Effect of particles shape on magnetohydrodynamics hybrid nanofluid flow and heat transfer in porous medium with slip condition

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Abstract

The objective of the present article is to analyze the magnetohydrodynamic hybrid nanofluid flow and heat transfer in a porous medium with the nanoparticles shape effect. The effect of thermal conductivity variation, slip condition and heat generation is also considered. The Sphere (spherical) and Lamina (non-spherical) shapes of Al_2O_3 and Cu nanoparticle are suspended in pure water to form hybrid nanofluid. The partial differential equations (PDEs) of motion are converted into ordinary differential equation (ODEs) by well-known similarity transformation. The resultant ordinary differential equation (ODEs) are solved analytically with the help of Homotopy analysis method (HAM). Furthermore, the coeffects of physical parameters on velocity profiles, temperature profiles and Nusselt number are also taken into consideration. It is found that the performance of the Lamina (non-spherical) shapes nanoparticles on temperature disturbance and heat transfer is better than Sphere (spherical) shape nanoparticles

Keywords: Nanoparticle shape,Nanofluid, Magnetic field, Homotopy analysis method.

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1. Introduction

Porous medium studied has many applications in medical sciences and industry. In field of medical science, porous medium phenomena used in transport process in kidneys and human lungs, clogging in arteries, gall bladder in the presence of stone, and it also used as blood vessel which cannot opposed. Several examples exist related natural porous medium such as seepage of water in river beds wood, limestone etc. Several researchers are attracted to discussed studied related porous medium because of technically and scientific importance such as metallurgy and earth science. These types of flow are examined at low Reynolds number in the existence of porous space. Many researchers discussed experimentally and analytically on the porous medium with respect to distinct aspects (1). Malik et al. (2) discussed the Carreau over a porous medium in the presence of pressure depend viscosity. Isfahani and Heyhat (3) performed experimental study of nanofluid flow in glass micromodel as porous medium.

Porous media is another interesting method for heat transfer in systems of industrial for example the use of metal-based exchanger i.e. copper form in the heat exchanger and channel. The technique of applying porous media and nanofluid has gained extensive attention and has led to a broad investigation in this field. Porous media enhance the contact surface area between solid surface and liquid also nanoparticles dispersed in nanofluid increase the effective thermal conductivity. Therefore, it looks that using nanofluid and porous media can enhance the ability of a typical thermal system intensely (4). Zuhra et al. (5) studied heat transfer and simultaneous flow of two (Williamson and Casson) liquid in the presence of cubic autocatalysis and gyrotactic microorganisms with chemical reaction through a porous medium.

Magnetohydrodynamic (MHD) flow over a flat surface has several significant industrial and technological applications for example micromixing of physiological samples, MHD pumps, drug delivery and biological transportation. Magnetic field application produces Lorentz force which can transport fluid in the mixing processes as an active micromixing technology method (6). Many researchers did work on MHD nanofluid. Matian et al. (7) discussed nanofluid flow past a stretching sheet. Also, he has been noted out by other, many applications of magnetic nanofluid: magnetogravimetric separation, aerodynamic sensor, micro/nano-structured magnetorheological fluid for semiactive vibration dampers, magnetofluidic leakage-free rotating seals, biological application in veterinary medicine and plant genetics. Recently, Umair et al. (8) discussed the effects of gold nanoparticles shape on MHD flow and heat transfer with the influence of radiation. Haroun et al. (9) examined numerical solution of hydromagnetic nanofluid flow over a porous stretching sheet. Rajput et al. (10) examined the magnetohydrodynamics fluid flow and heat transfer in a porous medium past over the exponentially stretching sheet.

Recently, the use of advanced nanofluid has gained considerable attention. The advance nanofluid is developed by combining dissimilar nanoparticles to improve the thermophysical characteristic of regular fluids. This group of nanofluid is known as hybrid nanofluid, experimentally has been analyzed by many scientists. In many researches, the mixture of metallic and metal oxide nanoparticles was suspended nanoparticles in base fluid (11). For example, Ghalamabaz et al. (12) studied the heat transfer of hybrid nanofluid flow over a vertical plate. The thermal conductivity of nanoparticles patronage the shape of nanoparticles (13). The motivation of the current study is to find the shape effect of Sphere (spherical) and Lamina (non-spherical) nanoparticles in hybrid nanofluid flow with considering the effect of thermal conductivity variation, slip condition and heat generation. Detailed discussion is explained over the subject with graphical description.

2. **Mathematical Formulation**

Consider that two-dimensional, laminar and steady Magnetohydrodynamic (MHD) hybrid nanofluid flow through stretching/shirking sheet amended in a porous medium. x,y are the coordinate direction along and perpendicular to the stretching/shirking sheet, respectively. $B=B_0\, e^{\frac{-x}{2L}}$ the magnetic field applied along y axis. $u_w=u_0\, e^{\frac{-x}{L}}$ is the surface velocity. The empirical shape factor m of nanoparticles is presented in Table 1. Also, Thermophysical characteristics of (Al_2O_3 and Cu)/pure water are presented in Table 2.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{u_{hn_f}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{hn_f}B^2}{\rho_{hnf}}u-\frac{u_{hn_f}}{\rho_{hnf}}\frac{\epsilon}{K1}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{hn_f}}{(\rho Cp)_{hnf}}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^{¿}}{3(\rho Cp)_{nf}K^{¿}}\frac{\partial^2 T}{\partial y^2}+\frac{1}{(\rho Cp)_{hnf}}\{\frac{\partial}{\partial y}[k¿¿hn_f(T)\frac{\partial T}{\partial y}]\}+\frac{Q_o}{(\rho Cp)_{hnf}}(T-T_\infty)¿$$

(3)

Where u and v are velocities of hybrid nanofluid along x and y axis respectively. v_w presents the suction/ injection parameter. T, T_w and T_∞ are temperature, wall temperature and ambient temperature respectively. Q_o represent heat absorption/generation. ε presents the porosity and $K1$ presents the permeability.

The boundary conditions are

$$u=Bu_w, v=v_w, T=T_w(x)+D_1(\frac{\partial T}{\partial y})\, at\, y=0$$

$$u=0, T\rightarrow T_\infty\, as\, y\rightarrow\infty.$$

(4)

The similarity transformations are

$$u=u_0 e^{\frac{-x}{L}}f'(\eta), v=-\sqrt{\frac{u_0 v_f}{2L}}e^{\frac{x}{2L}}[f(\eta)+\eta f'(\eta)],$$

$$\eta=y\sqrt{\frac{u_0}{2Lv_f}}e^{\frac{x}{2L}}, \psi=\sqrt{2u_0 v_f}Lf(\eta)e^{\frac{x}{2L}}, \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

(5)

The equations (1-3) become

$$\frac{B_1}{B_2}f'''+ff''-2f'^2-(\frac{B_3}{B_2}M-\frac{B_1}{B_2}K)f'=0, \tag{6}$$

$$\frac{1}{Pr}\left(B_4(2+\varepsilon\theta)+\frac{4}{3}R\right)\theta''+Pr B_5(f\theta'-f'\theta)+\varepsilon B_4\theta'^2+Pr S\theta=0 \tag{7}$$

Where R is the radiation parameter, Pr is the Prandtl number, M is the magnetic parameter, δ is the slip parameter and ε is the thermal conductivity variation and S is the heat absorption/generation. Furthermore, In the current study, we consider

$$B_{1=¿\frac{1}{¿¿}¿¿}$$

$$B_4=\frac{[k_{s2}+(m-1)kb_f]-(m-1)\phi_2(k_{bf}-k_{s2})}{[k_{S2}+(m-1)kb_f]+\phi_2(kb_f-k_{s2})}, \frac{k_{bf}}{k_f}=\frac{[k_{s1}+(m-1)kb_f]-(m-1)\phi_1(k_{bf}-k_{s1})}{[k_{S1}+(m-1)kb_f]+\phi_1(kb_f-k_{s1})}, 1\rho\sigma\sigma\sigma k$$

$$k\rho Cp\rho Cp$$

$$B_3=\frac{\sigma_{n_f}}{\sigma_f}=1+\frac{3+\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi},\frac{\sigma_{n_f}}{\sigma_f}=1+\frac{3+\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\left(\frac{\sigma_s}{\sigma_f}-1\right)\phi}$$

$$B_{5}=¿\frac{(1-\phi_2)\left[(1-\phi_1)(\rho Cp)_f+\phi_1(\rho Cp)_{s1}\right]+\phi_2(\rho Cp)_{s2}}{(\rho Cp)_f},¿ \tag{8}$$

The relevant boundary conditions are described as a fellow

$$f(0)=A, f'(0)=B, f'(\infty)=0, \theta(0)=1+\delta\theta'(0), \theta(\infty)=0. \tag{9}$$

It is noted that the surface is static when $A=0$, the surface is dwindles when $A<0$ the surface is extended and when $A>0$.

The Nusselt number Nu is specified as

$$Nu_x,=\frac{2L(x)}{k_f[T_w-T_\infty]}$$

Where, $q_w(x)=-k_{n_f}\left(\frac{\partial T}{\partial y}\right)_{y=o}+q_r(y)$

$$Nu=-(A_4+\frac{4}{3}R)\theta'(0) \tag{10}$$

3. **Solution procedure**

For Homotopy analysis method (HAM), the initial guesses and auxiliary linear operator are chosen in following as

$$L_f=f'''-f', L_\theta=\theta''-\theta, \tag{11}$$

which satisfied the following properties

$$L_f\left[Z_1+Z_2 e^{\eta}+Z_3 e^{-\eta}\right]=0, L_\theta\left[Z_4 e^{\eta}+Z_5 e^{-\eta}\right]=0, \tag{12}$$

where Z_1, Z_2, Z_3, Z_4 and Z_5 are arbitrary constants.

Let pϵ $[0,1]$ denotes an embedding parameter and $\hbar$f,$\hbar\theta \neq 0$ indicates the non-zero auxiliary parameters.

$$(1-p)L_f\left[\hat{f}(\eta,p)-f_0(\eta)\right]=p\hbar_f N_f\left[\hat{f}(\eta,p),\hat{\theta}_0(\eta,p)\right],$$

(13)

$$\hat{f}(0,p)=A, \hat{f}'(0,p)=B, \hat{f}(\infty,p)=0,$$

$$(1-p)L_\theta\left[\hat{\theta}(\eta,p)-\theta_0(\eta)\right]=p\hbar_\theta N_\theta\left[\hat{\theta}_0(\eta,p),\hat{f}(\eta,p)\right], \tag{14}$$

$$\hat{\theta}(0,p)=1+\hat{\theta}'(0),\hat{\theta}(\infty,p)=0,$$

3.1 **mth-order deformation equations**

$$L_f\left[f_m(\eta)-\chi_m f_{m-1}(\eta)\right]=\hbar_f R_f^m(f_{m-1}(\eta),\theta_{m-1}(\eta)), \tag{15}$$

$$f_m(0)=0,f_m'(0)=0,f_m'(\infty)=0,$$

$$L\,L_f\left[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)\right]=\hbar_\theta R_\theta^m(\theta_{m-1}(\eta),f_{m-1}(\eta)), \tag{16}$$

$$\theta_m(0)=0,\theta_m(\infty)=0.$$

Here

$$\chi_m=\begin{cases}0\,m\leq 1\\ 1\,m>1\end{cases}, \tag{17}$$

where,

$$R_f^m f_m(\eta)=\frac{B_1}{B_2}f_{m-1}'''(\eta)+\sum_{z=0}^{m-1}f_{m-1}f_{m-1-z}''-2\sum_{z=0}^{m-1}f_z'f_{m-1-z}'-(\frac{B_3}{B_2}M-\frac{B_1}{B_2}K)f_{m-1}'=0, \tag{18}$$

$$R_f^m\theta_m(\eta)=\frac{1}{Pr}\left(B_4(2+\varepsilon\theta)+\frac{4}{3}R\right)\theta_{m-1}''+Pr\,B_5[\sum_{z=0}^{m-1}f_z\theta_{m-1-z}'-\sum_{z=0}^{m-1}f_z'\theta_{m-1}]+\varepsilon B_4\sum_{z=0}^{m-1}\theta_z'\theta_{m-1-z}'+Pr\,S\,\theta_{m-1}=0, \tag{19}$$

4. Graphically Illustration

The obtained analytical solution in the previous section has been examined graphically for various physical parameters here. Figure 1 displays the Geometry model of the problem. Figure 2 shows the temperature profile with the effect of nanoparticle shape, it is observed from figure 2 lamina (non-spherical) shapes have betters' performance in temperature disturbance as compared to the Sphere (spherical) nanoparticles. Figure 3 illustrates the effect of magnetic fields on velocity and temperature profiles. It is noted from the figure velocity of hybrid nanofluid decrease while temperature profile increase with increase the magnetic parameter. The impact of porous media parameter on velocity and temperature of hybrid nanofluid flow is portrayed in Figure 4, it is noted from figure 4 velocity of hybrid nanofluid is increasing function of porous media parameter and temperature of hybrid nanofluid decreases with enhancing the porous media parameter. Variation in velocity and temperature profiles in case of suction is depicting in figure 5, it is found that velocity and temperature of hybrid nanofluid decrease in case of suction. Figure 6 display the variation in velocity and temperature of hybrid nanofluid in case of stretching, figure 6 presents that the velocity of hybrid nanofluid increase on a case of stretching, while temperature decreases. Figure 7 explains the effect of slip parameter on the temperature of hybrid nanofluid of the slip parameter, it is noticed from figure 7 temperature of hybrid nanofluid has inverse relation with slip. Figure 8 portrayed the temperature disturbance of hybrid nanofluid under the influences of thermal conductivity variation, the temperature of hybrid nanofluid is decrease with the increase the thermal conductivity variation. The effect of thermal radiation on temperature distribution is illustrate in figure 9, it is noticed that the temperature of hybrid nanofluid is increasing function of thermal radiation. Figures 10-11 present the heat transfer rate in hybrid nanofluid with the variation of porous media parameter and slip parameter. It is observed from figure 10-11 heat transfer rate increase with increase the porous media parameter and slip parameter. Figure 12 presents the heat transfer rate of hybrid nanofluid decrease with increase radiation parameter.

5. **Mean finding**

Here we scrutinize the magnetohydrodynamic hybrid nanofluid flow and heat transfer in a porous medium with the nanoparticles shape effect. The behavior of thermal conductivity variation, slip condition and heat generation is also studied. The main key points of present analysis are listed below

- Sphere (spherical) shape nanoparticles play dramatic role in temperature disturbance.
- Lamina (non-spherical) shapes show lowest performance in temperature disturbance.
- Sphere (spherical) shape nanoparticles play dramatic role in heat transfer.
- Lamina (non-spherical) shapes show lowest performance in heat transfer.

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