Table 1 A summary of equations of Euler gas and granular phases and discrete particles

(I) Constitutive equations of Euler gas phase

$$\bar{\bar{\tau}}_g = \alpha_g \mu_{ge} \left[\nabla \boldsymbol{u}_g + \left(\nabla \boldsymbol{u}_g \right)^T \right] - \frac{2\alpha_g}{3} \mu_{ge} \left(\nabla \cdot \boldsymbol{u}_g \right) \tag{T1}$$

(a) Effective viscosity of Euler gas phase

$$\mu_{ge} = \mu_{gl} + \mu_{gt} = \mu_{gl} + C_\mu \rho_g \frac{k_g^2}{\varepsilon_g} \tag{T2}$$

(b) Turbulence interactions of Euler gas phase and Euler granular phase

$$k_{gs} = \frac{\alpha_g \rho_g \eta_t}{\alpha_g \rho_g + (\alpha_g \rho_g + \alpha_s \rho_s)\eta_t} (2k_g + 3\frac{\alpha_s \rho_s}{\alpha_g \rho_g} \theta) \tag{T3}$$

$$\eta_t = \frac{3}{2} \frac{C_\mu \beta_{\mathrm{gs}}}{\alpha_{\mathrm{s}} \rho_{\mathrm{s}}} [1 + C_\beta \frac{3\left| \mathrm{u_g} - \mathrm{u_s} \right|^2}{2k_{\mathrm{g}}}]^{-0.5} \frac{k_{\mathrm{g}}}{\varepsilon_{\mathrm{g}}} \tag{T4}$$

(II) Constitutive equations of Euler granular phase

$$\bar{\bar{\tau}}_s = \alpha_s \mu_s \left[\nabla \boldsymbol{u}_s + \left(\nabla \boldsymbol{u}_s \right)^T \right] + \alpha_s \left(\lambda_s - \frac{2}{3} \mu_s \right) \left(\nabla \cdot \boldsymbol{u}_s \right) \bar{\bar{I}} \tag{T5}$$

(a) Granular pressure

$$p_s = p_c + p_f \tag{T6}$$

$$p_c = \left[1 + 2\left(1 + e_s \right) \alpha_s g_o \right] \alpha_s \rho_s \theta \tag{T7}$$

$$p_f = \begin{cases} 10^{25} \left(\alpha_s - \alpha_{s,\max} \right)^{10} & \alpha_s > \alpha_{s,\max} \\ 0 & \alpha_s \le \alpha_{s,\max} \end{cases} \tag{T8}$$

(b) Granular shear viscosity

$$\mu_s = \mu_c + \mu_f \tag{T9}$$

$$\mu_c = \frac{10\rho_s d_s \sqrt{\pi\theta}}{96\alpha_s g_o (1+e_s)}\left[1+\frac{4}{5}\alpha_s g_o (1+e_s)\right]^2 + \frac{4}{5}\alpha_s \rho_s d_s g_o (1+e_s)\sqrt{\frac{\theta}{\pi}} \tag{T10}$$

$$\mu_f = \frac{p_f \sin\phi}{2\sqrt{I_{2D}}} \tag{T11}$$

(c) Granular bulk viscosity

$$\lambda_s = \frac{4}{3}\alpha_s \rho_s d_s g_o (1+e_s)\sqrt{\frac{\theta}{\pi}} \tag{T12}$$

(d) Collisional dissipation of Euler granular phase

$$\gamma_s = 3\left(1-e_s^2\right) g_o \rho_s \alpha_s^2 [\frac{4}{d_s}\sqrt{\frac{\theta}{\pi}} - \nabla \cdot \vec{u}_s] \tag{T13}$$

$$\gamma_w = \frac{\sqrt{3}\pi(1-e_w^2)\alpha_s \rho_s g_o \theta^{3/2}}{4\alpha_{s,\max}} \tag{T14}$$

(e) Conductivity of the fluctuating energy of Euler granular phase

$$k_s = \frac{150\rho_s d_s \sqrt{\pi\theta}}{384(1+e_s) g_o}\left[1+\frac{6}{5}\alpha_s g_o (1+e_s)\right]^2 + 2\alpha_s^2 \rho_s d_s g_o (1+e_s)\sqrt{\frac{\theta}{\pi}} \tag{T15}$$

(III) Contact forces of Lagrangian discrete particles

$$\boldsymbol{f}_{nij} = (-k_n \boldsymbol{\delta}_{nij}^{3/2} - \eta_n \boldsymbol{v}_{rij} \cdot \boldsymbol{n}_{ij})\boldsymbol{n}_{ij} \tag{T16}$$

$$\boldsymbol{f}_{tij} = \begin{cases} -k_t \boldsymbol{\delta}_{tij} - \eta_t \boldsymbol{v}_{tij} & (\left|\boldsymbol{f}_{tij}\right| < \mu_f \left|\boldsymbol{f}_{nij}\right|) \\ -\mu_f \left|\boldsymbol{f}_{nij}\right| \frac{\boldsymbol{v}_{tij}}{\left|\vec{v}_{tij}\right|} & (\left|\boldsymbol{f}_{tij}\right| \quad \mu_f \left|\boldsymbol{f}_{nij}\right|) \end{cases} \tag{T17}$$

(a) Relative velocity of Lagrangian discrete particles

$$\boldsymbol{v}_{nij} = \boldsymbol{v}_j - \boldsymbol{v}_i \tag{T18}$$

$$\boldsymbol{v}_{tij} = \boldsymbol{v}_{nij} - (\boldsymbol{v}_{nij} \cdot \boldsymbol{n})\boldsymbol{n} + 0.5(r_i + r_j)(\boldsymbol{\omega}_i + \boldsymbol{\omega}_j) \times \boldsymbol{n} \tag{T19}$$

(b) Spring and damping coefficients of Lagrangian discrete particles

$$k_n = \frac{4}{3}\left(\frac{1-E_i^2}{G_i} + \frac{1-E_j^2}{G_j}\right)^{-1} \sqrt{\frac{d_i d_j}{2(d_i + d_j)}} \tag{T20}$$

$$k_t = 8\left(\frac{2-E_i}{G_i} + \frac{2-E_j}{G_j}\right)^{-1} \sqrt{\frac{d_i d_j}{2(d_i + d_j)}} \delta_n^{1/2} \tag{T21}$$

$$\eta_n = \eta_t = \alpha \sqrt{k_n \frac{m_i m_j}{m_i + m_j}} \delta_n^{1/4} \tag{T22}$$

(c) Elastic potential energy of discrete particles and particles-wall

$$\Delta E_{ep} = \sum_{i=1}^{N} (f_{cn,ij} \cdot v_{cn,ij} + f_{ct,ij} \cdot v_{ct,ij}) \Delta t \tag{T23}$$

$$\Delta E_{ew} = \sum_{i=1}^{N_w} (f_{cn,iw} \cdot v_{cn,iw} + f_{ct,iw} \cdot v_{ct,iw}) \Delta t \tag{T24}$$

(d) Impact energy dissipations of discrete particles and particles-wall

$$\Delta E_{ip} = \sum_{i=1}^{N} (f_{dn,ij} \cdot v_{dn,ij} + f_{dt,ij} \cdot v_{dt,ij}) \Delta t \tag{T25}$$

$$\Delta E_{iw} = \sum_{i=1}^{N_w} (f_{dn,iw} \cdot v_{dn,iw} + f_{dt,iw} \cdot v_{dt,iw}) \Delta t \tag{T26}$$

(e) Friction energy dissipation of discrete particles and particles-wall

$$\Delta E_{fp} = \sum_{i=1}^{N} (f_{cs,ij} \cdot v_{ct,ij}) \Delta t \tag{T27}$$

$$\Delta E_{fw} = \sum_{i=1}^{N_w} (f_{cs,iw} \cdot v_{ct,iw}) \Delta t \tag{T28}$$

(f) Rolling energy dissipation of discrete particles and particles-wall

$$\Delta E_{rp} = \sum_{i=1}^{N} (T_{ij} \cdot \omega_{ij}) \Delta t \tag{T29}$$

$$\Delta E_{rw} = \sum_{i=1}^{N} (T_{iw} \cdot \omega_{iw}) \Delta t \tag{T30}$$

Table 2 Simulation parameters of spout fluidized bed

Parameter	Value	Parameter	Value
Bed width (m)	0.145	Bed transverse thickness (m)	0.02
Bed height (m)	1.0	Spout inlet width(m)	0.05
Spout inlet depth(m)	0.02	Background velocity (U_{bg}),	2.4
Spouting velocity (m/s)	43.5	Particle number	12000
Particles diameter (mm)	3	Particles density (kg/m^3)	2505
Elastic modulus of particle (Pa)	7×10^{10}	Poisson's ratio of particle	0.33
Elastic modulus of wall (Pa)	7×10^{10}	Poisson's ratio of wall	0.3
Friction coefficient of particles	0.1	Friction coefficient of wall	0.1
Yield stress of particles (MPa)	280	Specularity coefficient	0.1
Air temperature (K)	298	Gas viscosity (kg/(m s))	1.8×10^{-5}