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On Solving Multi - Objective Fractional Linear Programming Problem with Intuitionistic Fuzzy Coefficients

Kirti Sharma · Vishnu Pratap Singh

Abstract In this paper, an approach for solving Intuitionistic Fuzzy Multi Objective Linear Fractional Programming Problem (IFMOLFPP) has been presented. The coefficients of objective functions, resources and all the technological coefficients are taken to be triangular intuitionistic fuzzy number (TIFN). In this paper, component wise optimization method has been adopted to handle the impreciseness. First, each intuitionistic fuzzy fractional objective is transformed into an equivalent crisp multiple fractional objectives and then using fuzzy programming approach, an optimal solution is obtained for every intuitionistic fuzzy fractional objective. Then, optimal solution of each individual intuitionistic fuzzy fractional objective is used to transform it into a corresponding intuitionistic fuzzy linear objective and thus the original problem now transforms to Multi Objective Intuitionistic fuzzy LPP which is further transformed into an Intuitionistic Fuzzy LPP with single objective and the problem is then solved by using component wise optimization and fuzzy goal programming approach. A numerical example is also presented to explain the methodology of the approach defined in this paper.

Keywords Intuitionistic Fuzzy Number · Multi Objective Linear Programming Problem · Fractional Linear Programming Problem · Fuzzy Programming Approach · Fuzzy Goal Programming Problems.

1 Introduction

Various fields of optimization are concerned about the maximization (minimization) of one or several ratios which usually measure the efficiency of the system. Real life examples of such optimization problems exists in production planning where ratios like profit/cost, inventory/sales are to be optimized. Many researchers have investigated various methods for solving linear fractional programming problems (LFPP). In [1], [2], [3], [4] and [5], various authors attempted to solve Linear Fractional Programming Problem.

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In most of the businesses, usually more than one fractional objectives are to be optimized. Authors like Nuran Guzel [6], [7] Chakraborty and Gupta [8], D. Dutta et. al [9] and Luhandjula [10] proposed several methods for solving MOLFPP in recent decades. In real life situations, the technological coefficients of an optimization problem are not always exact and depend upon market conditions which are not always stable or may depend upon some uncontrollable factors. This gives rise to decision making in imprecise environment. In these situations, the normal LPP gets converted into fuzzy LPP. In some situations, the imprecise coefficients may be provided with some level of hesitation. Intuitionistic fuzzy numbers(IFNs) can be used to model such situations effectively. Some of the remarkable works include [11], [12] and [13] for solving IFLFPP. Singh and Yadav [14] also proposed a method for solving IFMOLPP.

Various organizations face problems of optimizing multiple fractional objectives. For example, in case of institute planning, better quality and better returns are the two major objectives. The quality of an institute is measured by Student/Faculty ratio and returns are measured by Revenue/Investment. In real life situations, the number of students and faculties can not be told precisely at the time of planning and there is always a scope of hesitation because of various uncontrollable factors. As far as the author's knowledge in the problem domain is considered, a multi-objective fractional LPP with TIFNs as its coefficients has not been modelled till date. A possible model of an IFMOLFPP can be presented in case of planning and production purposes. For instance, Let us assume the expansion process of an educational institute, say XYZ institute, which wants to open one more branch in a new city. The administrators of the institute wants to decide the number of batches of three different subjects, say S_1, S_2 and S_3 they should operate in the new branch. For example, if the information collected from the past experiences is given by:

*"**Around 50** students can be accommodated in a batch of subject S_1, **around 30** students can be accommodated in a batch of subject S_2 and **around 45** students can be accommodated in a batch of S_3 and the faculties required for proper functioning of the institute are **approximately 5** for S_1, **approximately 4** for subject S_2 and **approximately 6** for subjects S_3. The fees for subject S_1, S_2 and S_3 is \$20, \$40 and \$30 respectively and the salaries of faculties for these subjects in the same order are \$200, \$300 and \$250. An investment of around \$1000 is required for infrastructure and approximately 5 staff members are required for administration and other works in the new branch. The constraints for the problem are the upper bound and lower bound on the total number of batches and non-negativity restrictions implies that the branch of the institute should provide coaching for all the three subjects. "*

The information revealed above can be represented in the form of MOLFPP with intuituionistic fuzzy coefficients. To the best of author's knowledge, no solution procedure has been derived for solving IF-MOLFPP till date. In this paper, we have proposed a solution method for solving Intuitionistic fuzzy MOLFPP (IFMOLFPP) using fuzzy mathematical programming approach and various membership functions for defining aspiration levels of the decision maker. This paper is organized as follows: In Section 2, preliminaries and concepts regarding Intuitionistic fuzzy set theory, Fractional LPP and MOLPP have been reviewed. Some important theorems regrading Fractional LPP, Multi - Objective LPP has been reviewed. In section 3, an IFMOLPP has been formulated and its solution methodology has been discussed. In Section 4, an algorithm for the same has been presented. In Section 6, a numerical example has been presented to explain the adopted methodology and the last section comprises of the concluding remarks.

2 Preliminaries and Concepts

Definition 1 Let X be a universe of discourse. Then an intuitionistic fuzzy set $\tilde{A}^I$ in X [15] is defined by

$$\tilde{A}^I = \{< x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x) >: x \in X\} \tag{1}$$

where $\mu_{\tilde{A}^I}(x)$ and $\vartheta_{\tilde{A}^I}(x)$ represents the degree of membership and degree of non-membership of element x in $\tilde{A}^I$, respectively. $h(x) = 1 - \mu_{\tilde{A}^I}(x) - \vartheta_{\tilde{A}^I}(x)$ represents the degree of hesitation for x.

Definition 2 A Triangular Intuitionistic fuzzy number (TIFN) [15] $\tilde{A}^I$ is an IFN with the membership function and non-membership function given by Eq. 2 and Eq. 3 respectively.

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-a}{b-a} & if\, a \le x \le b \\ 1 & if\, x = b \\ \dfrac{c-x}{c-b} & if\, b \le x \le c \\ 0 & otherwise \end{cases} \tag{2}$$

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \dfrac{b-x}{b-a'} & if\, a' \le x \le b \\ 1 & if\, x = b \\ \dfrac{x-c'}{c'-b} & if\, b \le x \le c' \\ 0 & otherwise \end{cases} \tag{3}$$

where $a' \le a \le b \le c \le c'$. This TIFN is denoted by $(a, b, c; a', b, c')$. The set of all TIFNs is denoted by $IF(R)$.

Definition 3 Arithmetic Operations on TIFNs [15] Let $\tilde{A}^I = (a, b, c; a', b, c')$ and $\tilde{B}^I = (p, q, r; p', q, r')$ be two TIFNs, then

1. Addition: $\tilde{A}^I \oplus \tilde{B}^I = (a + p, b + q, c + r; a' + p', b + q, c' + r')$.
2. Subtraction: $\tilde{A}^I \ominus \tilde{B}^I = (a - r, b - q, c - p; a' - r', b - q, c' - p')$.
3. Multiplication: $\tilde{A}^I \odot \tilde{B}^I = (l_1, l_2, l_3; l_1', l_2, l_3')$
   where $l_1 = min\{ap, ar, cp, cr\}$, $l_3 = max\{ap, ar, cp, cr\}$, $l_1' = min\{a'p', a'r', c'p', c'r'\}$, $l_3' = max\{a'p', a'r', c'p', c'r'\}$ and $l_2 = bq$.
4. Scalar Multiplication:
$$k\tilde{A}^I = \begin{cases} (ka, kb, kc; ka', kb, kc') & if\, k \ge 0 \\ (kc, kb, ka; kc', kb, ka') & if\, k < 0 \end{cases}$$
5. Division: $\tilde{A}^I / \tilde{B}^I = \{\frac{a}{r}, \frac{b}{q}, \frac{c}{q}; \frac{a'}{r'}, \frac{b}{q}, \frac{c'}{p'}\}$

2.1 Linear Fractional Programming Problem (LFPP)

A general form of LFPP is discussed in this subsection. A brief description of Charnes and Cooper [1] linear transformation is presented and a linear programming problem equivalent to LFPP is also presented. The generalized LFPP can be written as:

$$\begin{aligned} & Maximize\ Z(x) = \frac{N(x)}{D(x)} = \frac{c^T x + \alpha}{d^T x + \beta} \\ & \text{subject to} \qquad x \in S = \{x \in R^n : Ax \leq b, x \geq 0\}. \end{aligned} \tag{4}$$

where $A \in R^{m*n}, c, d \in R^n$ and $\alpha, \beta \in R$

For some values of x, $D(x)$ may be zero. To avoid such cases, we require either $\{Ax \leq b, x \geq 0, D(x) > 0\}$ or $\{Ax \leq b, x \geq 0, D(x) < 0\}$. For convenience, we consider the first case i.e. $\{Ax \leq b, x \geq 0, D(x) > 0\}$.

Theorem 1 *[1] Consider the following LFPP*

$$\begin{aligned} & Maximize\ \frac{N(x)}{D(x)} \\ & subject\ to\ Ax \leq b \\ & \qquad\qquad x \geq 0 \end{aligned} \tag{5}$$

Then the problem (5) is equivalent to (6), where (6) is obtained from (5) by using the transformation $t = \dfrac{1}{D(x)}, y = tx$ and the denominator of the objective function is restricted to be lesser than 1.

$$\begin{aligned} \text{Maximize} \quad & tN(\frac{y}{t}) \\ \text{subject to } & A\frac{y}{t} - b \leq 0 \\ & tD(\frac{y}{t}) \leq 1 \\ & y \geq 0 \\ & t > 0 \end{aligned} \tag{6}$$

Theorem 2 *[8] Let for some $x \in S$, $N(x) > 0$ and if (5) reaches a (global) maximum at $x = x^*$, then (6) reaches a (global) maximum at a point $(t, y) = (t^*, y^*)$ where $y^*/t^* = x^*$ and objective function at these points are equal.*

Theorem 3 *[16], [6] A solution $z^* = \dfrac{N(x^*)}{D(x^*)}$ is said to be an optimal solution of eq. (5) if and only if it is a solution of $F(z^*, x^*) = 0$, where $F(z^*, x^*) = max\{N(x) - z^* D(x), x \in S\}$*

2.2 Multi Objective Linear Fractional Programming Problem (MOLFPP)

A MOLFPP can be written as

$$\begin{aligned} & \text{Maximize } Z_i(x) = \frac{\sum c_j x_j + \alpha_i}{\sum d_j x_j + \beta_i} = \frac{N_i(x)}{D_i(x)} \\ & \text{subject to } x \in S = \{x \in R^n : Ax \leq b, x \geq 0\} \end{aligned} \tag{7}$$

Theorem 4 *[14] [6] $x^* \in S$ is an efficient solution of MOLFPP if there is no $x \in S$ such that $Z_i(x) \geq Z_i(x^*)$ i = 1, 2, ... k; $Z_i(x) > Z_i(x^*)$ for at least i.*

3 Problem Formulation and Model Development

A standard form of MOIFLFPP[17] is given by:

$$\begin{aligned} &Maximize\ \tilde{Z}^I(x) && = [\tilde{Z}_1^I(x)\tilde{Z}_2^I(x), ..., \tilde{Z}_k^I(x)] && \\ &\text{subject to}\ \sum \tilde{a_{ij}^I} x_j \leq \tilde{b_i^I} && && i = 1,2,...,m \\ & x_j && \geq 0 && j = 1,2,...,n. \end{aligned} \tag{8}$$

$$\text{where} \qquad \tilde{Z}_s^I(x) = \frac{\tilde{N}_s^I(x)}{\tilde{D}_s^I(x)} = \frac{\tilde{c_s^I} x + \tilde{\alpha_s^I}}{\tilde{d_s^I} x + \tilde{\beta_s^I}} \qquad s = 1,2,...,k$$

and $\tilde{c_s^I}, \tilde{d_s^I} \in IF(R^n)), \tilde{\alpha_s^I}, \tilde{\beta_s^I} \in IF(R)$ for all $s = 1,2,...,k$, $x \in R^n, \tilde{b_i^I} \in IF(R)$ for $i = 1,2,...,m$. $\tilde{a_{ij}^I} \in IF(R^{m\times n})$ for all $i = 1,2,...,m; j = 1,2,...,n$. Therefore, the problem (8) can be written as

$$\begin{aligned} Maximize \quad & \tilde{Z}^I(x) = [\tilde{Z}_1^I(x), \tilde{Z}_2^I(x), ..., \tilde{Z}_k^I(x)] \\ where \quad & \tilde{Z}_s^I(x) = \frac{\sum(c_{sj1}, c_{sj2}, c_{sj3}; c'_{sj1}, c_{sj2}, c'_{sj3})x_j + (\alpha_{s1}, \alpha_{s2}, \alpha_{s3}; \alpha'_{s1}, \alpha_{s2}, \alpha'_{s3})}{\sum(d_{sj1}, d_{sj2}, d_{sj3}; d'_{sj1}, d_{sj2}, d'_{sj3})x_j + (\beta_{s1}, \beta_{s2}, \beta_{s3}; \beta'_{s1}, \beta_{s2}, \beta'_{s3})} \quad s = 1,2,...k \end{aligned}$$

subject to :

$$\begin{aligned} (a_{ij1}, a_{ij2}, a_{ij3}; a'_{ij1}, a_{ij2}, a'_{ij3})x_j &\leq (b_{i1}, b_{i2}, b_{i3}; b'_{i1}, b_{i2}, b'_{i3}) && i = 1,2,...,m \\ x_j &\geq 0 && j = 1,2,...,n \end{aligned} \tag{9}$$

First we use the concept of component wise optimization and find the optimal solution of each objective $\tilde{Z}_s^I, s = 1,2,...,k$ by reducing IFFLPP into a MOLPP. That is, an intuitionistic fuzzy linear fractional programming problem can be reduced into multi objective linear programming problem as:

$$\begin{aligned} Maximize \quad & Z_s^1(x) = \frac{\sum c_{sj1}x_j + \alpha_{s_1}}{\sum d_{sj3}x_j + \beta_{s_3}} && Z_s^2(x)\frac{\sum c_{sj2}x_j + \alpha_{s_2}}{\sum d_{sj2}x_j + \beta_{s_2}} \\ & Z_s^3(x) = \frac{\sum c_{sj3}x_j + \alpha_{s_3}}{\sum d_{sj1}x_j + \beta_{s_1}} && Z_s^4(x)\frac{\sum c'_{sj1}x_j + \alpha'_{s_1}}{\sum d'_{sj3}x_j + \beta'_{s_3}} \\ & Z_s^5(x) = \frac{\sum c'_{sj3}x_j + \alpha'_{s_3}}{\sum d'_{sj1}x_j + \beta'_{s_1}} \end{aligned}$$

subject to

$$\begin{aligned} &\sum a_{ij1}x_j \leq b_{i1} \\ &\sum a_{ij2}x_j \leq b_{i2} \\ &\sum a_{ij3}x_j \leq b_{i3} \\ &\sum a'_{ij1}x_j \leq b'_{i1} \\ &\sum a'_{ij3}x_j \leq b'_{i3} \\ &\quad x_j \geq 0 \qquad i = 1,2,...,m \\ &\qquad\qquad\quad j = 1,2,...,n \end{aligned} \tag{10}$$

Now, Charnes and Cooper Transformation can be applied on (10) and a corresponding MOLPP can be obtained. Then, solving each set of objectives $Z_s^q, q = 1,2,3,4,5$ separately gives their optimal solution, let it be denoted by Z_s^{q*}. Using Fuzzy Mathematical Programming [14] approach, the optimal solution for every individual intuitionistic fuzzy objective $\tilde{Z}_s$ can be obtained. In order to find an efficient solution for vector $\tilde{Z}^I$, each fractional objective $\tilde{Z}_s^I$ can be written as $\tilde{N}_s^I - \tilde{Z}_s^{I^*}\tilde{D}_s^I$ and multiple objectives can be optimized by optimizing the sum of all linear forms of given fractional objectives while giving an equal weight to every objective, i.e., in order to achieve efficient solution for

(8), the corresponding IFMOLPP can be converted to IFLPP by reducing the objective function to $\sum_{s=1}^{k} \tilde{N}_s^I(x) - \tilde{Z}_s^{I^*} \tilde{D}_s^I(x) = \sum_{j=1}^{n}(w_{j1}, w_{j2}, w_{j3}; w'_{j1}, w_{j2}, w'_{j3})x_j + (o_1, o_2, o_3; o'_1, o_2, o'_3)$ Again the concept of component wise optimization can be used to reduce IFLPP to an equivalent MOLPP. The corresponding equivalent MOLPP is given by (11) which is given by:

$$
\begin{aligned}
&Maximize \quad && O_1(x) = w_{j1}x_j + o_1 \quad O_2(x) = w_{j2}x_j + o_2\\
& && O_3(x) = w_{j3}x_j + o_3 \quad O_4(x) = w'_{j1}x_j + o'_1\\
& && O_5(x) = w'_{j3}x_j + o'_3\\
&\text{subject to constraints of (10)}
\end{aligned}
\tag{11}
$$

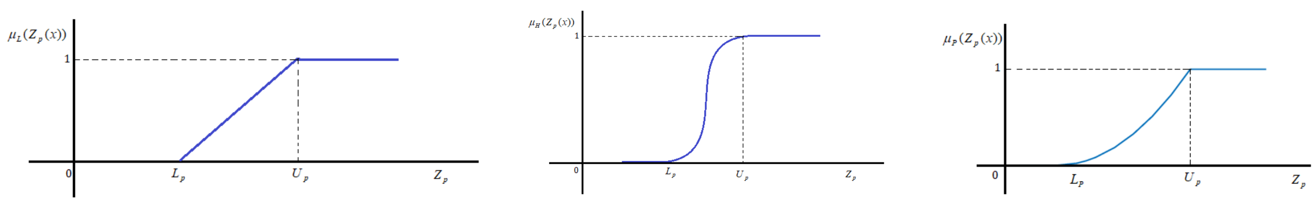
Now, we assign a suitable goal for each objective functions of MOLPP (11). The best way to assign a goal is to solve each objective separately subject to the same set of constraints and take it as the most desirable goal and we denote it by $U_p, p = 1, 2, ..., 5$. In this way, we find at max 5 different solutions for 5 linear programming problems, say $S = \{X_1, X_2, X_3, X_4, X_5\}$. Then we calculate the minimum value of all the 5 objectives at all the points of set S. That is, the minimum value of the objective $O_p, p = 1, 2, ..., 5$ on S and we denote it by L_p, i.e. $L_p = min\{O_p(X), X \in S, p = 1, 2, ..., 5\}$. Since the goal is a fuzzy goal, which means some deviations are allowed in the strict goal. To change the fuzzy goal programming model into a crisp LPP, we define different types of linear and non-linear membership functions. A linear, parabolic and hyperbolic membership functions can be defined by 12, 13 and 14 respectively :

$$
\mu_L(Z_p(x)) = \begin{cases} 0 & if\ Z_p \le L_p \\ \dfrac{Z_p - L_p}{U_p - L_p} & if\ L_p \le Z_p < U_p \\ 1 & if\ Z_p \ge U_p \end{cases} \tag{12}
$$

$$
\mu_P(Z_p(x)) = \begin{cases} 0 & if\ Z_p \le L_p \\ (\dfrac{Z_p - L_p}{U_p - L_p})^2 & if\ 0 < L_p \le Z_p < U_p \\ 1 & if\ Z_p \ge U_p \end{cases} \tag{13}
$$

$$
\mu_H(Z_p(x)) = \begin{cases} 0 & if\ Z_p \le L_p \\ \dfrac{1}{2} + \dfrac{1}{2}tanh(Z_p(x) - \dfrac{U_p + L_p}{2})\alpha_p & if\ 0 < L_p \le Z_p < U_p \\ 1 & if\ Z_p \ge U_p \end{cases} \tag{14}
$$

where $\alpha_p = \dfrac{6}{U_p - L_p}$. Now, various membership functions (Fig.1) can be used according to the satisfaction



(a) Linear Membership Function (b) Hyperbolic Membership Function (c) Parabolic Membership Function

Fig. 1: Linear/Non-Linear Membership Functions

level of the decision maker and a crisp programming model of the fuzzy goal programming model can be constructed. A corresponding crisp programming model for the fuzzy goal programming model (**??**) can be written as (15)

$$
\begin{aligned}
&Maximize \quad \lambda \\
&\text{subject to} \\
&\qquad \mu(O_p(X)) \geq \lambda \qquad p = 1, 2, ..., 5 \\
&\qquad \mu(O_p(X)) \leq 1 \qquad p = 1, 2, ..., 5 \\
&\qquad \sum a_{ij1}x_j \leq b_{i1} \\
&\qquad \sum a_{ij2}x_j \leq b_{i2} \\
&\qquad \sum a_{ij3}x_j \leq b_{i3} \\
&\qquad \sum a'_{ij1}x_j \leq b'_{i1} \\
&\qquad \sum a'_{ij3}x_j \leq b'_{i3} \\
&\qquad x_j \geq 0 \qquad i = 1, 2, ..., m \\
&\qquad\qquad\qquad j = 1, 2, ..., n
\end{aligned}
\tag{15}
$$

The crisp programming model (15) can then be solved with the help of various software packages and algorithms.

4 Algorithm

The whole solution procedure developed in Section 3 has been summarized as an algorithm in this section:
Step 1: Use component wise optimization and reduce every intuitionistic fuzzy linear fractional objective into 5 different linear fractional objectives and in the same way, one intuitionistic fuzzy constraint can also be further reduced to 5 constraints with crisp technological coefficients.
Step 2: Use Charnes and Cooper transformation to convert LFPP to LPP.
Step 3: Find the optimal solution of each linear objective separately.
Step 4: Use fuzzy programming approach to find the optimal solution of MOLPP.
Step 5: Using the obtained value of y and t, calculate vector x and thus the optimal value of each intuitionistic fuzzy objective can be calculated, separately.
Step 6: Using the optimal solution of each intuitionistic fuzzy objective, the IFMOLFPP can be converted to IFLPP by Maximizing $\sum_{s=1}^{k} \tilde{N}_s^I(x) - \tilde{Z}_s^{I^*} \tilde{D}_s^I(x)$.
Step 7: Component wise optimization can be used again to reduce IFLPP to MOLPP.
Step 8: Solve each objective separately and let the optimal solution be denoted by U_p and the optimal solution set be denoted by S.
Step 9: Find the minimum value of each objective function on the set S; let it be called L_p.
Step 10: Convert the MOLP model into a fuzzy goal programming model, where the goal is to find the value of the decision vector such that the optimal value of each objective is its desirable value and all the constraints are satisfied.
Step 11: Convert the goal programming into a crisp LPP, as given in (15).

Step 12: Using various membership functions, the obtained crisp LPP can be solved by using suitable software packages and an efficient solution of (8) can be obtained.

5 Example

$$
\begin{aligned}
Maximize \quad & Z(x) = [Z_1(x), Z_2(x)] \\
\text{where} \quad & \\
Z_1(x) &= \frac{(3,4,5;2,4,6)x_1 + (2,3,4;1,3,5)x_2}{(2,3,4;1,3,5)x_1 + (1,2,3;0,2,4)x_2 + (3,4,5;2,4,6)} \\
Z_2(x) &= \frac{(1,2,3;0,2,4)x_1 + (2,3,4;1,3,5)}{(3,4,5;2,4,6)x_1 + (6,7,8;5,7,9)x_2 + (2,3,4;1,3,5)}
\end{aligned}
$$

subject to

$$
\begin{aligned}
(1,2,3;0,2,4)x_1 + (4,5,6;3,5,7)x_2 &\leq (8,10,12;6,10,14) \\
(3,5,7;1,5,9)x_1 + (2,3,4;1,3,5)x_2 &\leq (13,15,17;11,15,19) \\
x_1, x_2 &\geq 0
\end{aligned}
\tag{16}
$$

Now, by using component wise optimization, every intuitionistic linear fractional objective can be further reduced to 5 linear fractional objectives and same happens with every intuitionistic fuzzy constraints. Using Charnes and Cooper transformation, every fractional objective can be solved separately and then fuzzy mathematical programming approach can be used and the optimal value of each intuitionistic fuzzy fractional objective can be obtained. Using the transformation $y = t{*}x$, we solve for the decision variables and the optimal value of both the objectives is obtained as $\tilde{Z_1}^I = (0.47015, 0.83590, 1.5361; 0.245432, 0.83590, 3.41656)$ and $\tilde{Z_2}^I = (0.5, 1, 2; 0.2, 1, 5)$ at $(1.863, 0.386835)$ and $(0,0)$ respectively. Now we convert the intuitionistic multi objective fractional linear programming problem to a intuitionistic single objective linear programming problem by using $Maximixe \sum(\tilde{N_i^I}(x) - \tilde{Z_i^{*I}}\tilde{D_i^I}(x))$ subject to the constraints as given in original problem. So, the new IFLPP is :

$$
\begin{aligned}
\text{Maximize} \;\; & (-12.144, -0.50769, 5.5597; -45.0828, -0.50769, 9.354569)x_1 + \\
& (-18.6083, -5.6718, 0.5298; -57.66624, -5.6718, 4)x_2 + \\
& (-13.6805, -3.3436, 1.58955; -44.499, -3.3436, 4.309136) \\
\text{subject to} \;\; & \\
& (1,2,3;0,2,4)x_1 + (4,5,6;3,5,7)x_2 \leq (8,10,12;6,10,14) \\
& (3,5,7;1,5,9)x_1 + (2,3,4;1,3,5)x_2 \leq (13,15,17;11,15,19) \\
& x_1, x_2 \geq 0
\end{aligned}
\tag{17}
$$

Now we again use component wise optimization and convert the IFLPP to a MOLPP with crisp constraints, and find their optimal solutions. With this we also find the optimal solution set and the least desirable values of each objectives. Solving each objectives gives the optimal solution as $U_1 = -13.6085, U_2 = -3.3436, U_3 = 13.32699, U_4 = -44.499$ and $U_5 = 24.05767$. The set S of optimal points is $\{(0,0), (2.11, 0)\}$. Using the set S,we obtain the set of least desirable values for each objectives as

$L_1 = -39.23234, L_2 = -4.41533, L_3 = 1.58955, L_4 = -139.668$ and $L_5 = 4.309136$. A fuzzy goal programming model can now be constructed whose goal is to achieve the most desirable values for each objective but an allowance upto the least desirable values is allowed. Using models 12 - 13, the goal programming is further reduced to a crisp LPP and then solved. The solution of (16) using various membership functions has been summarized in Table 1.

Table 1: Solutions

	Linear Function	**Hyperbolic Function**	**Parabolic Function**
Solution	(1.055282, 0)	(1.052314, 0)	(0.1650805, 0.1905678)
Z_1	(0.34332, 0.58906, 1.0324; 0.18716, 0.58906, 2.07237)	(0.34280, 0.58813, 1.03074; 0.18688, 0.58813, 2.06855)	(0.14062, 0.25265, 0.45095; 0.68628, 0.25265, 0.89757)
Z_2	(0.32936, 0.70772, 1.38715; 0.08824, 0.70772, 2.96445)	(0.32956, 0.70806, 1.38782; 0.08838, 0.70806, 2.96629)	(0.34096, 0.66679, 1.23541; 0.12977, 0.66679, 2.47933)
λ	0.499866	0.495395	0.00753415
Deviation from U_1	(-0.56230, 0.24683, 1.1927; -1.82694, 0.24683, 3.22939)	(-0.56059, 0.24776, 1.19329; -1.82312, 0.24776, 3.22967)	(0.01919, 0.58324, 1.3954; -0.6521, 0.5832, 3.3479)
Deviation from U_2	(-0.88715, 0.29227, 1.67063; -2.7644, 0.292276, 4.9117)	(-0.8878, 0.29193, 1.67043; -2.76629, 0.29193, 4.9116)	(-0.73541, 0.3332, 1.6590; -2.2793, 0.3332, 4.8702)

6 Conclusion

In this work, a method for solving IFMOLFPP has been discussed where all the technological coefficients are represented by TIFNs. In first phase of the problem solving, an optimal solution to each Intuitionistic fuzzy fractional objective is sought and in next phase that solution is used to transform the Multi - objective intuitionistic fuzzy fractional LPP into a corresponding IFLPP; which is further transformed to a fuzzy goal programming problem. In order to convert a goal programming problem to a crisp LPP, various linear/non-linear membership functions are used which represents the aspiration level or satisfaction level of the decision makers. These membership functions provide flexibility to the decision maker so as to choose the function which better fits the problem and provide more satisfaction. Here, we used Linear, hyperbolic and parabolic membership functions and discovered that the satisfaction level of DM follows the order, Linear $>$ Hyperbolic $>$ Parabolic, in the case of given numerical problem. In this work, the intuitionistic fuzzy coefficients have not been defuzzified by using accuracy function and thus no important information is lost during the entire solution finding process. The proposed approach can be further used to optimize planning or production in various other organizations. In future, the proposed approach can be extended for solving IFMOLFPP when coefficients are Trapezoidal intuitionistic fuzzy numbers and for solving non linear IFMOLFPP.

References

1. Abraham Charnes and William W Cooper. Programming with linear fractional functionals. *Naval Research logistics quarterly*, 9(3-4):181–186, 1962.
2. Kanti Swarup. Letter to the editor linear fractional functionals programming. *Operations Research*, 13(6):1029–1036, 1965.
3. Gabriel R Bitran and AG Novaes. Linear programming with a fractional objective function. *Operations Research*, 21(1):22–29, 1973.
4. P Pandian and M Jayalakshmi. On solving linear fractional programming problems. *Modern Applied Science*, 7(6):90, 2013.
5. Mohammad Babul Hasan and Sumi Acharjee. Solving lfp by converting it into a single lp. 2010.
6. Nuran Güzel. A proposal to the solution of multiobjective linear fractional programming problem. In *Abstract and applied analysis*, volume 2013. Hindawi, 2013.
7. N Guzel and Mustafa Sivri. Taylor series solution of multi-objective linear fractional programming problem. *Trakya Univ J Sci*, 6(2):80–87, 2005.
8. M Chakraborty and Sandipan Gupta. Fuzzy mathematical programming for multi objective linear fractional programming problem. *Fuzzy sets and systems*, 125(3):335–342, 2002.
9. D Dutta, RN Tiwari, and JR Rao. Multiple objective linear fractional programming—a fuzzy set theoretic approach. *Fuzzy sets and systems*, 52(1):39–45, 1992.
10. Monga Kalonda Luhandjula. Fuzzy approaches for multiple objective linear fractional optimization. *Fuzzy sets and systems*, 13(1):11–23, 1984.
11. Sujeet Kumar Singh and Shiv Prasad Yadav. Fuzzy programming approach for solving intuitionistic fuzzy linear fractional programming problem. *International Journal of Fuzzy Systems*, 18(2):263–269, 2016.
12. Sujeet Kumar Singh and Shiv Prasad Yadav. Modeling and optimization of multi objective non-linear programming problem in intuitionistic fuzzy environment. *Applied Mathematical Modelling*, 39(16):4617–4629, 2015.
13. Chinnadurai Veeramani and Muthukumar Sumathi. Fuzzy mathematical programming approach for solving fuzzy linear fractional programming problem. *RAIRO-Operations research*, 48(1):109–122, 2014.
14. Sujeet Kumar Singh and Shiv Prasad Yadav. Intuitionistic fuzzy multi-objective linear programming problem with various membership functions. *Annals of operations research*, 269(1-2):693–707, 2018.
15. Krassimir Atanassov. Intuitionistic fuzzy sets. *International Journal Bioautomation*, 20:1, 2016.
16. Werner Dinkelbach. On nonlinear fractional programming. *Management science*, 13(7):492–498, 1967.
17. R Parvathi and C Malathi. Intuitionistic fuzzy linear programming problems. *World applied sciences journal*, 17(12):1802–1807, 2012.