$

Now the rotational energy $E_{rot} = (M - M^*)c^2$ [12][13]where the apparent mass $M_{apparant}$ seems to decrease by a factor of $4\pi\beta^2$ although the absolute mass $M_{absolute}$ increases by a factor of $\sqrt{2}$ by the Penrose mechanism where the β^2 when takes the value of $\left({}^1/_{2\sqrt{2}\pi}\right)$, then $\left(4\pi * \left({}^1/_{2\sqrt{2}\pi}\right)\right)$ satisfies the increase of $M_{absolute}$ from J^* by a factor of $\sqrt{2}$ tallying with the Penrose process. [12][13] *Hence mass-energy equivalence has been satisfied.*

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