Stiffness behavior of sisal fiber reinforced foam concrete under flexural loading

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Abstract

This work investigates the static and fatigue behavior of foam concrete reinforced by sisal fibers under flexural loading. In particular, the effect of fiber content (0-0.3%) is determined by measuring the strains at two positions (mid-span and near the support of the beam) to relate with the bending stiffness. The results show that under static loading, the composite having a sisal fiber content of 0.15% has the lowest bending stiffness when the specimen fractures. This is especially the case at the mid-span of the beam where the bending stiffness degradation is more obvious. Under cyclic loading, sisal fibers are effective in decreasing the bending stiffness degradation of the composites. When the sisal fiber content is below 0.05%, a linear relation between the bending stiffness and the logarithmic fatigue life of the composite was found. But for a fiber content above 0.05%, a linear relationship is obtained when the fatigue life is used. It was also found that the bending stiffness obtained from two positions are linearly related. If the slopes of the fitting lines are fixed as 2, a mathematical model between the bending stiffness at the mid-span and the bending stiffness near the support is obtained.

KEYWORDS: Foam concrete; sisal fiber; flexural stiffness; fatigue life

1. INTRODUCTION

As a building material, concrete should have enough strength and toughness to withstand specific loads depending on its final application [1]. Furthermore, the stiffness of concrete must also be determined to quantify its resistance under deformation up to fracture. If the stiffness is too low, the deformation will be high and the load capacity of the structure will decrease. For most applications, concrete parts are under constant loading and their long term properties must be determined for safety reasons. This is why the time behavior is of high interest which can be determined via several characterizations like creep, relaxation and fatigue [2]. In most cases, stiffness (modulus/compliance) is quantified under different conditions.

The loss of stiffness and the residual stiffness of concrete are affected by several factors such as components [3], corrosion [4,5], structure [6], etc. For example, adding honeycomb structures inside reinforced concrete leads to a global density reduction but high stiffness loss with increasing volume content [7]. To enhance the flexural stiffness of concrete, constraints/reinforcements in the beam can be added. So the number/type of fibers/bars is important. To this end, a computational procedure was proposed to overcome the undesirable effect of limited ductility [8]. This methodology was used to optimize the loading capacity and the flexural stiffness of hyper-static beams. Furthermore, the effect of reinforcement content and their location on the bending stiffness of reinforced concrete was discussed [9]. In terms of experiments and numerical simulations, Sharaky et al. [10] investigated the effect of axial

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stiffness of near-surface mounted fiber reinforced polymer (NSM FRP) on the flexural performance of strengthened reinforced concrete. The results showed that a critical axial stiffness ratio ($1 + E_{FRP} A_{FRP} / E_{steel} A_{steel}$) of 1.25 was obtained. Furthermore, a new calculating method to achieve the effective stiffness of slender reinforced concrete column subjected to complex stress states including axial stress and bi-axial bending was presented [11]. Based on genetic programming (GP), an explicit formulation to determine the effective flexural stiffness of circular reinforced concrete columns was presented [12]. Compared with static loading, when the beam is subjected to a live load or fatigue loading, its flexural stiffness decreases (degradation) even under external loads below its ultimate strength. For the dynamic response of concrete, the relation between localized steel-concrete bond damage and dynamic stiffness is not obvious [13]. But bending cracks are directly affecting the dynamic response of reinforced concrete.

Combining the advantages of two materials, steel-concrete composites can produce higher flexural stiffness. Considering possible interfacial slip between the steel and concrete [14], a model was proposed to predict the stiffness of these composites under a negative moment. To achieve a significant integral behavior, a reinforcing truss and an inverted U-shaped reinforcement were adopted to produce stiffer steel-concrete beams [15]. For reinforced concrete columns, the flexural stiffness is also of interest. So a new expression able to accurately predict the stiffness of reinforced concrete was proposed [16].

For foam concrete, the main characteristics are lower density and strength. Youssef et al. [17] discussed the effect of sand, pores and cement paste on the flexural stiffness of foam concrete. But other components, such as palm oil ash and eggshell powder were also included to improve the flexural performance of foam concrete [18].

As reported above, steel fibers can be added into reinforced concrete to improve the initial and residual flexural stiffness [19]. Similarly, the flexural properties of foam concrete can be improved by adding steel fibers or other reinforcements like polystyrene, glass fiber reinforced polymer and steel [20-23]. In a previous work [24], the flexural performance of foam concrete under static and cyclic loading was investigated. It was shown that the addition of sisal fiber can efficiently improve the fatigue life of foam concrete when the fibers can be homogeneously dispersed in the matrix. In this study, the focus is made on the bending stiffness of foam concrete with different sisal fiber content. The work also includes a comparison between the beam bending stiffness at different locations.

2. EXPERIMENTAL WORK

To investigate the bending stiffness behavior of foam concrete, static flexural and fatigue tests were conducted. To determine the effect of sisal fiber content on the bending stiffness of foam concrete, six concentrations (0, 0.05, 0.1, 0.15, 0.2 and 0.3%) were studied to form 6 groups of tests. For the fatigue tests, three stress levels (0.75, 0.8 and 0.9), which are defined as the ratio of the maximum fatigue load to the bending strength of the specimen, were used. Each stress level includes 3 specimens. So a total of 54 specimens were used to complete the fatigue tests.

Materials

Although different fibers can be used to improve the mechanical properties of foam concrete [25-28], sisal is widely used in concrete engineering such as sandwich structures and bio-composites because of its biosourced origin, biodegradability and low cost [29, 30]. Sisal fiber reinforced foam concrete contains several components such as: cement, aggregate, water, etc.

Except for the basic materials, fly ash and silica fume will be used in this study. Fly ash can substitute the cement and improve the strength. Silica fume has a smaller particle diameter (0.1-0.3 μm), so it can be used as a filler to improve the compactness and the bending strength of foam concrete. Similarly, a water reducing agent is used to improve the mechanical properties of foam concrete. Hydrogen peroxide (foaming agent) and calcium stearate (foam stabilizer) are used to produce the foam structure. As a natural material, sisal fiber was shown to efficiently improve the mechanical properties of concrete [31, 32]. In this study, the sisal fibers were obtained from Jiangxi Poyang Changjiang textile co. Ltd. (China). The original fiber length ranged from 90 cm to 130 cm and the fiber purity was above 97%. To conveniently add the sisal fibers into the matrix, they were cut to 10 mm in length. The properties of the sisal fibers are listed in Table 1.

Table 1. Properties of sisal fibers.

Length after cutting (mm)	Diameter (μm)	Moisture regain (%)	Fiber tension (N)	Elongation (%)	Density (g/cm^3)
10	100-200	9.5	>800	5	1.34

The preparation of foam concrete includes the following steps: mixing of the mortar compositions, addition of the foaming agent, high intensity mixing and pouring in a mold (300 mm x 100 mm x 100 mm). Before testing, each component was weighed as reported in Table 2. In this study, the experimental steps to produce sisal fiber reinforced foam concrete are as follows: 1) cleaning of each equipment (mixer, mold, etc.) and coating the mold with engine oil to facilitate sample demolding; 2) weigh the required amount of raw materials. Then, place the cement, sand, fly ash and silica fume into the mixer and dry mix (JJ-5 planetary cement mortar mixer, China) for 1.5 min and then sisal fibers are slowly added into the mixer to continue dry mixing for another 2 min. Finally, the hydrogen peroxide and water are slowly added into the mixer for another 3 min of mixing; 3) The mixture is placed inside the mold, compacted and smoothed with a shovel. The mold is finally placed on a vibrating table (South China testing instrument Co. Ltd., China), vibrated for 1 min and left to rest; 4) After 24 h, the sample is placed in a curing room (20°C and 90% humidity). Finally, the specimen can be removed from the mold after 28 days and used for flexural testing.

Table 2. Mixing ratio of the foam concrete.

Cement (kg)	Sand (kg)	Water (kg)	Water reducer (kg)	Fly ash (kg)	Silica fume (kg)	Foaming agent (l)	Foam stabilizer (kg)
588	360	378	3	168	84	30	2

Mechanical characterization

For each type of test (static or fatigue), a three-point bending geometry was selected (Figure 1). The size of the beam was 300 mm x 100 mm x 100 mm and the span was fixed at 200 mm. The load was applied on the top surface at the mid-section of the beam. The static bending tests were carried out on a hydraulic universal testing machine WEW-300B (Jinan testing machine factory, Jinan, China) with computer control. The loading rate was fixed at 0.02 kN/s. The fatigue tests were carried out on an electro-hydraulic serve fatigue testing machine HYS-100 (Changchun Hao Yuan testing machine Co., Ltd., Changchun, China), and the sampling frequency was 100 Hz. The bending load (P_u) can be used to determine the stress levels for fatigue tests. Three stress levels ($S_R = P_{max}/P_u$) of 0.75, 0.8 and 0.9 were applied to the middle of the specimen for each sisal fiber content.

To measure the axial strains, five strain gauges were installed at different locations. Three stain gauges were pasted at the upper, lower and central positions, while the other two strain gauges were pasted close to the hinge supports. The specific positions of the strain gauges are reported in Figure 1. The strains were acquired via dynamic resistance strain gauges TMR7200 (Tokyo measuring instruments laboratory Co., Ltd., Tokyo, Japan). The size of the specimens in the fatigue tests were the same as for the static tests and again five strain gauges for each specimen were installed at the same positions (Figure 1).

The parts located at different positions are under different stress states, so they will display different bending stiffness. Since the strains for two positions (mid-span (M) and near the support (S)) are measured by the strain gauges, the bending stiffness of the specimen can be determined in terms of the tensile stresses related to their respective positions. With the theory of mechanics of materials, the bending stiffness at both positions (M and S) can be obtained as a function of the number of cycle (N) with respect to the load and deformation measured/imposed.

3. RESULTS

Considering the weak bending strength of foam concrete, the sisal fibers were added to improve their flexural performance. The rupture of a typical specimen is shown in Figure 2. From this figure, it can be seen that the sisal fibers are well embedded in the matrix to prevent cracking and premature rupture. But the fiber distribution in the matrix is uneven and agglomerations can be seen. For all the flexural tests, fiber pull-out from the matrix occurs and there is no evidence of fiber break-up. So interfacial slipping is highly present which dissipates a large amount of energy when the specimen cracks, thus improving the ductility of foam concrete.

In order to conveniently compare the experimental data, the results are plotted in Figure 3A. The bending strength of the specimens with increasing sisal fiber content is presented by the blue lines. Due to the bridging effect [33], increasing the sisal fiber content increases the bending strength up to 0.15%. Above 0.15%, the bending strength decreases mainly due to fiber agglomeration creating some defects in the structure. In fact, the dispersion of sisal fibers in the matrix was severely affected by their shape, morphology and surface treatment [34,35]. Furthermore, sisal fiber addition improves the ductility of the foam concrete. For example, two positions located at the mid-span (position 1, denoted by M) and near the support (position 2, denoted by S) are selected to depict the deformation of the specimen. In Figure 3A, the tensile strains at position 1 (red lines) and position 2 (black lines) are plotted, showing the axial deformation as a function of sisal fiber content. Whatever the position (1 or 2), the tensile strain is maximum when the fiber content is 0.15%, similar to the bending strength trend. This means that the specimens have their maximum strength and maximum flexibility under the same conditions.

Comparing the tensile strains located at both positions (S and M), under any conditions, the tensile strain at position 1 is larger than that at position 2 when the specimen fractures. However, when the fiber content reaches 0.15%, the tensile strain at position 1 increases rapidly and is much higher than that for position 2. Because the sisal fiber efficiently improves the ductility of the foam concrete, when the specimen fractures in the middle of the beam, large deformation was produced.

Based on the theory of the mechanics of materials and the bending strength of the foam concrete with different sisal fiber contents, the ultimate stresses at both positions (M and S) can be obtained. The axial modulus is calculated by the ratio of the ultimate stress over the ultimate strain, while the bending stiffness is obtained by the product of the axial modulus and the axial moment of inertia. Because the axial modulus is related to the stress state, the bending stiffness for different positions is different. Here, the bending stiffness calculated at position 1 and position 2 are compared. The required values are computed by Equations (1-5) and the results are listed in Table 3.

$$\sigma_M = \frac{y}{y_{\max}} \sigma_b = \frac{35}{50} \sigma_b \quad (1)$$

$$\sigma_S = \frac{M_S}{M_M} \sigma_M \quad (2)$$

$$k_M = E_M I = \frac{\sigma_M}{\varepsilon_M} I \quad (3)$$

$$k_S = E_S I = \frac{\sigma_S}{\varepsilon_S} I \quad (4)$$

$$I = \frac{1}{12} h^3 b \quad (5)$$

where σ and ε refer to the stress and the strain respectively, M_M and M_S are the bending moments at the mid-span and near the support respectively, I is the axial moment of inertia, while h and b refer to the height and the width of the beam section.

Table 3. Mechanical parameters at position 1 (M) and 2 (S).

Sisal fiber content (V_f) (%)	0	0.05	0.10	0.15	0.20	0.30
Bending strength σ_b (MPa)	0.83	0.94	1.00	1.07	0.95	0.89
M-Ultimate stress σ_M (MPa)	0.581	0.658	0.700	0.749	0.665	0.623
M-Ultimate strain ε_M (με)	93	123	175	582	368	231
M-Axial modulus E_M (GPa)	6.247	5.349	4.000	1.287	1.807	2.697
M-Bending stiffness k_M (10^6 kNmm^2)	52.06	44.58	33.33	10.72	15.06	22.47
S-Ultimate stress σ_S (MPa)	0.0872	0.0987	0.1050	0.1120	0.0998	0.0935
S-Ultimate strain ε_S (με)	41	78	106	131	120	101
S-Axial modulus E_S (GPa)	2.126	1.265	0.991	0.858	0.831	0.925
S-Bending stiffness k_S (10^6 kNmm^2)	17.71	10.54	8.25	7.15	6.93	7.71

Table 3 shows that whatever the position (1 = mid-span or 2 = near the support), the bending stiffness decreases with increasing sisal fiber content below 0.15%, but slightly increases above 0.15%. For any fiber volume fraction, the bending stiffness of the foam concrete at position 1 is higher than for position 2. This means that the part at position 1 is more difficult to deform than that at position 2. To explain this phenomenon, three micro-elements are chosen to analyze the stress states (Figure 1). Micro-element 1 (Figure 3B) is taken from position 1 and micro-element 2 is taken from position 2. Micro-element 1 is under unidirectional tension and micro-element 3 is under unidirectional compression. This case is different with the direct tension because the upper side of the beam will resist the tensile deformation of the lower side of the beam. In contrast, the micro-element 2 is under a complex stress state including tensile and shear stresses. From Figure 3B, the shear stress in the micro-element 2 is beneficial to the tensile deformation of the specimen. From the stress-strain analysis, the bending stiffness obtained from the tensile stress and the tensile strain at position 2 is less than that at position 1. The experimental data are consistent with the results of Mohammadhassani et al. [9] who also reported that the stiffness away from the mid-span is lower than that at the mid-span.

To conveniently compare the bending stiffness of the foam concrete with the sisal fiber volume fraction, the results obtained from two positions are plotted in Figure 4. Figure 4A shows that the normalized bending strength varies with the fiber content. When the position 1 is selected to calculate the bending strength, the optimal sisal fiber content leading to the minimum stiffness is again 0.15%; i.e. the specimen has a maximum flexibility at 0.15%. However, the bending stiffness of the specimen at the position 2 is minimum when the sisal fiber content is 0.2%. Comparing these two cases, the optimal sisal fiber content for position 1 is 0.15% which is slightly lower than that for position 2 to get the maximum flexibility. Figure 4B shows that the bending stiffness at position 2 rapidly decreases at the beginning and then slowly decreases with increasing sisal fiber content below 0.15% (black curves). When the fiber content is higher than 0.15%, this is not the case for position 1 where the bending stiffness increases faster than that at the position 2 (red curves in Figure 4B). From the analysis performed, the addition of sisal fiber can improve the fracture flexibility of the foam concrete under the static load at low fiber content, which is more efficient at the beam mid-span.

Bending stiffness vs. fatigue life

The bending stiffness of the samples can be illustrated with increasing time due to mechanical fatigue loading. These changes in terms of the concrete beam bending stiffness can reflect the differences to resist the deformation between each material. For sisal fiber reinforced foam concrete, Figure 5A shows the relationships between the bending stiffness and the fatigue life with different sisal fiber content under a stress level of 0.75. From this figure, the mid-span of the beam has higher bending stiffness than that near the support. At low fiber content (0-0.05%), three stages can be found: the bending stiffness initially decreases rapidly (crack initiation), then a second stage has a constant slope (crack propagation), finally the bending stiffness tends to zero in the last stage (complete fracture). When the sisal fiber content is larger than 0.05%, the initially decreasing stage is not obvious. The curves mainly include the stable stage and the last decreasing stage. When a higher stress level is applied (0.8 or 0.9), similar behaviors are obtained in Figures 5B and 5C. Whatever the stress level, all the three stages are very clear without sisal fiber. However, when sisal fibers are present, they are playing a bridging role restricting the deformation of the composites and improving the stiffness degradation of the specimens. As the fiber volume fraction is 0.05%, the three stages of the curves are still observed. But when the fiber content is higher than 0.05%, the stiffness degradation of the specimens becomes slow and the initial decreasing stage disappears.

To investigate the effect of sisal fiber content on the flexural stiffness degradation of foam concrete, normalized bending stiffness-fatigue life curves under different stress levels are plotted in the same figure. For example, when the stress level of 0.75 is selected, the relationships between the bending stiffness and the fatigue life for different sisal fiber contents and for the mid-span of the beam are plotted in Figure 6A. It is clear that the sisal fibers improved the flexural stiffness of the foam concrete. When the sisal fiber content is 0.05%, the normalized stiffness vs. fatigue life curves are close to that of the neat foam concrete which contains three stages. At the first stage, the bending stiffness decreases rapidly. When the sisal fiber content exceeds 0.05%, the bending stiffness degradation curves decrease slowly. A similar trend can be seen in Figure 6B in which the bending stiffness is measured near the support. For higher stress levels (0.8 or 0.9), the results are also gotten and all the curves decrease slower because the fatigue life is reduced. In general, whatever the stress level, the presence of sisal fibers can efficiently resist the loss of bending stiffness when a fatigue load is applied. This is especially the case when the fiber content reaches 0.3% where an optimal effect is observed.

Bending stiffness degradation curves

For the foam concrete, the bending stiffness degradation curves include three stages. To find a relationship between both properties, a linear regression analysis was conducted. Under the three stress levels, the bending stiffness curves have the same trends, so all the points at each position are used to perform the regression analysis. By taking the logarithm of the fatigue life, the fitting curves related to the bending stiffness are obtained. Figure 7A shows the fitting results for the position 1 and 2. The bending stiffness and the logarithm of the fatigue life of the neat foam concrete have a good correlation with an adj. R^2 (coefficient of determination) of 0.891 and 0.831. Generally speaking, the bending stiffness at the mid-span is higher than that near the support. When sisal fibers are added at low content, the stiffness degradation of the composites is similar with that of the neat foam concrete. The two regression lines are plotted in Figure 7B. However, the adj. R^2 is lower than the former. To make a more quantitative analysis, a relationship between the bending stiffness (k_M or k_S) and the fatigue life (N) of the neat concrete is proposed as:

$$k = a + b\log_{10} N \quad (6)$$

where a and b are fitting parameters.

The quality of linear regression can be measured by the coefficient of determination (COD), or R^2, which can be computed as [36]:

$$R^2 = 1 - \frac{RSS}{TSS} \quad (7)$$

where *TSS* is the total sum of square and *RSS* is the residual sum of square. In addition, the adj. R^2 can be defined as:

$$\overline{R}^2 = 1 - \frac{RSS / df_{Error}}{TSS / df_{Total}} \quad (8)$$

where df_{Error} and df_{Total} are the degree of freedom for *RSS* and *TSS*, respectively.

Because the intercept is included in the linear model, df_{Total} equals n-1 and df_{Error} equals n-2 where n is the number of points (36 here). *RSS* is actually the sum of the square of the vertical deviations from each data point to the fitted line. *RSS* and *TSS* can be computed as:

$$RSS = \sum_{i=1}^{n} \omega_i \left[y_i - (\beta_0 + \beta_1 x_i) \right]^2 \quad (9)$$

$$TSS = \sum_{i=1}^{n} \omega_i (y_i - \overline{y})^2 \text{ and } \omega_i = \frac{1}{\sigma_i^2} \quad (10)$$

where ω_i are the weights and σ_i are the measurement errors. Here, both are set at 1. β_0 and β_1 are fitting parameters.

For sisal fiber contents of 0% and 0.05%, all the fitting parameters are listed in Table 4. It can be seen that when the sisal fibers are added, the linear correlation between the bending stiffness and the logarithm of the fatigue life is decreasing. This means that higher sisal fiber contents are less well represented by the model and some modification (improvement) must be made.

Table 4. Fitting parameters for the relationship between the bending stiffness and the fatigue life.

Sisal fiber content (%)		0.00		0.05	
Position		M	S	M	S
Number of data		36	36	36	36
Degrees of freedom		34	34	34	34
Intercept *a*	Value	323.4	131.2	203.6	93.4
	Standard error	10.5	6.6	11.6	5.7
Slope *b*	Value	-51.0	-24.8	-23.7	-14.0
	Standard error	3.0	1.9	3.2	1.6
Residual sum of squares		15190.4	5032.8	12721.9	3297.3
Coefficient of determination R^2		0.8937	0.8362	0.6209	0.7018
Adj. R^2		0.891	0.831	0.610	0.693
Pearson's r		-0.945	-0.914	-0.788	-0.838

Note: M and S refer to mid-span and support position; R is the correlation coefficient.

When the sisal fiber content is above 0.05%, the three stages of the bending stiffness behavior are not obvious, and only two stages are clearly seen: the stable decreasing stage and the last stage. It was difficult to find a suitable fitting curve to represent the whole process including all three stress levels (0.75, 0.8 and 0.9). To get a good relationship between the bending stiffness and the fatigue life, the stable decreasing stage under each stress level was linearly fitted. When the sisal fiber content is in the 0.1%-0.3% range, the fitting curves under each stress level (0.75, 0.8 or 0.9) are plotted in Figures 7C, 7D, 7E and 7F. Whatever the position

(1 or 2), the slopes decrease with increasing the stress level. In general, all the regression lines can well reflect the relationship between the bending stiffness and the fatigue of composites. Similarly, Equation (11) can be used as a first approximation:

$$k = c + d \cdot N \quad (11)$$

where *c* and *d* are fitting parameters.

To conveniently compare the effects of the position and the stress level on the results, the fitting parameters and their coefficients of determination are listed in Table 5.

Table 5. Fitting parameters of the relationship between the bending stiffness and the fatigue life.

Fiber content (position)	$S_R = 0.75$			$S_R = 0.80$			$S_R = 0.90$		
	c	*d*	R^2	*c*	*d*	R^2	*c*	*d*	R^2
0.10% (M)	193.98	-3.96	0.96	183.89	-10.14	0.90	213.60	-64.74	0.83
0.15% (M)	152.50	-2.65	0.97	156.79	-5.44	0.84	182.61	-15.71	0.98
0.20% (M)	125.18	-1.75	0.95	135.21	-8.62	0.93	156.68	-16.78	0.98
0.30% (M)	123.23	-1.88	0.98	136.29	-7.90	0.96	156.41	-28.09	0.82
0.10% (S)	56.25	-1.18	0.88	57.25	-5.93	0.89	74.46	-33.79	0.86
0.15% (S)	45.49	-0.92	0.89	43.00	-1.70	0.96	61.31	-10.08	0.78
0.20% (S)	50.90	-1.22	0.90	52.80	-5.04	0.84	62.42	-10.16	0.87
0.30% (S)	50.91	-1.48	0.86	63.70	-9.11	0.87	70.59	-34.61	0.97

These linear regressions are not only used for foam concrete, but also for ordinary concrete and high strength concrete. By normalizing the variables, Zhang et al. [37] also reported a linear relationship between the dynamic stiffness of ordinary concrete and the number of cycles under fatigue loading. For the second stage of the fatigue curve [38], it can be shown that the logarithm of the stiffness is correlated with the logarithm of the fatigue life. Comparing the reports, the relationships between the flexural stiffness of foam concrete and the fatigue life are consistent with the results from the literature in which the parameters are different due to the different formulation used.

Bending stiffness obtained from two positions

At the mid-span or near the support of the beam, because the stress states are different, the bending stiffness calculated by the mechanics of materials is also different. From the previous analyses at the mid-span, the lower part of the beam is elongated and the upper is compressed under the bending moment, which is different compared to purely tensile deformation. But concrete has a much higher compressive strength than its tensile strength. Since the stress state under bending moment includes tensile, compressive and shear stresses, the tensile deformation of the lower part is not isolated and limited by the compressive deformation of the upper part, which results in larger bending stiffness of the mid-span of the beam. Despite this difference between both type of solicitation, a relationship between the bending stiffness at the mid-span and near the support was searched. To find a quantitative relationship between them, linear regression analyses were conducted. Figure 8A shows the linear fitting results on both bending stiffness without sisal fiber. As the correlation coefficient is 0.94, the bending stiffness of the beam at the mid-span is linearly related to the bending stiffness near the support. The slope of the fitting line is 2.0, which means that the bending stiffness from the mid-span increases more rapidly than that from the support with increasing cyclic loading. Keeping the same slope (2.0) of the fitting line, when the sisal fiber content is 0.05%, the fitting curve between both bending stiffness is plotted in Figure 8B. In this case, the correlation coefficient between both bending stiffness is 0.91. If the sisal fiber content is increased to 0.1%, 0.15%, 0.2% or 0.3%, similar trends can be observed. For all these fitting curves, the correlation coefficients are higher than 0.8 (slopes fixed at 2.0). The related fitting parameters are listed in Table 6.

To build a quantitative relationship between both bending stiffness at the different positions, a general equation is proposed as:

$$k_M = A + Bk_S \quad (12)$$

where k_M and k_S refer to the bending stiffness at the mid-span and near the support respectively, while *A* and *B* are fitting parameters.

Table 6. Fitting parameters for the linear regression analyses of Equation (12).

Sisal fiber content (%)		0.00	0.05	0.10	0.15	0.20	0.30
A	Value	58.0	31.7	65.7	54.0	29.5	24.8
	Standard error	3.5	2.5	2.5	2.4	4.4	2.3
B (Fixed value)		2.0	2.0	2.0	2.0	2.0	2.0
Residual sum of squares		15819.3	7648.5	14289.7	10678.9	3587.5	6549.9
Adj. R^2		0.89	0.77	0.83	0.85	0.92	0.84
Pearson's r		0.94	0.91	0.93	0.94	0.96	0.92

From Equation (12), the parameter *B* was fixed at 2.0 and the parameter *A* was determined based on the experimental data, which varies with the sisal fiber content. If a relationship between the parameter *A* and the sisal fiber content is possible, a model between both bending stiffness can be built. Based on the results obtained, a linear model is proposed and the best fit is plotted in Figures 9. Although the parameter *A* is discrete with increasing sisal fiber content, they have a good interdependence (Equation 13). The related parameters *C* and C_1 are also obtained and the coefficient of determination is 0.93. All the fitting data are listed in Table 7.

$$A = C + C_1 V_f \quad (13)$$

Table 7. Fitting parameters for Equation (13).

C (fixed value)	C_1		Number of points	Degrees of freedom	Residual sum of squares	R^2	Adj. R^2
	Value	Standard error					
56.67	-95.4	34.22	6	5	966.15	0.93	0.91

Based on the parameters *C*, C_1 and *B*, as well as combining Equations (12-13), Equation (14) is obtained. This means that when a cyclic loading is applied, the bending stiffness obtained from both positions (mid-span or near the support) has a direct relationship; i.e. one parameter can be inferred by the other if the sisal fiber content is fixed to give:

$$k_M = 56.67 - 95.4V_f + 2.0k_S \quad (14)$$

4. CONCLUSIONS

In this work, sisal fibers were used as a component in building materials to improve the stiffness of foam concrete. In particular, the effect of fiber content on the bending stiffness of foam concrete under static and fatigue loading was investigated under three-point bending conditions. For a more complete analysis, the bending stiffness at different positions of the beam (mid-span and near the support) was also measured and discussed. The main results under the range of conditions investigated are:

1. The foam concrete had different bending stiffness at different positions. In general, the bending stiffness at the mid-span is higher than that away from the mid-span whatever the sisal fiber volume fraction.
2. In static tests, the residual bending stiffness before the foam concrete fractures decreased with increasing sisal fiber content. From a normalized bending stiffness analysis, when the sisal fiber content reached 0.15%, the bending stiffness was minimal. At this volume fraction, the foam concrete had the maximum residual flexibility.
3. Under cyclic loading, the bending stiffness of foam concrete will degrade. For the neat foam concrete and foam concrete with low volume fraction of sisal fibers (0.05%), the bending stiffness degradation contained three stages: initial decrease, stable stage and a final rapid decreasing. If the sisal fiber content is above 0.05%, the initial stage was not obvious. In this case, the bending stiffness variation only contained two stages: a stable decrease followed by a final stage.
4. Using linear regression analysis, the bending stiffness of the neat foam concrete was linearly correlated to the logarithm of the fatigue life independent of the stress level. When the sisal fiber content was only 0.05%, the relationship between the bending stiffness and the fatigue life was similar as for the neat foam concrete. But above 0.05%, the bending stiffness degradation included two stages and the bending stiffness during the stable stage was linearly correlated to the fatigue life of the sisal fiber reinforced foam concrete.
5. The bending stiffness degradation obtained from both positions (mid-span (M) and near the support of the beam (S)) under cyclic loading are different. Using regression analyses, a mathematical model was proposed to represent the relationship between both bending stiffness (k_M and k_S).

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AUTHOR CONTRIBUTIONS

Conceptualization, J Huang. and PY Huang; methodology, J Huang. and D Rodrigue; software, J

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CONFLICT OF INTEREST

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

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REFERENCES

1. Liu W, Xu S, Li Q. Flexural behaviour of UHTCC-layered concrete composite beam subjected to static and fatigue loads. *Fatigue Fract Eng M*. 2013;36(8):738-749.
2. Luo S, Qian Z, Yang X, Lu Q. Fatigue behavior of epoxy asphalt concrete and its moisture susceptibility from flexural stiffness and phase angle. *Constr Build Mater*. 2017;145:506-517.
3. McGinnis MJ, Davis M, de la Rosa A, Weldon BD, Kurama YC. Strength and stiffness of concrete with recycled concrete aggregates. *Constr Build Mater*. 2017;154:258-269.
4. Zhong JQ, Gardoni P, Rosowsky D. Stiffness Degradation and Time to Cracking of Cover Concrete in Reinforced Concrete Structures Subject to Corrosion. *J Eng Mech-Asce*. 2010;136(2):209-219.
5. Liu Y, Jiang N, Deng Y, Ma Y, Zhang H, Li M. Flexural experiment and stiffness investigation of reinforced concrete beam under chloride penetration and sustained loading. *Constr Build Mater*. 2016;117:302-310.
6. Castel A, Vidal T, François R. Finite-Element Modeling to Calculate the Overall Stiffness of Cracked Reinforced Concrete Beams. *J Struct Eng-Asce*. 2012;138(7):889-898.
7. Monajemi H, Mazinani I, Ong ZC, Khoo SY, Kong KK, Karim R. Using local stiffness indicator to examine the effect of honeycombs on the flexural stiffness of reinforced concrete beams. *Measurement*. 2015;64:157-162.
8. Sebastian WM. Optimisation of flexural stiffness profiles to compensate for reduced ductility in hyperstatic reinforced concrete structures. *Eng Struct*. 2006;28(7):893-902.
9. Mohammadhassani M, Jumaat MZB, Chemrouk M, Maghsoudi AA, Jameel M, Akib S. An experimental investigation on bending stiffness and neutral axis depth variation of over-reinforced high strength concrete beams. *Nucl Eng Des*. 2011;241(6):2060-2067.
10. Sharaky IA, Baena M, Barris C, Sallam HEM, Torres L. Effect of axial stiffness of NSM FRP reinforcement and concrete cover confinement on flexural behaviour of strengthened RC beams: Experimental and numerical study. *Eng Struct*. 2018;173:987-1001.
11. Bonet JL, Romero ML, Miguel PF. Effective flexural stiffness of slender reinforced concrete columns under axial forces and biaxial bending. *Eng Struct*. 2011;33(3):881-893.
12. Caglar N, Demir A, Ozturk H, Akkaya A. A simple formulation for effective flexural stiffness of circular reinforced concrete columns. *Eng Appl Artif Intel*. 2015;38:79-87.
13. Xu T, Castel A. Modeling the dynamic stiffness of cracked reinforced concrete beams under low-amplitude vibration loads. *J Sound Vib*. 2016;368:135-147.
14. Nie J, Fan J, Cai CS. Stiffness and Deflection of Steel–Concrete Composite Beams under Negative Bending. *J Struct Eng-Asce*. 2004;130(11):1842-1851.
15. Liu J, Zhao Y, Yang Y, Chen YF. Bending Capacity and Elastic Stiffness for a Novel Configuration of Cold-Formed U-Shaped Steel-and-Concrete Composite Beams. *J Struct Eng-Asce*. 2019;145(10).
16. Roeder CW, Lehman DE, Bishop E. Strength and Stiffness of Circular Concrete-Filled Tubes. *J Struct Eng-Asce*. 2010;136(12):1545-1553.
17. Youssef MB, Lavergne F, Sab K, Miled K, Neji J. Upscaling the elastic stiffness of foam concrete as a three-phase composite material. *Cement Concrete Res*. 2018;110:13-23.
18. Rahman AF, Goh WI, Jhatial AA. Flexural Study of Reinforced Foamed Concrete Beam Containing Palm Oil Fuel Ash (POFA) and Eggshell Powder (ESP) as Partial Cement Replacement. *International Journal of Sustainable Construction Engineering Technology*. 2019;10(1).
19. Kytinou VK, Chalioris CE, Karayannis CG. Analysis of Residual Flexural Stiffness of Steel Fiber-Reinforced Concrete Beams with Steel Reinforcement. *Materials (Basel)*. 2020;13(12).
20. Amran YHM, Rashid RSM, Hejazi F, Safiee NA, Ali AAA. Response of precast foamed concrete sandwich

panels to flexural loading. *J Build Eng*. 2016;7:143-158.

21. Abd SM, Ghalib D. Flexural Behaviour of Lightweight Foamed Concrete Beams Reinforced with GFRP Bars. *Civil Engineering Journal*. 2018;4(2).
22. Liu Q, Du W, Uddin N, Zhou Z. Flexural Behaviors of Concrete/EPS-Foam/Glass-Fiber Composite Sandwich Panel. *Adv Mater Sci Eng*. 2018;2018:1-10.
23. Prabha P, Palani GS, Lakshmanan N, Senthil R. Flexural Behaviour of Steel-Foam Concrete Composite Light-Weight Panels. *Ksce J Civ Eng*. 2018;22(9):3534-3545.
24. Huang J, Tian G, Huang P, Chen Z. Flexural Performance of Sisal Fiber Reinforced Foamed Concrete under Static and Fatigue Loading. *Materials (Basel)*. 2020;13(14).
25. Falliano D, De Domenico D, Ricciardi G, Gugliandolo E. Compressive and flexural strength of fiber-reinforced foamed concrete: Effect of fiber content, curing conditions and dry density. *Constr Build Mater*. 2019;198:479-493.
26. Keerio MA, Lakhiar MT, Sohu S. Comparative Study on Flexural Performance of Foamed Concrete Beam Containing Plastic Fibers. *International Journal of Sustainable Construction Engineering Technology*. 2019;10(1).
27. Othuman Mydin MA, Zamzani NM, Ghani ANA. Experimental data on compressive and flexural strengths of coir fibre reinforced foamed concrete at elevated temperatures. *Data Brief*. 2019;25:104320.
28. Falliano D, De Domenico D, Ricciardi G, Gugliandolo E. Improving the flexural capacity of extrudable foamed concrete with glass-fiber bi-directional grid reinforcement: An experimental study. *Compos Struct*. 2019;209:45-59.
29. Frazao C, Barros J, Toledo R, Ferreira S, Goncalves D. Development of sandwich panels combining Sisal Fiber-Cement Composites and Fiber-Reinforced Lightweight Concrete. *Cement Concrete Comp*. 2018;86:206-223.
30. Ribeiro SLM, Oliveira PR, Vieira LMG, Panzera TH, Freire RTS, Scarpa F. Hybrid bio-composites reinforced with sisal-glass fibres and Portland cement particles: A statistical approach. *Compos Part B-Eng*. 2018;149:58-65.
31. Naraganti SR, Pannem RMR, Putta J. Impact resistance of hybrid fibre reinforced concrete containing sisal fibres. *Ain Shams Engineering Journal*. 2019;10(2):297-305.
32. Sabarish KV, Paul P, Bhuvaneshwari, Jones J. An experimental investigation on properties of sisal fiber used in the concrete. *Materials Today: Proceedings*. 2020;22:439-443.
33. Silva FD, Mobasher B, Toledo RD. Cracking mechanisms in durable sisal fiber reinforced cement composites. *Cement Concrete Comp*. 2009;31(10):721-730.
34. Silva FD, Mobasher B, Soranakom C, Toledo RD. Effect of fiber shape and morphology on interfacial bond and cracking behaviors of sisal fiber cement based composites. *Cement Concrete Comp*. 2011;33(8):814-823.
35. Fernandes EM, Mano JF, Reis RL. Hybrid cork–polymer composites containing sisal fibre: Morphology, effect of the fibre treatment on the mechanical properties and tensile failure prediction. *Compos Struct*. 2013;105:153-162.
36. Gujarati DN. *Linear regression : a mathematical introduction*. Los Angeles: Sage; 2019.
37. Zhang b, Wu k. Residual fatigue strength and stiffness of ordinary. *Cement Concrete Res*. 1997;27(1):115-126.
38. Oneschkow N. Fatigue behaviour of high-strength concrete with respect to strain and stiffness. *Int J Fatigue*. 2016;87:38-49.